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Other separation axioms in soft bi-topological spaces

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Abstract

The main objective of this article is to introduce soft generalized separation axioms in soft bi topological spaces relative to crisp points and soft points. Further we will address the behavior of soft semi T_3 and soft normal semi T_4 spaces at different angles with respect to ordinary points as well as with respect to soft points. Hereditary properties are also discussed.

Keywords

Soft sets, soft points,soft topology,soft bi topological space,soft semi open set, soft semi T_i spaces(i=1,2,3,4), soft semi regular and soft semi normal spaces.

AMS Subject Classification

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Contents

1. Introduction

In real life condition the problems in economics,engineering, social sciences, medicalscience etc. We cannot beautifully use the traditional classical methods because of different types of uncertainties presented in these problems. To overcome these difficulties,some kinds of theories were put forwarded like theory of fuzzy set, intuitionistic fuzzy set,rough set and bi polar fuzzy sets, in which we can safely use a mathematical techniques for businessing with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999,Russian scientist Molodtsov [5], initiated the above complications. Molodtsov [5] and B.Ahmad and A Kharal [6], successfully applied the soft set theory in different directions, such as smoothness of functions, game theory,operation research, Riemann integration, Perron integration, probability, theory of measurement and so on.After presentation of the operations of soft sets [7], the properties and applications of the soft set theory have been studied increasingly [7–9]. Xiao et al [10] and D. Pei and D. Maio [11] discusse the linkage between soft sets and information systems. They showed that soft sets are class of special information system. In the recent year, many interesting applications of soft sets theory have been extended by embedding the ideas of fuzzy sets [12–19, 21–23] industrialized soft set theory , the operations of the sets are redefined and in indecision making method was constructed by using their new operations [24]. Recently, in 2011, Shabir and Naz

[24] launched the study of soft Topological spaces, they beautiful defined soft Topology as a collection of τ of soft sets over X. They also defined basic conception of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several behaviors. A. Kandil et al. [26] scrutinized some belongings of these soft separation axioms. Kandil et al [27] introduced some soft operations such as semi open soft,pre-open soft,α-open soft and β-open soft and examined their properties in detail. Kandil et al [28] introduced the concept of soft semi - separation axioms, in particular soft semi-regular spaces. The concept of soft ideal was discussed for the first time by Kandil et al. [29].They also introduced the concept of soft local function .there concepts are discussed with a view to find new soft topological from the original one,called soft topological spaces with soft ideal(X,τ,E,I) Application to different zone were further discussed by Kandil et al. [29–31, 33, 34],S.A El-Sheikh and A.M Abd-e-Latif [35] and I.Zorlutana et al.[36]. The notion of super soft topological spaces was initiated for the first time by L.A.Zadeh [37]. They also introduced new different types of subsets of supra soft topological spaces and study the dealings between them in great detail.Bin Chin [42] introduced the concept of soft semi open set and studied their related properties, Hussain [43] discussed soft separation axioms. Mahanta % [40] introduced semi open and semi closed soft sets. [41] A.Lancy and Arockiarani, On Soft β-Separation Axioms, Arockiarani. I.Arokialancy in [44] generalized soft g β closed and soft gs β closed sets in soft topology are exposed.In this present paper the concept of soft b T_0, soft b T_1, soft b T_2, and soft b T_3 spaces in Soft topological spaces is introduced with respect to soft points. space the concept of soft semi T_0, semi $T-1$,semi T_2, soft semi $T-3$ and soft semi T_4 spaces are introduced in soft bi topological space with respect to ordinary as well as soft points. Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set,soft b-open set,soft semi-open set,softα-open set and soft β-open set. They also worked over the hereditary properties of different soft topological structures in soft topology.In this present work hand is tried and work is encouraged over the gap that exists in soft bi-topology.Related to soft semiT_3 and soft semi T_4 spaces, some proposition in soft by topological spaces are discussed with respect to ordinary points as well as with respect to soft points.When we talk about the distances between the points in soft topology then the concept of soft separation axioms will automatically come in play. That is why these structures are catching our attentions.We hope that these results will be valuable for the future study on soft bi topological spaces to accomplish general framework for the practical applications and to solve the most intricate problems containing scruple in economics, engineering, medical,environment and in general mechanic system of various kinds.

2. Preliminaries

The following Definitions which are pre-requisites for present study

Definition 2.1. *[1, 2] Let X be an initial universe of discourse and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty sub-set of E. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(X)$ In other words, a set over X is a parameterized family of sub set of universe of discourse X.For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F,A) and if $e \notin A$ then $F(e) = \phi$ that is $F_A = \{F(e) : e \in A \subseteq E, F : A \to P(X)\}$ the family of all these soft sets over X denoted by $SS(X)_A$.*

Definition 2.2. *[3] Let $F_A, G_B \in SS(X)_E$ then F_A is a soft subset of G_B denoted by $F_A \tilde{\subseteq} G_B$, if*
1. $A \subseteq B$
2. $F(e) \subseteq G(e), \forall\ e \in A$
In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super set of F_A,$G_B \tilde{\supseteq} F_A$.

Definition 2.3. *[4] Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4. *[5] The complement of soft subset (F,A) denoted by $(F,A)^c$ is defined by $(F,A)^c = (F^c,A)$. $F^c \to P(X)$ is a function given by $F^c(e) = U - F(e)\ \forall\ e \in A$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.*

Definition 2.5. *[6] The difference between two soft subset (F,E) and (G,E) over common universe of discourse X denoted by $(F,E) \setminus (G,E)$ is the soft set (H,E) which is defined a $H(e) = (F,E) \setminus (G,E)\ \forall\ e \in E$*

Definition 2.6. *[7] Let (G,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ and read as x belong to the soft set (F,E) whenever $x \in F(e)\ \forall\ e \in E$. The soft set (F,E) over X such that $F(e) = \{x\}\ \forall\ e \in E$ is called singleton soft point and denoted by x_E or (x,E).*

Definition 2.7. *[8] A soft set (F,A) over X is said to be null soft set denoted by $\overline{\phi}$ or ϕ_A if $\forall\ e \in A$, $F(e) = \overline{\phi}$.*

Definition 2.8. *[9] A soft set (F,A) over X is said to be an absolute soft denoted by $\overline{A}$ or X_A if $\forall\ e \in A$, $F(e) = X$.*
Clearly, we have. $X_A^C = \phi_A$ and $\phi_A^C = X_A$

Definition 2.9. *[10] Let (G,E) be a soft set over X and $e_G \in X_A$, we say that $e_G \in (F,E)$ and read as e_G belong to the* *soft set(F,E) whenever $e_G \in F(e)\ \forall\ e \in E$. the soft set (F,E) over X such that $F(e) = \{e_G\}, \forall e \in E$ is called singleton soft point and denoted by e_G or (e_G,E).*

Definition 2.10. *[11] The soft set $(F,A) \in SSX_A$ is called a soft point in X_A, denoted by e_F,if for the element $e \in A, F(e) \neq \phi$ and $F(e') = \overline{\phi}$ if for all $e' \in A - \{e\}$*

Definition 2.11. *[12] The soft point e_F is said to be in the soft set (G,A) , denoted by $e_F \in (G,E)$ if for the element $e \in A, F(e) \subseteq G(e)$.*

Definition 2.12. *[13] Two soft sets $(G,A),(H,A)$ in X_A are said to be soft disjoint, written $(G,A) \cap (H,A) = \phi_A$ If $G(e) \cap H(e) = \overline{\phi}\ \forall\ e \in A$.*

Definition 2.13. *[14] The soft point $e_G, e_H \in X_A$ are disjoint, written $e_G \neq e_H$, if there corresponding soft sets (G,A) and (H,A) are disjoint.*

Definition 2.14. *[15] The union of two soft sets $(F,A)$and(G,B) over the common universe of discourse X is the soft set (H,C), whe. $A \cup B\ \forall\ e \in C$*

$$H(e) = \begin{cases} F(e) & if\ \ e \in A - B \\ G(e) & if\ \ e \in B - A \\ F(e) \cup G(e) & if\ \ e \in (A \cap B) \end{cases}$$

Written as $(F,A) \cup (G,B) = (H,C)$

Definition 2.15. *[16] The intersection (H,C) of two soft sets (F,A) and (G,B) over common universe X, denoted $(F,A)\overline{\cap}(G,B)$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)\ \forall\ e \in E$.*

Definition 2.16. *[17] Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F,E) over Y denoted by (Y_F,E), is defined as follows $Y_F(\alpha) = Y \cap F(\alpha), \forall$ $\alpha \in E$ in other words*
$(Y_F,E) = Y \cap (F,E)$.

Definition 2.17. *[18] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if*
1. ϕ, X belong to τ
2. The union of any number of soft sets inτ belongs to τ
3. The intersection of any two soft sets in τ belongs to τ
The triplet (X,F,E) is called a soft topological space.

Definition 2.18. *[19] Let (X,τ,E) be a soft topological space over X then the member of τ are said to be soft open sets in X.*

Definition 2.19. *[20] Let (X,τ,E) be a soft topological space over X. A soft set (F,A) over X is said to be a soft closed set in X,if its relative complement $(F,E)^C$ belong to τ.*

Definition 2.20. *[21] A soft set (B,E) in a soft topological space (X,τ,E) will be termed soft semi open (written S.S.O) if and only if there exists a soft open set (O,E) such that $(O,E)\overline{\subseteq}(A,E) \subseteq Cl(O,E)$.*

Definition 2.21. *[22] A soft set (A,E) in a soft topological space (X,τ,E) will be termed soft semi closed (written S.S.C) if its relative complement is soft semi-open, e.g., exists a soft closed set (F,E) such that $int(F,E)\overline{\subseteq}(B,E) \subseteq (F,E)$.*

Proposition 2.22. *[1, 2] Let (X,τ,E) be a soft topological space over X. If (X,τ,E) is soft semi T_3-space, then for all $x \in X$,$x_E = (x,E)$ is semi-closed soft set.*

3. Separation axioms of soft topological spaces with respect to crisp and soft points

Definition 3.1. *[23] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$ if there exist at least one soft open set (F_1,A) OR (F_2,A) such that $x \in (F_1,A)$, $y \notin (F_1,A)$ or $y \in (F_2,A)$,$x \notin (F_2,A)$ then (X,τ,A) is called a soft T_0 space.*

Definition 3.2. *[24] Let (X,τ,A) be a soft topological space over X and $x,y \in X$ such that $x \neq y$ if there exist soft open set (F_1,A) OR (F_2,A) such that $x \in (F_1,A)$, $y \notin (F_1,A)$ and Then $y \in (F_2,A)$, $x \notin (F_2,A)$ Then (X,τ,A) is called a soft T_1 space.*

Definition 3.3. *[25] Let (X,τ,A) be a soft topological space over X and $x,y \in X$ such that $x \neq y$ if there exist soft open set (F_1,A) OR (F_2,A) such that $x \in (F_1,A)$, $y \notin (F_1,A)$ and $F_1 \cap F_2 = \overline{\phi}$ Then (X,τ,A) is called soft T_2 spaces.*

Definition 3.4. *[26] Let (X,τ,A) be a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search it least one soft open set (F_1,A) or (F_2,A) such that $e_G \in (F_1,A)$ $e_H \notin (F_1,A)$ or $e_H \in (F_2,A)$, $e_G \notin (F_2,A)$ Then (X,τ,A) is called a soft T_0 space.*

Definition 3.5. *[27] Let (X,τ,A) be a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft sets (F_1,A) and (F_2,A) such that $e_G \in (F_1,A)$ $e_H \notin (F_1,A)$ and $e_H \in (F_2,A)$, $e_G \notin (F_2,A)$ Then (X,τ,A) is called a soft T_1 space.*

Definition 3.6. *[28] Let (X,τ,A) be a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open set (F_1,A) and (F_2,A) such that $e_G \in (F_1,A)$ and $e_H \in (F_2,A)$ and $(F_1,A)\cap(F_2,A) = \phi_A$ Then (X,τ,A) is called a soft T_2 space.*

Definition 3.7. *[29] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$. If we can find soft semi open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ Then (X,τ,A) is called soft semi T_0 space.*

Definition 3.8. *[30] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$. If we can find two soft semi open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$ Then (X,τ,A) is called soft semi T_1 space.*

Definition 3.9. *[31] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$. If we can find two soft semi open soft such that $x \in (F,E)$ and $y \in (F,E)$ more over $(F,E)\cap(G,E) = \phi$. Then (X,τ,A) is called soft semi T_2 space.*

Definition 3.10. *[32] Let (X,τ,E) be a soft topological space (G,E) and semi closed soft set in X and $x \in X$ such that $x \notin (G,E)$. If there occurs soft semi open sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$,$(G,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$. Then (X,τ,E) is called soft semi regular spaces.A soft b regular T_1 space is called soft semi T_3 space.*

Definition 3.11. *[33] Let (X,τ,E) be a soft topological space and (F,E),(G,E) be soft b closed sets in X such that $(F,E)\cap(G,E) = \phi$. If there occurs b open soft sets (F_1,E) and (F_2,E) such that ,$(F,E) \subseteq (F_1,E)$,$(G,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$ then (X,τ,E) is called a soft b normal space. A soft semi normal T_1 space is called soft semi T_4 space.*

4. Soft semi separation axioms of bi-topological spaces

Let X is an initial set and E be the non-empty set of parameter. In [1] soft bi topological space over the soft set X is introduced. Soft separation axioms in soft bi topological spaces were introduced by Basavaraj and Ittangi [1]. In this section we introduced the concept of soft semi T_3 and semi T_4 spaces in soft bi topological spaces with respect to ordinary as well as soft points and some of its basic properties are studied and applied to different results in this section.

Definition 4.1. *[34] Let (X,τ_1,E) and (X,τ_2,E) be two different soft topologies on X. Then (X,τ_1,τ_2,E) is called a soft bi topological space. The two soft topologies (X,τ_1,E) and(X,τ_2,E) are independently satisfy the axioms of soft topology. The member of τ_1 are called τ_1 soft open set. And complement of τ_1. Soft open set is called τ_1 soft closed set. Similarly, the member of τ_2 are called τ_2 soft open set and the complement of τ_2 soft open sets are called τ_2 soft closed set.*

Definition 4.2. *[35] Let (X,τ_1,τ_2,E) be a soft topological space over X and Y be a non-empty subset of X. Then $\tau_{1Y} = \{(F_E,E) : (F,E) \in \tau_1\}$ and $\tau_{2Y} = \{(G_E,E) : (G,E) \in \tau_2\}$ are said to be relative topologies on Y. Then $(Y,\tau_{1Y},\tau_{2Y},E)$ is called relative soft bi-topological space of (X,τ_1,τ_2,E).*

5. Soft semi separation axioms of soft bi-topological spaces with respect to ordinary points

In this section we introduced soft b separation axioms in soft bi topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 5.1. *[36] In a soft bi topological space (X,τ_1,τ_2,E)*
1) τ_1 is said to be semi T_0 space with respect to τ_2 if for each pair of distinct points $x,y \in X$ there exists τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ similarly, τ_2 is said to be soft semi T_0 space with respect to τ_1 if for each pair of distinct points $x,y \in X$ there exists τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$. Soft bi topological spaces (X,τ_1,τ_2,E) is said to be pair wise soft semi T_0 space if τ_1 is soft semi T_0 space with respect to τ_2 and τ_2 is soft semi T_0 space with respect to τ_1.
2) τ_1 is said to be soft semi T_1 space with respect to τ_2 if for each pair of distinct points $x,y \in X$ there exists a τ_1 soft semi open set (F,E) and τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$. Similarly, τ_2 is said to be soft semi T_1 space with respect to τ_1 if for each pair of distinct points $x,y \in X$ there exist a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$. Soft bi topological spaces (X,τ_1,τ_2,E) is said to be pair wise soft semi T_1 space if τ_1 is soft semi T_1 space with respect to τ_2 and τ_2 is soft semi T_1 space with respect to τ_1.
3) τ_1 is said to be soft semi T_2 space with respect to τ_2 if for each pair of distinct points $x,y \in X$ there exists a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \in (G,E)$, $(F,E)\cap(G,E) = \phi$. Similarly, τ_2 is said to be soft semi T_2 space with respect to τ_1 if for each pair of distinct points $x,y \in X$ there exists a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $x \in (F,E)$ and $y \in (G,E)$ and $(F,E)\cap(G,E) = \phi$. The soft bi topological space (X,τ_1,τ_2,E) is said to be pair wise soft semi T_2 space if τ_1 is a soft semi T_2 space with respect to τ_2 and τ_2 is soft semi T_2 space with respect to τ_1.

Definition 5.2. *[37] In a soft bi topological space (X,τ_1,τ_2,E)*
1) τ_1 is said to be soft semi T_3 space with respect to τ_2 if τ_1 is

soft semi T_1 space with respect to τ_2 and for each pair of distinct points $x, y \in X$ there exists a τ_1 semi closed soft set (G,E) such that $x \notin (G,E)$, τ_1 soft semi open set (F_1,E) and τ_2 soft semi open set (F_2,E) such that $x \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Similarly, τ_2 is said to be soft semi T_3 space with respect to τ_1 if τ_2 is soft semi T_1 space with respect to τ_1 and for each pair of distinct points $x, y \in X$ there exist a τ_2 soft semi closed set (G,E) such that $x \notin (G,E)$, τ_2 soft semi open set (F_1,E) such that $x \in (F_1,E), (F_2,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. (X, τ_1, τ_2, E) is said to be pair wise soft semi T_3 space if τ_1 is soft semi T_3 space with respect to τ_2 and τ_2 is a soft semi T_3 space with respect to τ_1.
2) τ_1 is said to be soft semi T_4 space with respect to τ_2 if τ_1 is soft semi T_1 space with respect to τ_2, there exists $\alpha\tau_1$ soft semi closed set (F_1,E) and τ_2 soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$ also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_1 semi open set, (G_1,E) is a soft τ_2 semi open set such that $(F_1,E) \subseteq (F_3,E)$, $(F_2,E) \subseteq (G_1,E)$. Similarly, τ_2 is said to be soft semi T_4 space with respect to τ_1 if τ_2 is soft semi T_1 space with respect to τ_1, there exist τ_2 soft semi closed set (F_1,E) and τ_1 soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$ also there exist (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set, (G_1,E) is soft τ_1 semi soft set such that $(F_1,E) \subseteq (F_3,E)$,$(F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Thus, (X, τ_1, τ_2, E) is said to be pair wise soft semi T_4 space if τ_1 is soft semi T_4 space with respect to τ_2 and τ_2 is soft semi T_4 space with respect to τ_1.

Proposition 5.3. *[3] Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and $(F,E) \in SS(X)$ then*
1. If (F,E) is semi open soft set i Y and $Y \in \tau$, then $(F,E) \in \tau$.
2. (F,E) is semi open soft set in Y if and only $(F,E) = Y \cap (G,E)$ for some $(G,E) \in \tau$.
3. (F,E) is semi closed soft set in Y if and only if and only if $(F,E) = Y \cap (H,E)$ for some (H,E) is τ soft semi closed set.

Proof. [2] 1) Let (F,E) be a soft semi open set in Y,then there does exists a soft semi open set (G,E) in X such that $(F,Y) = Y \cap (G,E)$. Now, if $Y \in \tau$ then $Y \cap (G,E) \in \tau$ by third condition of the definition of a soft topological space and hence $(F,E) \in \tau$.
2) Fallow from the definition of a soft subspace.
3) If (F,E) is soft semi closed in Y then we have $(F,E) = Y \backslash (G,E)$, for some $(G,E) \in \tau_Y$.Now,$(G,E) = Y \backslash cap(H,E)$ for semi open set $(H,E) \in \tau$.for any $\alpha \in E$. $F(\alpha) = Y(\alpha) \backslash G(\alpha) = Y \backslash G(\alpha) = Y \backslash (Y(\alpha) \cap H(\alpha)) = Y \backslash (Y \cap H(\alpha)) = Y \backslash H(\alpha) = Y \cap (X \backslash H(\alpha)) = Y \cap (H(\alpha))^c = Y(\alpha) \cap (H(\alpha))^c$. Thus $(F,E) = Y \cap (H,E)'$ is soft semi closed in X as $(H,E) \in \tau$. Conversely, suppose that $(F,E) = Y \cap (G,E)$ for some soft semi closed set (G,E) in X. This qualifies us to say that $(G,E)' \in \tau$.Now, if $(G,E) = (X,E) \backslash (H,E)$ where (H,E) is soft semi open set in τ then for any $\alpha \in E$,$F(\alpha) = Y(\alpha) \cap G(\alpha) = Y \cap G(\alpha) = Y \cap (X(\alpha) \backslash H(\alpha) = Y \cap (X(\alpha) \backslash H(\alpha)) = Y \cap (X \backslash H(\alpha)) Y \backslash H(\alpha) = Y \backslash (Y \cap H(\alpha)) = Y(\alpha) \backslash (Y(\alpha) \cap H(\alpha))$. Thus $(F,E) = Y \backslash (Y \cap \backslash (H,E))$. Since $(H,E) \in \tau$, So $(Y \cap (H,E) \in \tau_Y$ and hence (F,E) is soft semi closed in Y.

□

Proposition 5.4. *[4] Let (X, τ_1, τ_2, E) be a soft bi topological space over X. Then,if (X, τ_1, E) and (X, τ_2, E) are soft semi T_3 space then (X, τ_1, τ_2, E) is pair wise soft semi T_2 space.*

Proof. [5] Suppose (X, τ_1, E) is a soft semi T_3 space with respect to (X, τ_2, E) then according to definition for $x, y \in X, x \neq y$,by using Theorem 1, (y,E) is soft semi closed set in τ_2 and $x \notin (y,E)$ there exist a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $x \in (F,E)$,$y \in (y,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence τ_1 is soft semi T_2 space with respect to τ_2. Similarly, if (X, τ_2, E) is a soft semi T_3 space with respect to (X, τ_1, E) then according to definition for $x, y \in X, x \neq y$,by using Theorem 1, (x,E) is semi closed soft set in τ_1 and $y \notin (x,E)$ there exists a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $y \in (F,E)$, $x \in (x,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence τ_2 is soft semi T_2 space.this implies that (X, τ_1, τ_2, E) is a pair wise soft semi T_2 space.

□

Proposition 5.5. *[5] Let (X, τ_1, τ_2, E) be a soft bi topological space over X. if (X, τ_1, E) and (X, τ_2, E) are soft semi T_3 space then (X, τ_1, τ_2, E) is a pair wise soft semi T_3 space.*

Proof. [2] Let (X, τ_1, E) is a soft semi T_3 space with respect to (X, τ_2, E) then according to definition for $x, y \in X, x \neq y$ there exists a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ and for each point $x \in X$ and each τ_1 semi closed soft set (G_1,E) such that $x \notin (G_1,E)$ there exists a τ_1 soft semi open set $F_1,E)$ and τ_2 soft semi open set $F_2,E)$ such that $x \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Similarly, (X, τ_2, E) is a soft semi T_3 space with respect to (X, τ_1, E). So according to definition for $x, y \in X, x \neq y$,there exists τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ and for each $x \in X$ and each τ_2 semi closed soft set (G_1,E) such that $x \notin (G_1,E)$, there exists a τ_2 soft semi open set (F_1,E) and a τ_1 semi open set (F_2,E) such that $x \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence (X, τ_1, τ_2, E) is pair wise soft semi T_3 space.

□

Proposition 5.6. *[6] If (X, τ_1, τ_2, E) be a soft bi topological space over X. if (X, τ_1, E) and (X, τ_2, E) are soft semi T_4 space then (x, τ_1, τ_2, E) is pair wise soft semi T_4 space.*

Proof. Suppose (X, τ_1, E) is soft semi T_4 space with respect to (X, τ_2, E). So according to definition for $x, y \in X, x \neq y$ there exist a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and

$x \notin (G,E)$ each τ_1 soft semi closed set (F_1,E) and a τ_2 soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$. There exist (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set (G_1,E) is soft τ_1 semi open set $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Similarly, τ_2 is soft semi T_4 space with respect to τ_1 so according to definition for $x,y \in X, x \neq y$ there exist a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ and for each τ_2 soft semi closed set (F_1,E) and τ_1 soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set (G_1,E) is soft τ_1 semi open set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$ hence (X, τ_1, τ_2, E) is pair wise soft semi T_4 space.

□

Proposition 5.7. *[7] Let (X, τ_1, τ_2, E) be a soft bi topological space over X and Y be a non-empty subset of X.if $(X, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft semi T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft semi T_3 space.*

Proof. First we prove that $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft semi T_1 space. Let $x,y \in X, x \neq y$ if (x, τ_1, τ_2, E) is pair wise space then this implies that (X, τ_1, τ_2, E) is pair wise soft τ_1 space. So there exists τ_1 soft semi open (F,E) and τ_2 soft semi open set (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ now $x \in Y$ and $x \notin (G,E)$. Hence $x \in Y \cap (F,E) = (Y_F,E)$ then $y \notin Y \cap (\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$.
Therefore,$y \notin Y \cap (F,E) = (Y_F,E)$. Now $y \in Y$ and $y \in (G,E)$ hence $y \in Y \cap (G,E) = (G_Y,E)$ where $(G,E) \in \tau_2$. Consider $x \notin (G,E)$ this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$.Therefore $x \notin Y \cap (G,E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft semi T_1 space.
Now we prove that (X, τ_1, τ_2, E) is pair wise soft semi T_3 space then $(X, \tau_1, \tau_2,)$ is pair wise soft semi regular space.
Let $y \in Y$ and (G,E) be a soft semi closed set in Y such that $y \notin (G,E)$ where $(G,E) \in \tau_1$ then $(G,E) = (Y,E) \cap (F,E)$ for some soft semi closed set in τ_1. Hence $y \notin (Y,E) \cap (F,E)$ but $y \in (Y,E)$,so $y \notin (F,E)$ since (X, τ_1, τ_2, E) is soft semi T_3 space.
(X, τ_1, τ_2, E) is soft semi regular space to there exists τ_1 soft semi open set (F_1,E) and τ_2 soft semi open set $F_2,E)$ such that

$$y \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E)(F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_2,E)$ then $(G_1,E), (G_2,E)$ are soft semi open set in Y such that

$$y \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$
$$(G_1,E) \cap (G_2,E) = \phi$$

Therefore τ_{1Y} is soft semi regular space with respect to τ_{2Y}.similarly, Let $y \in Y$ and (G,E) be a soft semi closed subset in Y. such that $\notin (G,E)$, where $(G,E) \in \tau_2$ then $(G,E) = (Y,E) \cap (F,E)$ where (F,E) is some soft semi closed set in τ_2.$y \notin (y,E) \cap (F,E)$ But $y \in (Y,E)$ so $y \notin (F,E)$ since (X, τ_1, τ_2, E) is soft semi regular space so there exists τ_2 soft semi open set (F_1,E) and τ_1 soft semi open set (F_2,E) such that

$$y \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E) \cap (F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_1,E)$

$(G_1,E) = (Y,E) \cap (F_1,E)$ Then (G_1,E) and (G_2,E) are soft semi open set in Y such that

$$y \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$

Therefore τ_{2Y} is soft semi regular space with respect to τ_{1Y}.
$\Rightarrow (X, \tau_1, \tau_2, E)$ is pair wise soft semi T_3 space.

□

Proposition 5.8. *[8] Let (X, τ_1, τ_2, E) be a soft bi topological over X and Y be a soft b closed sub space of X. if (X, τ_1, τ_2, E) is pair wise soft b T_4 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_4 space.*

Proof. Since (X, τ_1, τ_2, E) is pair wise soft b T_4 space so this implies that (X, τ_1, τ_2, E) is pair wise soft b T_1 space as proved above.
We prove (X, τ_1, τ_2, E) is pair wise soft semi normal space.
Let $(G_1,E), (G_2,E)$ be soft semi closed sets in Y such that

$$(G_1,E) \cap (G_2,E) = \phi$$
$$(G_1,E) = (Y,E) \cap (F_1,E)$$

And $(G_2,E) = (Y,E) \cap (F_2,E)$
For some soft semi closed sets such that (F_1,E) is soft semi closed set in τ_1 (F_2,E) is soft semi closed set in τ_2.

$$\text{And } (F_1,E) \cap (F_2,E) = \phi$$

From proposition 2. Since, Y is soft semi closed sub set of X then $(G_1,E), (G_2,E)$ are soft semi closed set in X such that

$$(G_1,E) \cap (G_2,E) = \phi$$

Since (X, τ_1, τ_2, E) is pair wise soft semi normal space. So there exists soft semi open sets (H_1,E) and (H_2,E) such that (H_1,E) is soft semi open set in τ_1 and (H_2,E) is soft semi open set in τ_2 such that

$$(G_1,E) \subseteq (H_1,E)$$
$$(G_2,E) \subseteq (H_2,E)$$
$$(H_1,E) \cap (H_2,E) = \phi$$
$$\text{Since } (G_1,E), (G_2,E) \subseteq (Y,E)$$
$$\text{Then } (G_1,E) \subseteq (Y,E) \cap (H_1,E)$$
$$(G_2,E) \subseteq (Y,E) \cap (H_2,E)$$
$$\text{And } [(Y,E) \cap (H_1,E)] \cap [(Y,E) \cap (H_2,E)] = \phi$$

Where $(Y,E)\cap(H_1,E)$ and $(Y,E)\cap(H_2,E)$ are soft semi open sets in Y therefore τ_{1Y} is soft semi normal space with respect to τ_{2Y}. Similarly, let $(G_1,E),(G_2,E)$ be soft semi closed subset in Y such that

$$(G_1,E)\cap(G_2,E)=\phi$$
$$\text{Then } (G_1,E)=(Y,E)\cap(F_1,E)$$
$$\text{And } (G_2,E)=(Y,E)\cap(F_2,E)$$

For some soft semi closed sets such that (F_1,E) is soft semi closed set in τ_2 (F_2,E) soft semi closed set in τ_1 and

$$(F_1,E)(F_2,E)=\phi$$

From **proposition 2.** Since, Y is soft semi closed subset in X then $(G_1,E),(G_2,E)$ are soft semi closed sets in X such that

$$(G_1,E)\cap(G_2,E)=\phi$$

Since (X,τ_1,τ_2,E) is pair wise soft semi normal space so there exists soft semi open sets (H_1,E) and (H_2,E)
Such that (H_1,E) is soft semi open set in τ_2 and (H_2,E) is soft semi open set in τ_1 such that

$$(G_1,E)\subseteq(H_1,E)$$
$$(G_2,E)\subseteq(H_2,E)$$
$$(H_1,E)\cap(H_2,E)=\phi$$

Since

$$(G_1,E),(G_2,E)\subseteq(Y,E)$$
$$\text{Then } (G_1,E)\subseteq(Y,E)\cap(H_1,E)$$
$$(G_2,E)\subseteq(Y,E)\cap(H_2,E)$$
$$\text{And } [(Y,E)\cap(H_1,E)]\cap[(Y,E)\cap(H_2,E)]=\phi$$

Where $(Y,E)\cap(H_1,E)$ and $(Y,E)\cap(H_2,E)$ are soft semi open sets in Y therefore τ_{2Y} is soft semi normal space with respect to τ_{1Y}.
$\Rightarrow (Y,\tau_{1Y},\tau_{2Y},E)$ is pair wise soft semi T_4 space.

□

6. Soft semi separation axioms of soft bi-topological spaces with respect to soft points

In this section, we introduce soft semi separation axioms in soft topology and in soft bi topology with respect to soft points. With the application of these soft semi separation axioms different result are discussed.

Definition 6.1. *[38] Let (X,τ,A) be a soft topological space over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen at least one soft semi open set (F_1,A) OR (F_2,A) such that $e_G\in(F_1,A)$,$e_H\notin(F_1,A)$ or $e_H\in(F_1,A)$,$e_G\notin(F_2,A)$ then (X,τ,A) is called a soft semi T_0 space.*

Definition 6.2. *[39] Let (X,τ,A) be a soft topological spaces over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen soft semi open sets (F_1,A) and (F_2,A) such that $e_G\in(F_1,A)$,$e_H\notin(F_1,A)$ and $e_H\in(F_2,A)$,$e_G\notin(F_2,A)$ then (X,τ,A) is called soft semi T_1 space.*

Definition 6.3. *[40] Let (X,τ,A) be a soft topological space over X and $e_G,e_H\in X_A$ such that $e_G\neq e_H$ if there can happen soft semi open sets (F_1,A) and (F_2,A) such that $e_G\in(F_1,A)$,and $e_H\in(F_2,A)$ and $(F_1,A)\cap(F_2,A)=\phi_A$. Then (X,τ,A) is called soft semi T_2 space.*

Definition 6.4. *[41] Let (X,τ,E) be a soft topological space (G,E) be semi closed soft set in X and $e_G\in X_A$ such that $e_G\notin(G,E)$. If there occurs soft semi open sets (F_1,E) and (F_2,E) such that $e_G\in(F_1,E),(G,E)\subseteq(F_2,E)$ and $(F_1,E)\cap(F_2,E)=\phi$. Then (X,τ,E) is called soft semi regular spaces. A soft semi regular T_1 Space is called soft semi T_3 space.*

Definition 6.5. *[42] In a soft bi topological spaces (X,τ_1,τ_2,E)*
1) τ_1 said to be soft semi T_0 space with respect to τ_2 if for each pair of distinct points $e_G,e_H\in X_A$ there happens τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_H\notin(G,E)$, Similarly, τ_2 is said to be soft semi T_0 space with respect to τ_1 if for each pair of distinct points $e_G,e_H\in X_A$ there happens τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_H\notin(F,E)$ or $e_H\in(G,E)$ and $e_G\notin(G,E)$. Soft bi topological spaces (X,τ_1,τ_2,E) is said to be pair wise soft semi T_0 space if τ_1 soft semi τ_0 space with respect to τ_2 and τ_2 is soft semi T_0 spaces with respect to τ_1.
2) τ_1 is said to be soft semi T_1 space with respect to τ_2 if for each pair of distinct points $e_G,e_H\in X_A$ there happens a τ_1 soft semi open set (F,E) and τ_2 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_H\notin(G,E)$ and $e_H\in(G,E)$ and $e_G\in(G,E)$,Similarly, τ_1 is said to be soft semi T_1 space with respect to τ_1 if for each pair of distinct points $e_G,e_H\in X_A$ there exist a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_H\notin(G,E)$ and $e_H\in(G,E)$ and $e_G\notin(G,E)$. Soft bi topological spaces (X,τ_1,τ_2,E) is said to be pair wise soft semi T_1 space if τ_1 is soft semi T_1 space with respect to τ_2 and τ_2 is soft semi T_1 spaces with respect to τ_1.
3) τ_1 is said to be soft semi T_2 space with respect to τ_2, if for each pair of distinct points $e_G,e_H\in X_A$ there happens a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_H\notin(G,E)$ and $e_H\in(G,E)$ and $e_G\notin(G,E)$ and $(F,E)\cap(G,E)=\phi$. Similarly, τ_2 is said to be soft semi T_2 space with respect to τ_1 if for each pair of distinct points $e_G,e_G\in X_A$ there happens a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_G\in(F,E)$ and $e_G\in(G,E)$ and $(F,E)\cap(G,E)=\phi$.The soft bi topological space (X,τ_1,τ_2,E) is said to be pair wise soft semi T_2 space if τ_1 is soft semi T_2 space with respect to τ_2 and τ_2 is a soft semi T_2 space with respect to τ_1.

Definition 6.6. *[43] In a soft bi topological space (X,τ_1,τ_2,E). 1) τ_1 is said to be soft semi T_3 space with respect τ_2 if τ_1 is soft semi T_1 space with respect to τ_2 and for each pair of distinct points $e_G, e_H \in X_A$, there exist a τ_1 semi closed soft set (G,E) such that $e_G \notin (G,E)$, τ_1 soft semi open set (F_1,E) and τ_2 soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$. Similarly, τ_2 is said to be soft semi T_3 space with respect to τ_1 if τ_2 is soft semi T_1 space with respect to τ_1 and for each pair of distinct points $e_G, e_H \in X_A$ there exists a τ_2 soft semi closed set (G,E) such that $e_G \notin (G,E)$, τ_2 soft semi open set (F_1,E) and τ_1 soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$.*
(X,τ_1,τ_2,E) is said to be pair wise soft semi T_3 space if τ_1 is soft semi T_3 space with respect to τ_2 and τ_2 is soft semi T_3 space with respect to τ_1.
2) τ_1 is said to be soft semi T_4 space with respect to τ_2 if τ_1 is soft semi T_1 space with respect to τ_2, there exists a τ_1 soft semi closed set (F_1,E) and τ_2 semi closed set (F_2,E) such that $(F_1,E)\cap(F_2,E) = \phi$ also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_1 semi open set, (G_1,E) is soft τ_2 semi open set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$. Similarly, τ_2 is said to be soft semi T_4 space with respect to τ_1 if τ_2 is soft semi T_1 space with respect to τ_1, there exists τ_2 soft semi closed set (F_1,E) and τ_1 soft semi closed set (F_2,E) such that $(F_1,E)\cap(F_2,E) = \overline{\phi}$. Also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set, (G_1,E) is soft τ_1 semi soft set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E)\cap(G_1,E) = \overline{\phi}$. Thus, (X,τ_1,τ_2,E) is said to be pair wise soft semi T_4 space if τ_1 is soft semi T_4 space with respect to τ_2 and τ_2 is soft semi T_4 space with respect to τ_1.

Proposition 6.7. *[9] Let (X,τ,E) be a soft topological space over X. If (X,τ,E) is soft semi T_3 space, then for all $e_G \in X_E, e_G = (e_G,E)$ is semi-closed soft set.*

Proof. We want to prove e_G is semi-closed set, which is sufficient to prove that e_G^c is semi-open soft set for all $e_H \in \{e_G\}^c$. Since (X,τ,E) is soft semi T_3-space, then there exist soft semi sets $(F,E)_{e_H}$ and (G,E) such that $e_{H_E} \subseteq (F,E)_{e_H}$ and $e_{G_H} \cap (F,E)_{e_H} = \overline{\phi}$ and $e_{G_E} \subseteq (G,E)$ and $e_{H_E} \cap (G,E) = \overline{\phi}$. It follows that $\cup_{e_H \in (e_G)^c (F,E)_{e_H} \subseteq e_{G_E^c}}$. Now, we want to prove that $e_{G^C} \subseteq \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$. Let $\cup_{e_H \in (e_G)^c} (F,E)_{e_H} = (H,E)$. Where $H(e) = \cup_{e_H \in (e_G)^c (F(e)_{e_H})}$ for all $e \in E$. Since $e_{G_E^C}(e) = (e_G)^c$ for all $e \in E$ from definition 9, so, for all $e_H \in \{e_G\}^c$ and $e \in E$ $e_{G_E^C}(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c F(e)_{e_H}} = H(e)$. Thus, $e_{G_E^C} \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$ from definition 2, and so, $e_{G_E^C} = \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$. This means that, $e_{G_E^C}$ is soft semi-open set for all $e_H \in \{e_{G_E}\}^c$. Therefore, e_{G_E} is semi-closed soft set.

□

Proposition 6.8. *[10] Let (X,τ_1,τ_2,E) be a soft bi topological space over X. Then, if (X,τ_1,E) and (X,τ_2,E) are soft semi T_3 space, then (X,τ_1,τ_2,E) is pair wise soft semi T_2 space.*

Proof. Suppose (X,τ_1,E) is a soft semi T_3 space with respect to (X,τ_2,E), then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_H,E) is soft semi closed set in τ_2 and $e_G \notin (e_H,E)$ there exists τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $e_G \in (F,E), e_H \in (y,E) \subseteq (G,E)$ and $(F_1,E)\cup(F_2,E) = \phi$. Hence τ_1 is soft semi T_2 space with respect to τ_2. Similarly, if (X,τ_2,E) is a sot semi T_3 space with respect to (X,τ_1,E), then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_G,E) is semi closed soft set in τ_1 and $y \notin (x,E)$ there exists a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_H \in (F,E), e_G \in (x,E) \subseteq (G,E)$ and $(F_1,E)\cap(F_2,E) = \phi$. Hence, τ_2 is a soft b T_2 space. Thus (X,τ_1,τ_2,E) is a pair wise soft semi T_2 space.

□

Proposition 6.9. *[11] Let (X,τ_1,τ_2,E) be a soft bi topological space over X. if (X,τ_1,E) and (X,τ_2,E) are soft semi T_3 space then (X,τ_1,τ_2,E) is a pair wise soft semi T_3 space.*

Proof. Suppose (X,τ_1,E) is a soft semi T_3 space with respect to (X,τ_2,E) then according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$ and each τ_1 semi closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there happens a τ_1 soft semi open set (F_1,E) and τ_2 soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$. Similarly, (X,τ_2,E) is a soft semi T_3 space with respect to (X,τ_1,E). So according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there exist τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_H \in (F,E)$ and $e_G \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$ and each τ_2 semi closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there exists a τ_2 soft semi open set (F_1,E) and a τ_1 soft semi open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E)\cap(F_2,E) = \phi$. Hence (X,τ_1,τ_2,E) is pair wise soft semi T_3 space.

□

Proposition 6.10. *[12] If (X,τ_1,τ_2,E) be a soft bi topological space over X if (X,τ_1,E) and (X,τ_2,X) are soft b T_4 space then (X,τ_1,τ_2,E) is pair wise soft semi T_4 space.*

Proof. Suppose (X,τ_1,E) is soft semi T_4 space with respect to (X,τ_2,E). So according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a τ_1 soft semi open set (F,E) and a τ_2 soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ each τ_1 soft semi closed set (F_1,E) and a τ_2 soft semi closed set (F_2,E) such that $(F_1,E)\cap(F_2,E) = \phi$. There occurs (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set (G_1,E) is soft τ_1 semi open set $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Similarly, τ_2 is soft semi T_4 space with respect to τ_1 so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a τ_2 soft semi open set (F,E) and a τ_1 soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each τ_2 soft b closed set (F_1,E) and τ_1 soft semi closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.There occurs (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 semi open set (G_1,E) is soft τ_1 semi open set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$ hence (X,τ_1,τ_2,E) is pair wise soft semi T_4 space.

□

Proposition 6.11. *[13] Let (X,τ_1,τ_2,E) be a soft bi topological space over X and Y be a non-empty subset of X. if $(X,\tau_{1Y},\tau_{2Y},E)$ is pair wise soft semi T_3 space. Then $(Y,\tau_{1Y},\tau_{2Y},E)$ is pair wise soft semi T_3 space.*

Proof. First we prove that $(Y,\tau_{1Y},\tau_{2Y},E)$ is pair wise soft semi T_1 space.
Let $e_G, e_H \in X_A, e_G \neq e_H$ if (X,τ_1,τ_2,E) is pair wise space then this implies that (X,τ_1,τ_2,E) is pair wise soft τ_1 space. So there exist τ_1 soft semi open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ now $e_G \in Y$ and $e_G \notin (G,E)$. Hence $e_G \in Y \cap (F,E) = (Y_F,E)$ then $e_H \notin Y \cap F(\alpha)$ foe seme $\alpha \in E$. This means that α. This means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$.
Therefore $e_H \notin Y \cap (F,E) = (Y_F,E)$. Now $e_H \in Y$ and $e_H \in (G,E)$.Hence, $e_H \in Y \cap (G,E) = (G_Y,E)$ where $(G,E)\tau_2$. Consider $x \notin (G,E)$.this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. Therefore $e_G \notin Y \cap (G,E) = (G_Y,E)$ thus $(Y,\tau_{1Y},\tau_{2Y},E)$ is pair wise soft semi T_1 space.
Now, we prove that (X,τ_1,τ_2,E) is pair wise soft semi T_3 space then (X,τ_1,τ_2,E) is pair wise soft semi regular space.
Let $e_H \in Y$ and (G,E) be soft semi closed set in Y such that $e_h \notin (G,E)$ where $(G,E) \in \tau_1$ then $(G,E) = (Y,E) \cap (F,E)$ for some soft semi closed set in τ_1.Hence $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$, so $e_H \notin (F,E)$ since (X,τ_1,τ_2,E) is soft semi T_3 space (X,τ_1,τ_2,E) is soft semi regular space so there happens τ_1 soft semi open set (F_1,E) and τ_2 soft semi open set (F_2,E) such that

$$e_H \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E)(F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_2,E)$ then $(G_1,E), (G_2,E)$ are soft semi open sets in Y such that

$$e_H \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$
$$(G_1,E) \cap (G_2,E) = \phi$$

Therefore, τ_{1Y} is soft semi regular space with respect to τ_{2Y}.
Similarly, Let $e_H \in Y$ and (G,E) be a soft semi closed subset Y such that $e_H \notin (G,E) \in \tau_2$ then $(G,E) = (Y,E) \cap (F,E)$ where (F,E) is some soft semi closed set in τ_2. $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$ so $e_H \notin (F,E)$ since (X,τ_1,τ_2,E) is soft semi regular space so there happens τ_2 soft semi open set (F_1,E) and τ_1 soft semi open set (F_2,E).Such that

$$e_H \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E) \cap (F_2,E) = \phi$$
$$\text{Take } (G_1,E) = (Y,E) \cap (F_1,E)$$
$$(G_1,E) = (Y,E) \cap (F_1,E)$$

Then (G_1,E) and (G_2,E) are soft semi open set in Y such that

$$e_H \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$

Therefore τ_{2Y} is soft semi regular space.

□

7. Conclusion

Topology is the most important branch of mathematics which deals with mathematical structures. Recently,many researchers have studied the soft set theory which is initiated by Molodtsov [5] and safely applied to many problems which contain uncertainties in our social life.Shabir and Naz in [24] introduce and deeply studied the concept of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.
In the present work, we have continued to study the properties of soft semi separation axioms in soft bi topological spaces with respect to soft points as well as ordinary points of a soft topological spaces. We defined soft T_0, T_1, T_2, T_3 spaces with respect to soft points and studied their behaviors in soft bi topological spaces. We also extended these axioms to different results. These soft separation axioms would be useful for the growth of the set theory of soft topology to solve complex problems, compressing doubts in economics, engineering, medical etc. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. We hope that these results in this paper will help the researchers for strengthening the toolbox of soft topology. In the next study, we extend the concept of α-open, Pre-open and b**open soft sets in soft bi topological spaces with respect to ordinary as well as soft points.

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