

Peristaltic pumping of a Jeffrey fluid in an asymmetric channel with permeable walls

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Abstract

This paper deals with the peristaltic pumping of a Jeffrey fluid in an asymmetric channel with permeable walls under long wave length and low Reynolds number assumptions. The channel asymmetry is produced by choosing the peristaltic wave trains with phase difference on the walls of the channel. The flow is investigated in a wave frame of reference with the velocity of the wave. The effect of various parameters on the flow characteristics are discussed through graphs.

Keywords: Peristalsis, Jeffrey fluid, Asymmetric channel, Permeable walls.

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1 Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube. Peristaltic pumping has been the object of scientific and engineering research during the past few decades. The pumping of fluids through muscular tubes by means of peristaltic waves is an important biological mechanism. Study of the mechanism of peristalsis from both the mechanical and physiological viewpoints has been the object of scientific research. The waves can be short, local reflexes or long, continuous contractions along the length of the organ. In the esophagus, peristaltic waves push food into the stomach. In the stomach, they help mix stomach contents and propel food to the small intestine, where they expose food to the intestinal wall for absorption and move it forward. Peristalsis in the large intestine pushes waste towards the anal canal and is important in removing gas and dislodging potential bacterial colonies.

Peristalsis plays an indispensable role in transporting many physiological fluids in the body such as urine transport from kidney to bladder, the movement of chyme in the gastrointestinal tract, the transport of spermatozoa in the ductus efferentes of the male reproductive tract, the movement of ovum in the fallopian tubes, the swallowing of food through oesophagus and the vasomotion of small blood vessels. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting fluids without internal moving parts. The problem of mechanism of peristaltic transport has attracted the attention of many researchers since the experimental investigation of Latham [4]. Subsequently a number of analytical, numerical and experimental studies of peristaltic flow of different fluids have been reported under different conditions with reference to physiological and mechanical situations.

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The importance of the study of peristaltic transport in an asymmetric channel has been brought out by Eytan and Elad [1] with an application to uterine fluid flow in a non - pregnant uterus. Mishra and Rao [6] studied the peristaltic transport of a Newtonian fluid in an asymmetric channel. Srinivas [10] investigated the nonlinear peristaltic transport in an inclined asymmetric channel. Peristaltic motion of a power-law fluid in an asymmetric channel was investigated by Sreenadh et al. [11]. Ravi Kumar et al. [9] studied the unsteady peristaltic pumping in a finite length tube with permeable wall.

Kothandapani and Srinivas [3] have analyzed the MHD peristaltic flow of a viscous fluid in an asymmetrical channel with heat transfer. Wang et al. [14] have studied the MHD peristaltic motion of a Sisko fluid in an asymmetric channel. Peristaltic motion of a Carreau fluid in an asymmetric channel is studied by Ali and Hayat [7]. They used perturbation method to find the solution.

The study of peristaltic transport through and past porous media has become the important area of research because of its vast applications in the study of biofluids. Misra and Ghosh [5] proposed a mathematical model to study the blood flow taking the channel bounded by permeable walls. Gopalan [2] modeled the tissue region in the blood vessels as porous medium. Ravi Kumar et al. [8] studied the peristaltic transport of a power - law fluid in an asymmetric channel bounded by permeable walls. Ravi Kumar et al. [14] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Many of the physiological fluids are known to be non - Newtonian. Peristaltic transport of blood in small vessels is investigated by considering various non - Newtonian fluids such as power - law, Casson, Herschel - Bulkley, Micropolar. Krishna Kumari et al. [12, 13] considered Jeffrey fluid in their study. Jeffrey model is a relatively simpler linear model using time derivatives instead of convected derivatives.

The present paper deals with peristaltic pumping of Jeffrey fluid, in an asymmetric channel with permeable walls. The channel asymmetry is produced by choosing the peristaltic wave trains with phase difference on the walls. The governing equations are solved subject to relevant boundary conditions. The results are numerically evaluated and discussed through graphs.

2 Mathematical Formulation of the Problem

We consider the motion of an incompressible viscous fluid in a two dimensional channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls. The wall deformations are given by

$$\begin{aligned}\bar{h}_1(\bar{X}, \bar{t}) &= d_1 + a_1 \cos \frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) \quad (\text{Upper wall}) \\ \bar{h}_2(\bar{X}, \bar{t}) &= d_2 + a_2 \cos \frac{2\pi}{\lambda}[(\bar{X} - c\bar{t}) + \theta] \quad (\text{Lower wall})\end{aligned}\quad (2.1)$$

where a_1, a_2 are the amplitudes of waves, λ is the wave length, $d_1 + d_2$ is the width of the channel. The phase difference θ varies in the range $0 \leq \theta \leq \pi$, $\theta = 0$ corresponds to symmetric channel with waves out of phase and for $\theta = \pi$ the waves are in phase and further a_1, a_2, d_1, d_2 and θ satisfy the condition

$$a_1^2 + a_2^2 + 2a_1a_2 \cos \theta \leq (d_1 + d_2)^2. \quad (2.2)$$

Equations of motion

The constitutive equations for an incompressible Jeffrey fluid are

$$\begin{aligned}\bar{T} &= -\bar{p}\bar{I} + \bar{S} \\ \bar{S} &= \frac{\mu}{1 + \lambda_1} \left(\frac{\partial \bar{\gamma}}{\partial t} + \lambda_2 \frac{\partial^2 \bar{\gamma}}{\partial t^2} \right)\end{aligned}\quad (2.3)$$

where \bar{T} and \bar{S} are Cauchy stress tensor and extra stress tensor, \bar{p} is the pressure, \bar{I} is the identity tensor, λ_1 is the ratio of the relaxation to retardation times, λ_2 is the retardation time and $\bar{\gamma}$ is the shear rate.

In laboratory frame, the equations governing two dimensional motion of an incompressible Jeffrey fluid are

$$\rho \left[\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{U} = -\frac{\partial \bar{p}}{\partial \bar{X}} + \frac{\partial}{\partial \bar{X}}(\bar{S}_{\bar{X}\bar{X}}) + \frac{\partial}{\partial \bar{Y}}(\bar{S}_{\bar{X}\bar{Y}}) \quad (2.4)$$

$$\rho \left[\frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right] \bar{V} = -\frac{\partial \bar{p}}{\partial \bar{Y}} + \frac{\partial}{\partial \bar{X}} (\bar{S}_{\bar{X}\bar{Y}}) + \frac{\partial}{\partial \bar{Y}} (\bar{S}_{\bar{Y}\bar{Y}}) \quad (2.5)$$

and the equation of continuity is

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad (2.6)$$

where S_{xx} , S_{yy} and S_{xy} are the stress components in laboratory frame.

We introduce a wave frame of reference (\bar{x}, \bar{y}) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of wavelength and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference (\bar{X}, \bar{Y}) to wave frame of reference (\bar{x}, \bar{y}) is given by

$$\bar{x} = \bar{X} - ct, \quad \bar{y} = \bar{Y}, \quad u = \bar{U} - c, \quad \bar{v} = \bar{V}, \quad \bar{p}(x) = \bar{P}(X, t) \quad (2.7)$$

where \bar{u}, \bar{v} are the velocity components in the wave frame (\bar{x}, \bar{y}) , \bar{p}, \bar{P} are pressures in wave and fixed frame of references respectively.

Non - dimensionalisation of the flow quantities

Now introducing the non-dimensional quantities,

$$x = \frac{2\pi\bar{X}}{\lambda}, \quad y = \frac{\bar{Y}}{d}, \quad u = \frac{\bar{U}}{c}, \quad v = \frac{\bar{V}}{c\delta}, \quad \delta = \frac{2\pi d}{\lambda}, \quad p = \frac{2\pi d^2 \bar{p}}{\mu c \lambda},$$

$$h_1 = \frac{\bar{h}_1}{d}, \quad h_2 = \frac{\bar{h}_2}{d}, \quad S = \frac{\bar{S}d}{\mu c}, \quad \phi_1 = \frac{a_1}{d_1}, \quad \phi_2 = \frac{a_2}{d_2}. \quad (2.8)$$

Using conditions (2.3) in (2.4) and (2.5), the equations of motion reduces to

$$\rho \left[u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] u = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} (S_{xx}) + \frac{\partial}{\partial y} (S_{xy}) \quad (2.9)$$

$$\rho \left[u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] v = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} (S_{xy}) + \frac{\partial}{\partial y} (S_{yy}) \quad (2.10)$$

and the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.11)$$

Eliminating pressure from equations (2.9) and (2.10), we get

$$\delta Re \left[\left(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \nabla^2 \psi \right] = \left[\left(\frac{\partial^2}{\partial y^2} - \delta^2 \frac{\partial^2}{\partial x^2} \right) S_{xy} \right] + \delta \left[\frac{\partial^2}{\partial x \partial y} (S_{xx} - S_{yy}) \right] \quad (2.12)$$

in which

$$S_{xx} = \frac{2\delta}{1 + \lambda_1} \left[1 + \frac{\delta \lambda_2 c}{d} \left(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \right] \frac{\partial^2 \psi}{\partial x \partial y} \quad (2.13)$$

$$S_{xy} = \frac{1}{1 + \lambda_1} \left[1 + \frac{\delta \lambda_2 c}{d} \left(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \right] \left(\frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) \quad (2.14)$$

$$S_{yy} = \frac{2\delta}{1 + \lambda_1} \left[1 + \frac{\delta \lambda_2 c}{d} \left(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \right] \frac{\partial^2 \psi}{\partial x \partial y} \quad (2.15)$$

$$\nabla^2 = \delta^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}, \quad \delta = \frac{2\pi d}{\lambda}, \quad Re = \frac{\rho c d}{\mu}. \quad (2.16)$$

Using the long wave length approximation and neglecting the wave number δ , we get

$$\frac{\partial^2}{\partial y^2} S_{xy} = 0 \quad (2.17)$$

$$\frac{\partial}{\partial y} \left(\frac{1}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} \right) = \frac{dp}{dx} \quad (2.18)$$

or

$$\frac{\partial}{\partial y} \left(\frac{1}{1 + \lambda_1} \frac{\partial u}{\partial y} \right) = \frac{dp}{dx}. \quad (2.19)$$

The corresponding boundary conditions are (Saffman slip conditions)

$$\frac{\partial u}{\partial y} = \frac{-\alpha}{\sqrt{k}} u \quad \text{at } y = h_1 \text{ (upper wall)} \quad (2.20)$$

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k}} u \quad \text{at } y = h_2 \text{ (lower wall)} \quad (2.21)$$

After non dimensionalisation the governing equations and the boundary conditions become

$$\frac{\partial}{\partial y} \left(\frac{1}{1 + \lambda_1} \frac{\partial u}{\partial y} \right) = \frac{dp}{dx}. \quad (2.22)$$

The corresponding boundary conditions are

$$\frac{\partial u}{\partial y} = -\alpha \sigma \cdot u \quad \text{at } y = h_1 \text{ (Upper wall)} \quad (2.23)$$

$$\frac{\partial u}{\partial y} = \alpha \sigma \cdot u \quad \text{at } y = h_2 \text{ (Lpper wall)} \quad (2.24)$$

3 Solution of the Problem

Solving the equation (2.22) together with the boundary conditions (2.23) and (2.24), we get the velocity as

$$u = \frac{P(1 + \lambda_1)}{2} y^2 + c_1 y + c_2, \quad P = \frac{dp}{dx} \quad (3.1)$$

where

$$c_1 = \frac{-P(1 + \lambda_1)((h_1 + h_2))}{2} \quad c_2 = \frac{P((1 + \lambda_1)(h_2 - h_1) + \alpha P \sigma ((1 + \lambda_1)h_1 h_2))}{2\alpha \sigma}.$$

The volume flow rate 'q' in the wave frame of reference is given by

$$q = \int_{h_2}^{h_1} u dy = P(1 + \lambda_1)(h_2 - h_1) \left[\frac{h_1^2 + h_1 h_2 + h_2^2}{6} + \frac{(h_1 + h_2)^2}{4} + \frac{(h_2 - h_1) + \alpha \sigma \cdot h_2 h_1}{2\alpha \sigma} \right]. \quad (3.2)$$

From (3.2), we get

$$\frac{dp}{dx} = \frac{q}{(1 + \lambda_1)(h_2 - h_1)D} \quad (3.3)$$

where

$$D = \left[\frac{h_1^2 + h_1 h_2 + h_2^2}{6} + \frac{(h_1 + h_2)^2}{4} + \frac{(h_2 - h_1) + \alpha \sigma \cdot h_2 h_1}{2\alpha \sigma} \right].$$

The instantaneous flux at any axial station is

$$Q(x, t) = \int_{h_2}^{h_1} (u + 1) dy = q + h_1 - h_2. \quad (3.4)$$

The average volume flow rate over one period ($T = \frac{\lambda}{c}$) of the peristaltic wave is defined as

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = \frac{1}{T} \int_0^T (q + h_1 - h_2) dt = q + 1 + d. \quad (3.5)$$

The dimensionless frictional forces at $y = h_1$ and $y = h_2$ are given by

$$F_1 = \int_0^1 h_1^2 \left(\frac{-dp}{dx} \right) dx$$

$$F_2 = \int_0^1 h_2^2 \left(\frac{-dp}{dx} \right) dx \quad (3.6)$$

4 Discussion of the Results

From the Eq. (3.6), we have calculated the pressure difference P as a function of time average flow rate \bar{Q} to study the effects of various parameters on pumping characteristics.

Figs. (2) - (4) are drawn to study the effect of Jeffrey parameter on pumping characteristics for the values of $\theta = 0, \pi/4, \pi/6$. It is observed that the pumping rate decreases with the increase in the Jeffrey parameter λ_1 for pumping ($\Delta P > 0$) and as well as for free pumping ($\Delta P = 0$). Further, observed that the pumping is more for a Jeffrey fluid when compared with a Newtonian fluid. Fig. 2 corresponds to symmetric channel. From Figs. (2) - (4) it is also observed that the pumping rate decreases as the symmetry of the channel increases.

The variation of pressure rise with time averaged flow rate (\bar{Q}) is calculated from equation (30) for different values of α (slip parameter) and is shown in Figs. (5),(6) and (7). We observe that the lesser the slip parameter, the greater the pressure rise against which the pump works. For a given ΔP , the flux \bar{Q} decreases with increasing α . For a given flux \bar{Q} , the pressure difference ΔP increases with increasing α .

The variation of \bar{Q} with ΔP for different values of phase difference θ is shown in Fig. 8. It is observed that the pumping decreases as the phase difference θ increases. For a fixed ΔP , \bar{Q} decreases as θ increases, this is due to the asymmetry of the channel. Fig. 9 is drawn for the variation of the axial velocity u with y for varying Jeffrey parameter λ_1 . It is observed that maximum velocity decreases as λ_1 decreases. It is observed that the velocity increases as slip parameter decreases from Fig. 10.

5 Conclusions

In this paper, peristaltic pumping of a Jeffrey fluid in an asymmetric channel with permeable walls has been studied. The effect of various parameters on the pumping characteristics is discussed. The following conclusions have been found and summarized as follows.

- (1) Pumping rate decreases with the increasing Jeffrey parameter.
- (2) The pressure rise decreases as the slip parameter increases.
- (3) The axial velocity decreases as Jeffrey parameter increases.
- (4) The axial velocity increases as slip parameter decreases.

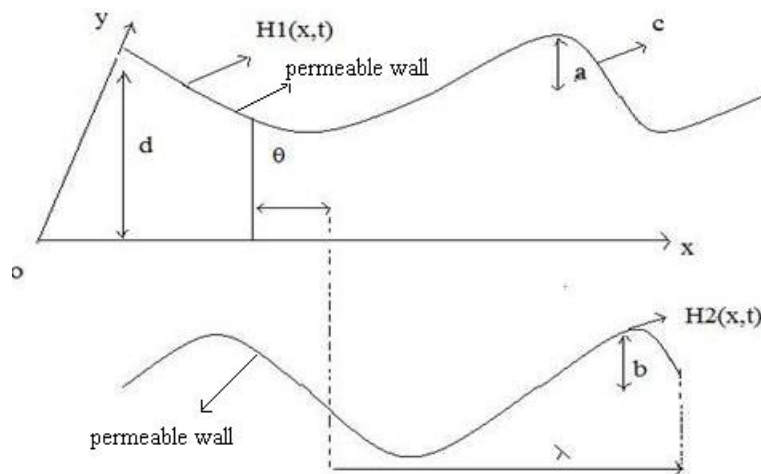


Fig. 1 : Physical Model

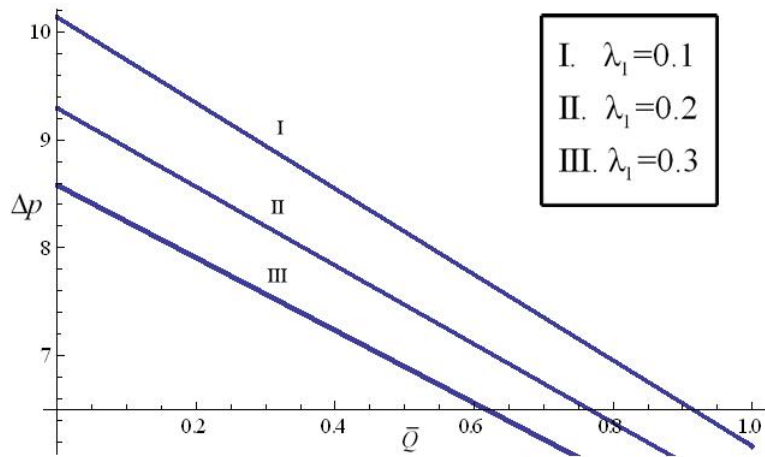


Fig. 2 : The variation of Δp with \bar{Q} for different values of λ_1 with $\theta = 0$.

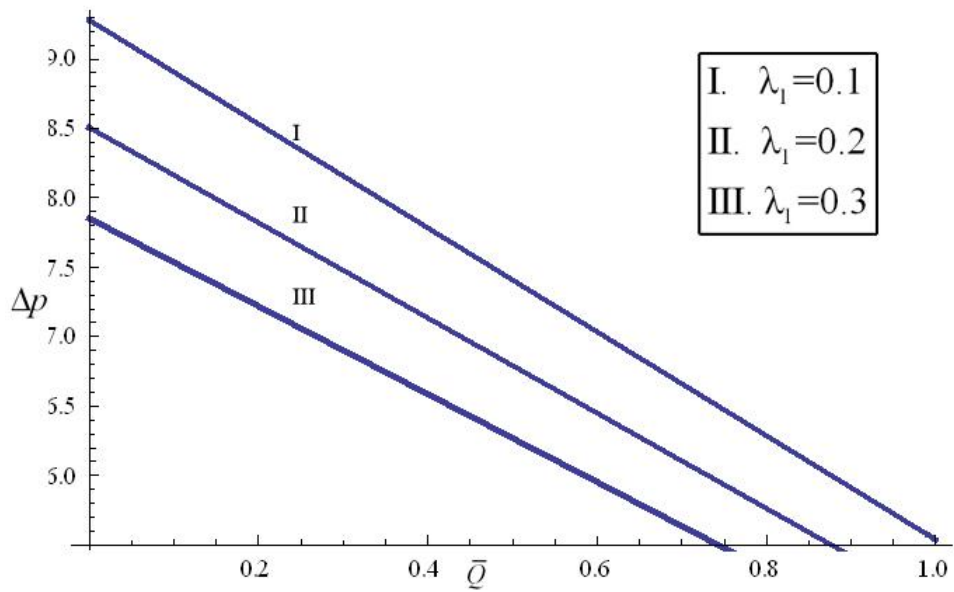


Fig. 3 : The variation of Δp with \bar{Q} for different values of λ_1 with $\theta = \pi/4$.

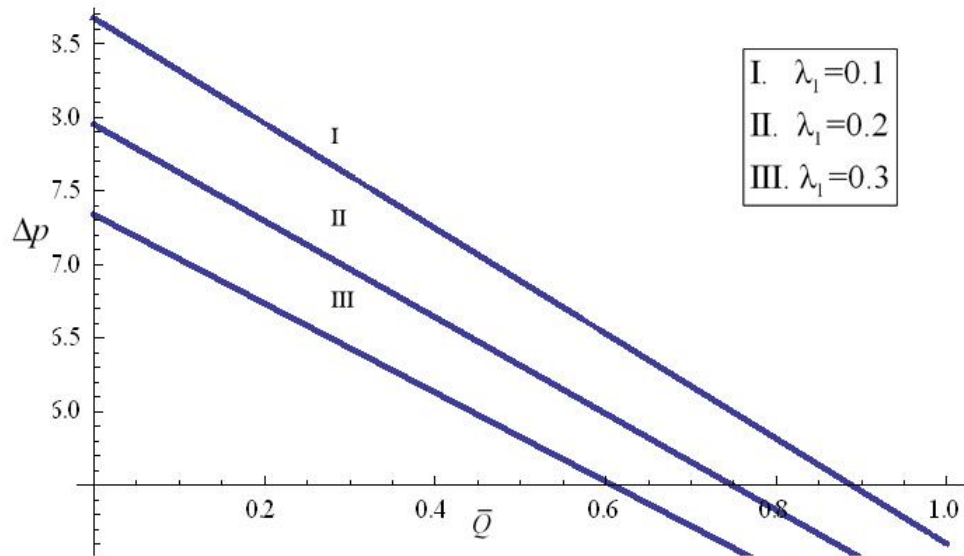


Fig. 4 : The variation of Δp with \bar{Q} for different values of λ_1 with $\theta = \pi/3$.

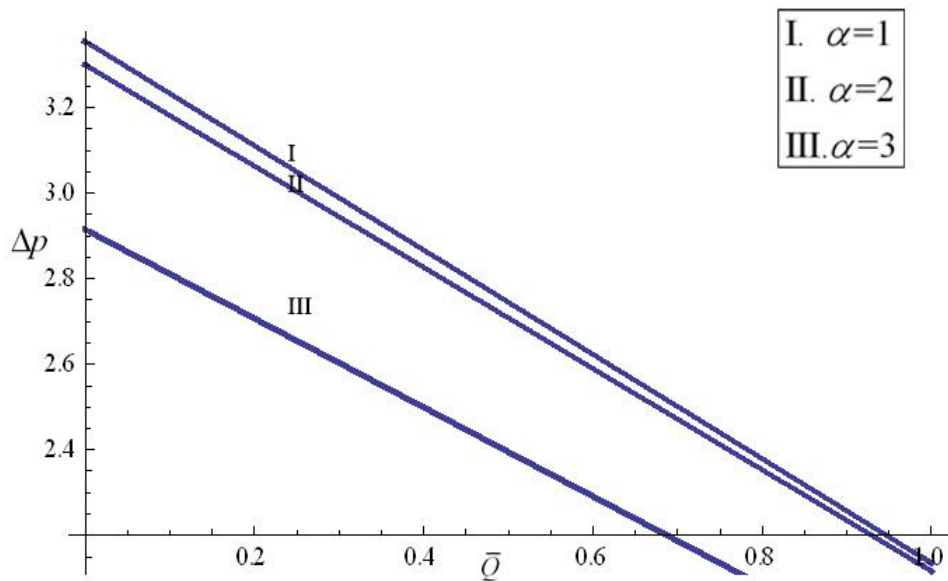


Fig. 5 : The variation of Δp with \bar{Q} for different values of α with $\theta = 0$.

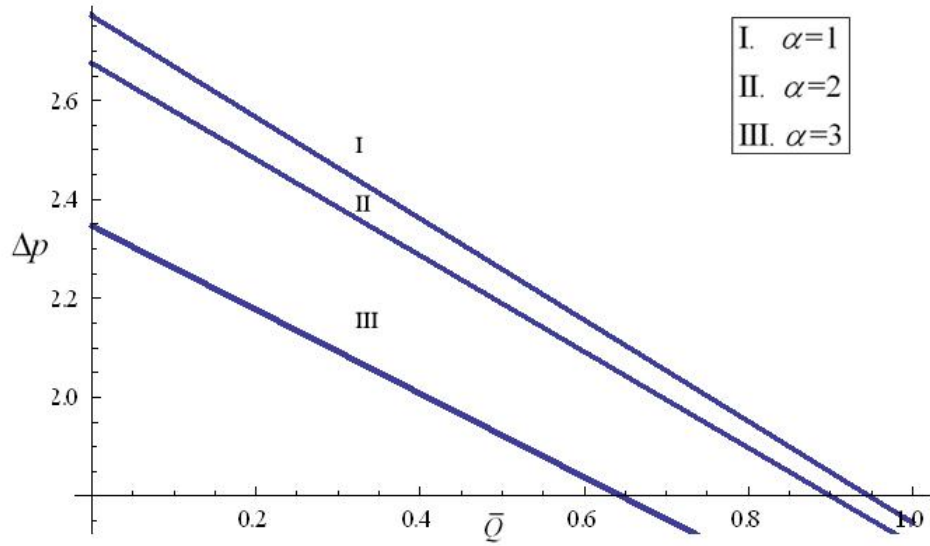


Fig. 6 : The variation of Δp with \bar{Q} for different values of α with $\theta = \pi/4$.

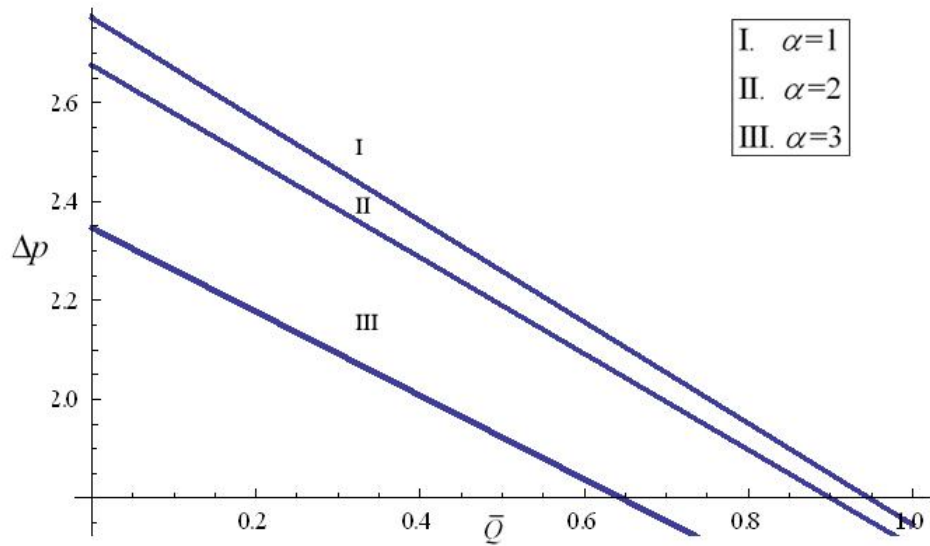


Fig. 7 : The variation of Δp with \bar{Q} for different values of α with $\theta = \pi/3$.

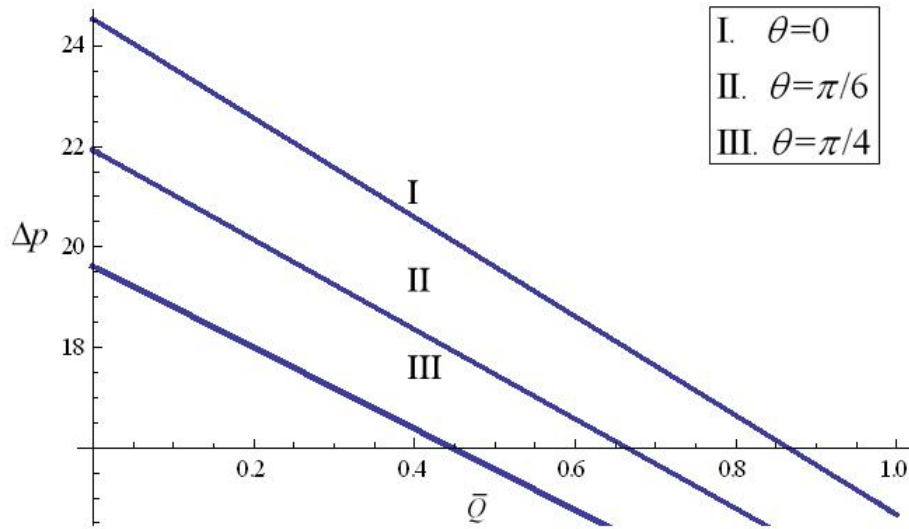


Fig. 8 : The variation of Δp with \bar{Q} for different values of τ .

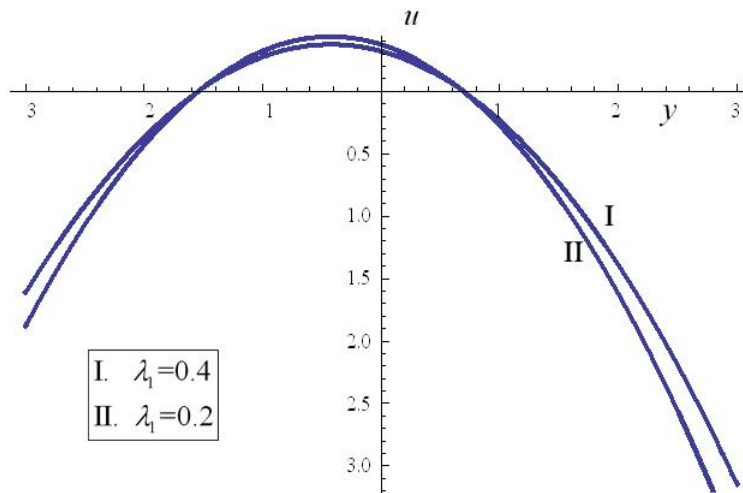


Fig. 9 : The variation of the velocity u with y for different values of λ_1 with $\theta = \pi/4$.

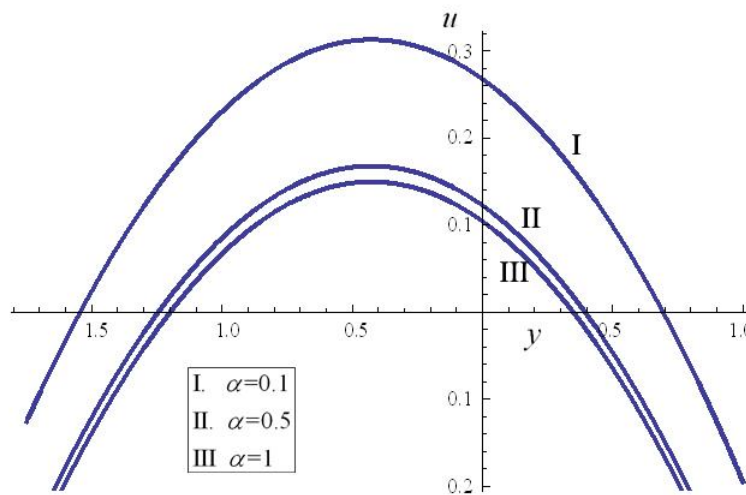


Fig. 10 : The variation of the velocity u with y for different values of α with $\theta = \pi/4$.

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