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A note on mixed super quasi Einstein manifold

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Abstract
Mixed super quasi Einstein manifold ($MS(QE)_n$) is a generalization of Einstein manifold. In this paper we have studied some geometric properties of $MS(QE)_n$. Also we have studied $MS(QE)_n$)satisfying some curvature restriction and obtained the form of Riemannian curvature tensor. We have studied conformally flat and conformally conservative $MS(QE)_n$. We have deduced a necessary condition for a $MS(QE)_n$, to be conformally conservative. Some basic properties of $MS(QE)_n$ on viscous fluid $MS(QE)_n$ spacetimes are discussed. We have proved that if a viscous fluid $MS(QE)_n$ spacetime admitting heat flux obeys Einstein equation with a cosmological constant then none of the energy density and isotropic pressure of the fluid can be a constant.

Keywords
Mixed super quasi-Einstein manifold, conformally flat, conformally conservative, viscous fluid, heat flux, cosmological constant, energy density, isotropic pressure.

AMS Subject Classification
Primary 53C50, 53C25, 53B30; Secondary 53C80, 53B50.

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1. Introduction

Let $U_s = \{x \in M : S \neq \frac{r}{n} g, atx\}$, where S and r are respectively the Ricci tensor and scalar curvature of a Riemannian manifold (M^n, g), $(n \geq 3)$. Then the manifold is said to be a quasi Einstein [4] manifold if on U_s, we have

$$S - ag = bA \otimes A,$$

where A is a 1-form on U_s and a, b are some functions on U_s. It is clear that the 1-form A as well as the function b are non zero at every point on U_s. From the above definition it follows that every Einstein manifold is quasi- Einstein. The scalars a, b are known as the associated scalars of the manifold. Also the 1-form A is called the associated 1-form of the manifold defined by $g(X,U) = A(X)$ for any vector field X; U being a unit vector field, called the generator of the manifold. Such an n-dimensional quasi Einstein manifold is denoted by $(QE)_n$. There are many generalization of $(QE)_n$ in literature([1], [2], [3], [4], [5], [7]). One of them is mixed super quasi-Einstein manifold introduced by A. Bhattacharaya, M. Tarafdar and D. Debnath [2]. According to them a non flat Riemannian manifold is said to be *mixed super quasi-Einstein manifold* if it satisfies the condition

$$\begin{aligned} S(X,Y) &= ag(X,Y) \\ &+ bA(X)A(Y) + cb(X)B(Y) \\ &+ d[A(X)B(Y) + A(Y)B(X)] \\ &+ eD(X,Y), \end{aligned} \tag{1.1}$$

where a, b ,c ,d, e are real valued functions on (M^n, g) of which $b \neq 0$,$c \neq 0$,$d \neq 0$,$e \neq 0$ and A, B are two non zero 1-forms such that
$g(X,U) = A(X), g(X,V) = B(X), g(U,U) = 1$,
$g(V,V) = 1, g(U,V) = 0$, D is a symmetric tensor of type $(0,2)$ with zero trace such that $D(X,U) = 0\ \forall X \in \chi(M)$. Here

a,b,c,d,e are called the associated scalars, A,B are called the main and the auxilliary generators and D is called the structure tensor. Such a space is denoted by $MS(QE)_n$. The paper is organized as follows. Section 2 is concerned with preliminaries. In section 3 we have obtained some geometric properties of a $MS(QE)_n$. Section 4 deals with conformally flat and conservative $MS(QE)_n$. In section 5 we have studied some properties of pseudo Ricci symmetric $MS(QE)_n$. In the last section 6 we studied viscous fluid $(MSQE)_n$ spacetimes.

2. Preliminaries

Putting $X = Y = e_i$ where $\{e_i : 1 \leq i \leq n\}$ is an orthonormal basis of the tangent space of the manifold in (1.1) and summing from 1 to n we get,

$$r = na+b+c. \tag{2.1}$$

Putting $X = Y = U$ in (1.1)

$$S(U,U) = a+b. \tag{2.2}$$

Setting $X = Y = V$ in (1.1) we get,

$$S(V,V) = a+c+eD(V.V). \tag{2.3}$$

Again putting $X = U, Y = V$ in (1.1) we get,

$$S(U,V) = d. \tag{2.4}$$

From above we ca state the following

Theorem 2.1. *In $MS(QE)_n$ the scalars a+b and a+c+eD(V,V) are the Ricci curvatures along the generators U and V respectively.*

Suppose $S(X,Y) = g(QX,Y)$,$D(X,Y) = g(LX,Y)$, $s^2 = \sum_1^n S(Qe_i,e_i)$, $f^2 = \sum_1^n D(Le_i,e_i)$
From (1.1) we get

$$\begin{aligned} \sum_1^n S(Qe_i,e_i) &= a(an+b+c) \\ &+ b(a+b)+c(a+c+eD(V,V)) \\ &+ d(d+d)+e\sum_1^n D(Qe_i,e_i) \\ &= (n-2)a^2+(a+b)^2+(a+c)^2+2d^2 \\ &+ ceD(V,V)+e\sum_1^n S(Le_i,e_i). \end{aligned} \tag{2.5}$$

Again from (1.1)

$$\begin{aligned} \sum_1^n S(Le_i,e_i) &= cD(V,V)+e\sum_1^n D(Le_i,e_i) \\ &= cD(V,V)+ef^2 \end{aligned} \tag{2.6}$$

Using (2.5) and (2.6) we get

$$\begin{aligned} s^2 &= (n-2)a^2+(a+b)^2+(a+c)^2+2d^2 \\ &+ ceD(V,V)+ceD(V,V)+e^2f^2. \end{aligned} \tag{2.7}$$

From (2.3) it is clear that

$$\begin{aligned} s^2 &= na^2+b^2+c^2+2ab+2ac \\ &+ 2ceD(V,V)+e^2f^2+2d^2 \\ &= na^2+b^2+c^2+2ab+2ac \\ &+ 2ce(S(V,V)-a-c)+e^2f^2+2d \\ &= na^2+b^2-c^2+2d^2+2cS(V,V)+e^2f^2. \end{aligned} \tag{2.8}$$

Now, $e > \frac{s}{f}$ $(res < 0 \text{or} = 0)$ according as $na^2+b^2-c^2+2d^2+2cS(V,V) < 0$ $(res > \text{or} = 0)$. Hence we can state the following

Theorem 2.2. *In a $MS(QE)_n$ $(n>2)$ the associated scalar e is less than or equal to or greater than the ratio which the length of the Ricci tensor S bears to the length of the structure tensor D according as,* $na^2+b^2-c^2+2d^2+2cS(V,V) > 0$ $(res = 0 or < 0)$.

3. Some geometric properties

Let us suppose that in a $MS(QE)_n$ the generator U is parallel vector field . Then $\nabla_X U = 0\ \forall X$. So $R(X,Y)U = 0$ and $S(X,U) = 0\ \forall X$
From (1.1), $0 = (a+b)A(X)+dB(X)\ \forall X$
Putting $X = V$ we obtain $d = 0$. Again putting $X = U$ we obtain $a+b = 0$. Hence we have the following

Theorem 3.1. *If the generator U of a $MS(QE)_n$ is a parallel vector vector field then either $d = 0$ or $a+b = 0$.*

Theorem 3.2. *In a $MS(QE)_n$ QU,V are orthogonal iff $d = 0$.*

Proof. $S(U,V) = d$ i.e., $g(QU,V) = d$, which is 0 if and only if $d = 0$. Hence the theorem. ☐

Theorem 3.3. *In a $MS(QE)_n$ QV, V are orthogonal iff $a+c+eD(V,V) = 0$.*

Proof.

$S(V,V) = a+c+eD(V,V)$ i.e.,
$g(QV,V) = a+c+eD(V,V)$.
So $g(QV,V) = 0$, iff $a+c+eD(V,V) = 0$.

Hence the theorem. ☐

Theorem 3.4. *An $MS(QE)_n$ is a $P(GQE)_n$ if either of the vector field is a parallel vector field.*

Proof. If the vector field U is a parallel vector field, then we have $\nabla_X U = 0\ \forall X$ .So $R(X,Y)U = 0$ and eventually $S(X,U) = 0\ \forall X$
From (1.1), $0 = (a+b)A(X)+dB(X), \forall X$
Putting $X = V$ we obtain $d = 0$, i,e the manifold is $P(GQE)_n$ [6] .
Again if the vector field V is parallel then $R(X,Y)V = 0$, consequently $S(Y,V) = 0$ i,e $aB(Y)+cB(Y)+d[A(Y)]+eD(Y,V) = 0$. Putting $Y = U$we get $d = 0$. i,e the manifold is $P(GQE)_n$. Hence the theorem. ☐

Theorem 3.5. *In a $MS(QE)_n$ 0 is an eigen value of L in the direction of the eigen vector U i,e $LU = 0$, where L is the symmetric endomorphism of the tangent space at any point of the manifold corresponding to the structure tensor D .*

Proof. We have $g(LX,Y) = D(X,Y)\ \forall X,Y \in \chi(M)$. Putting $X = U$, we get, $g(LU,Y) = D(U,Y) = 0\ \forall Y$. So $LU = 0$ i,e 0 is an eigen value of L in the direction of U. ☐

We now consider a compact orientable $MS(QE)_n$ $(n > 2)$ without boundary. From (1.1) we get,

$$\begin{aligned} S(X,X) &= ag(X,X) \\ &+ bA(X)A(X) + cB(X)B(X) \\ &+ d[A(X)B(X) + A(X)B(X)] + eD(X,X). \end{aligned} \tag{3.1}$$

Let us assume that θ_u be the angel between U and any vector X, θ_v be the angel between V and any vector field X then

$$\cos\theta_u = \frac{g(X,U)}{g(X,X)^{\frac{1}{2}}}, \cos\theta_v = \frac{g(X,V)}{g(X,X)^{\frac{1}{2}}} \tag{3.2}$$

Further we assume that $\theta_u \geq \theta_v$, then we have $\cos\theta_u \geq \cos\theta_v$, i.e., $g(X,U) \geq g(X,V)$. Therefore,

$$S(X,X) \geq [a+b+c+2d][g(X,U)]^2,$$

when $a,b,c,d,e,D(X,X)$ are positive.

Definition 3.6. *A vector field H in a Riemannian manifold (M^n, g) $(n > 2)$ is said to be harmonic [8] if $d\tau = 0$ and $\delta\tau = 0$ where $\tau(X) = g(X,H)\ \forall X$.*

It is known from a compact orientable Riemannian manifold the following relations holds $\int_M [S(X,X) - \frac{1}{2}(d\tau)^2 + (\nabla X)^2 - (\delta\tau)^2]dv = 0$, for any vector field X where dv denotes the volume element of M . Now let $X \in \chi(M)$ be harmonic vector field then $\int_M [S(X,X) + (\nabla X)^2]dv = 0$ for any X. Hence if each $a,b,c,d,e,D(X,X)$ is positive then $\int_M [(a+b+c+2d)g(X,U)^2 + (\nabla X)^2]dv \geq 0$, by virtue of $a+b+c+2d > 0$, $g(X,U) = 0$ and $\nabla X = 0$ for any vector field X. This follows that X is orthogonal to U and X is a parallel vector field. Similarly if $\theta_v \geq \theta_u$, assuming as before it can be shown $g(X,V) = 0$ and $\nabla X = 0$ for any vector field X. Thus we have the following theorem

Theorem 3.7. *In a compact orientable $MS(QE)_n$ $(n > 2)$ without boundary any harmonic vector field X is parallel and orthogonal to one of the generators of the manifold which makes greatest angle with vector X provided $a,b,c,d,e,D(X,X)$ are positive scalars.*

Let us now investigate whether a $MS(QE)_n$ $(n > 2)$ is projectively flat or not.

Theorem 3.8. *A $MS(QE)_n$ $(n > 2)$ can not be projectively flat.*

Proof. Let if possible a $MS(QE)_n (n > 2)$ is projectively flat. Then the Riemannian curvature tensor is given by

$$R(X,Y,Z,W) = \frac{1}{n-1}[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)].$$

Contracting Y and Z and putting $W = U$ we get

$$S(X,U) = \frac{1}{n-1}[rA(X) - S(X,U)].$$

Or,

$$S(X,U) = \frac{r}{n}A(X).$$

Putting $X = V$, in above we get $d = 0$, a contradiction. Hence the theorem. ☐

4. Conformally flat and Conformally conservative $MS(QE)_n$

Theorem 4.1. *If the main generator of a conformally flat $MS(QE)_n$ is parallel vector field then it is a $(GQE)_n$*

Proof. We recall that in a $MS(QE)_n$ the scalar curvature is given by $r = an + b + c$. Now if the manifold is conformally flat then its Riemannian curvature tensor is given by

$$\begin{aligned} R[X,Y,Z,W] &= \frac{1}{n-2}[S(Y,Z)g(X,W) \\ &- S(X,Z)g(Y,W) + S(X,W)g(Y,Z) \\ &- S(Y,W)g(X,Z)] - \frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,W) \\ &- g(X,Z)g(Y,W)]. \end{aligned} \tag{4.1}$$

Now using definition of $MS(QE)_n$ and using $r = an + b + c$ and putting $Z = U$ we get

$$\begin{aligned} R(X,Y)U &= \frac{a+b}{n-1}[A(Y)X - A(X)Y] \\ &- \frac{c}{n-2}[VB(Y) + \frac{1}{(n-1)}[UA(Y) - Y] \\ &+ \frac{d}{n-2}[B(Y)X - B(X)Y + B(X)A(Y)U - B(Y)A(X)U] \\ &+ \frac{e}{n-2}[A(Y)LX - A(X)LY], \end{aligned} \tag{4.2}$$

where $g(LX,Y) = D(X,Y)$. If U is a parallel vector field then $R(X,Y)U = 0$, $a+b = d = 0$, so the last equation becomes

$$\begin{aligned} &\frac{c}{n-2}[VB(Y) + \frac{1}{(n-1)}[UA(Y) - Y] \\ &+ \frac{e}{n-2}[A(Y)LX - A(X)LY]. \end{aligned} \tag{4.3}$$

Putting $Y = U$ we get

$$e[LX - A(X)LU] = 0. \tag{4.4}$$

But $LU = 0$, so we have $eLX = 0\ \forall X$, Hence $e = 0$. So, if U is parallel vector field in a conformally flat $MS(QE)_n$, then $a + b = d = e = 0$, i,e the manifold reduces to $(GQE)_n$. ☐

Theorem 4.2. *A necessary condition for a $MS(QE)_n$, to be conformally conservative is $(d((n-2)a+(2n-3)b+c)(V) = 2(n-1)(dd)(U)$*

Proof. A Riemannian manifold is said to be conformally conservative if the the divergence of its conformal curvature tensor is zero.i,e

$$\begin{aligned} &(\nabla_X S)(Y,Z) - (\nabla_Z S)(Y,X) \\ = &\frac{1}{2(n-1)}[dr(X)g(Y,Z) - (dr)(Z)g(X,Y)]. \end{aligned} \tag{4.5}$$

Now putting $X = Y = U$ and $Z = V$,in above we get,

$$\begin{aligned} &(\nabla_U S)(U,V) - (\nabla_V S)(U,U)) \\ = &\frac{1}{2(n-1)}[dr(U)g(U,V) - (dr)(V)g(U,U)]. \end{aligned} \tag{4.6}$$

Now using the relations $S(U,V) = d$, $S(U,U) = a+b$ and $r = an+b+c$ in above we get

$$\begin{aligned} &(dd)(U) - d(a+b)(V) \\ = &\frac{1}{2(n-1)}[n(da)(V) + (db)(V) + (dc)(V)]. \end{aligned}$$

On simplification

$$\begin{aligned} &(dd)(U) - d(a+b)(V) \\ = &\frac{1}{2(n-1)}[n(da)(V) + (db)(V) + (dc)(V)], \end{aligned} \tag{4.7}$$

or

$$\begin{aligned} &2(n-1)(dd)(U) - 2(n-1)d(a+b)(V) \\ = &-[n(da)(V) + (db)(V) + (dc)(V)], \end{aligned}$$

or

$$\begin{aligned} &2(n-1)(dd)(U) \\ = &(d((n-2)a + (2n-3)b + c)(V). \end{aligned} \tag{4.8}$$

Hence the theorem □

5. Ricci-pseudosymmetric $MS(QE)_n$

An n-dimensional Riemannian manifold (M^n, g) is called Ricci-pseudosymmetric if ,

$$(R(X,Y).S)(Z,W) = L_s Q(g,S)(Z,W;X,Y) \tag{5.1}$$

holds on $U_s = \{x \in M : S \neq \frac{r}{n} g, atx\}$ and L_s is a certain function on U_s. Then we have ,

$$\begin{aligned} &S(R(X,Y)Z,W) + S(Z,R(X,Y)W) \\ = &L_s[g(Y,Z)S(X,W) - g(X,Z)S(Y,W) \\ + &g(Y,W)S(Z,X) - g(X,W)S(Y,Z)] \end{aligned} \tag{5.2}$$

holds.

Theorem 5.1. *In a Ricci-pseudosymmetric $MS(QE)_n$ $n \geq 3$ the following results holds.*

$$R(V,U,U,V) = L_s, \tag{5.3}$$

$$D(R(V,U)V,V) = 0, \tag{5.4}$$

$$L_s = \frac{D(R(U,V)V,V)}{D(V,V)}, \tag{5.5}$$

provided $D(V,V) \neq 0$.

Proof. We consider Ricci-pseudosymmetric $MS(QE)_n$. Then we have

$$\begin{aligned} &S(R(X,Y)Z,W) + S(Z,R(X,Y)W) \\ = &L_s[g(Y,Z)S(X,W) - g(X,Z)S(Y,W) \\ + &g(Y,W)S(Z,X) - g(X,W)S(Y,Z)], \end{aligned} \tag{5.6}$$

or

$$\begin{aligned} &b[A(R(X,Y)Z)A(W) + A(Z)A(R(X,Y)W)] \\ + &c[B(R(X,Y)Z)B(W) + B(Z)B(R(X,Y)W)] \\ + &d[A(R(X,Y)Z)B(W) + A(W)B(R(X,Y)Z) \\ + &A(Z)B(R(X,Y)W) + A(R(X,Y)W)B(Z)] \\ + &e[D(R(X,Y)Z,W) + D(Z,R(X,Y)W)] \\ = &L_s[b\{g(Y,Z)A(X)A(W) - g(X,Z)A(Y)A(W) \\ + &g(Y,W)A(Z)A(X) - g(X,W)A(Y)A(Z)\} \\ + &c\{g(Y,Z)B(X)B(W) - g(X,Z)B(Y)B(W) \\ + &g(Y,W)B(Z)B(X) - g(X,W)B(Y)B(Z)\} \\ + &d\{g(Y,Z)[A(X)B(W) + A(W)B(X)] \\ - &g(X,Z)[A(Y)B(W) + A(W)B(Y)] \\ + &g(Y,W)[A(Z)B(X) + A(X)B(Z)] \\ - &g(X,W)[A(Y)B(Z) + A(Z)B(Y)]\} \\ + &e\{g(Y,Z)D(X,W) - g(X,Z)D(Y,W) \\ + &g(Y,W)D(X,Z) - g(X,W)D(Y,Z)\}]. \end{aligned} \tag{5.7}$$

Putting $Z = U$ and $W = V$ in (5.7), we get

$$\begin{aligned} &b[R(X,Y,V,U) \\ - &L_s\{A(X)B(Y) - A(Y)B(X)\}] \\ + &c[R(X,Y,U,V) - L_s\{A(Y)B(X) - A(X)B(Y)\}] \\ + &e[D(R(X,Y)U,V) - L_s\{A(Y)D(X,V) \\ - &A(X)D(Y,V)\}] = 0. \end{aligned} \tag{5.8}$$

Putting $Z = W = U$ in (5.7) we get

$$d[R(X,Y)U,V) - L_s\{A(Y)B(X) - A(X)B(Y)\}] = 0. \tag{5.9}$$

Since, $d \neq 0$ we get

$$R(X,Y)U,V) - L_s\{A(Y)B(X) - A(X)B(Y)\} = 0. \tag{5.10}$$

Similarly, if we take $Z = W = V$ in (5.7) we get,

$$\begin{aligned} & d[R(X,Y)V,V) - L_s\{A(Y)B(X) \\ - & A(X)B(Y)\}] - e[D(R(X,Y)V,V) \\ - & L_s\{B(Y)D(X,V) - B(X)D(Y,V)\}] = 0. \end{aligned} \tag{5.11}$$

Using (5.9) we get

$$\begin{aligned} & e[D(R(X,Y)V,V) \\ - & L_s\{B(Y)D(X,V) - B(X)D(Y,V)\}] = 0. \end{aligned}$$

Since $e \neq 0$, we have

$$\begin{aligned} & D(R(X,Y)V,V) \\ - & L_s\{[B(Y)D(X,V) - B(X)D(Y,V)\} = 0. \end{aligned} \tag{5.12}$$

Putting $X = V, Y = U$ in (5.10) we get (5.3). Again putting $X = V, Y = U$ in (5.12) we get (5.4). Using (5.12) in (5.11) we get

$$\begin{aligned} & D(R(X,Y)U,V) \\ - & L_s\{A(Y)D(X,V) - A(X)D(Y,V)\} = 0. \end{aligned} \tag{5.13}$$

Putting $X = U, Y = V$ in above we get (5.5). □

6. General relativistic viscous fluid spacetime admitting heat flux [6]

Let (M^n, g) be a connected semi-Riemannian viscous fluid spactime admitting heat flux and satisfying Einstein's equation with a cosmoloical constant λ. Also let U be the unit timelike velocity vector field, V be the unit heat flux vector and D be the anisotropic pressure tensor of the fluid. The we have

$$g(U,U) = -1, g(V,V) = 1, g(U,V) = 0 \tag{6.1}$$

$$D(X,Y) = D(Y,X), Tr.D = 0, D(X,U) = 0 \,\forall\, X. \tag{6.2}$$

Let

$$g(X,U) = A(X), g(X,V) = B(X) \,\forall\, X. \tag{6.3}$$

Also let T be the energy-momentum tensor of type (0,2) describing the matter distribution of such fluid and it be of the following form

$$\begin{aligned} T(X,Y) &= pg(X,Y) \\ &+ (\sigma + p)A(X)A(Y) + B(X)B(Y) \\ &+ [A(X)B(Y) + A(Y)B(X)] + D(X,Y), \end{aligned} \tag{6.4}$$

where σ, p are the energy density and isotropic pressure respectively. General relativity flows from Einstein equation given by

$$S(X,Y) = -\frac{r}{2}g(X,Y) + \lambda g(X,Y) = kT(X,Y), \tag{6.5}$$

for all vector fields X, Y. S is the Ricci tensor of type of type (0,2) and r is the scalar curvature, λ is a cosmological constant. Thus by virtue of (6.4) above equation can be written as

$$\begin{aligned} & S(X,Y) - \frac{r}{2}g(X,Y) + \lambda g(X,Y) \\ = & k[pg(X,Y) + (\sigma + p)A(X)A(Y) + B(X)B(Y) \\ + & \{A(X)B(Y) + A(Y)B(X)\} + D(X,Y)]. \end{aligned} \tag{6.6}$$

Putting this in (1.1) we get

$$\begin{aligned} & \frac{2kp - 2\lambda + 2a + b + c}{2} g(X,Y) \\ = & [b - k(\sigma + p)]A(X)A(Y) + (c - k)B(X)B(Y) \\ + & (d - k)[A(X)B(Y) + A(Y)B(X)] + (e - k)D(X,Y). \end{aligned} \tag{6.7}$$

Putting $X = U$, $Y = V$ in above we get $d = k$
Putting $X = U$, $Y = U$ we get

$$\sigma = \frac{2a + 3b + c - 2\lambda}{2k}, \tag{6.8}$$

or,

$$\sigma = \frac{2a + 3b + c - 2\lambda}{2d}. \tag{6.9}$$

Again contracting (6.6) we get

$$r - 2r + 4\lambda = k[3p - \sigma + 1), \tag{6.10}$$

or,

$$p = \frac{6\lambda - 6a + b - c - 2d}{6d}. \tag{6.11}$$

Hence we can state the following

Theorem 6.1. *If a viscous fluid $MS(QE)_4$ spacetime admitting heat flux obeys Einstein equation with cosmological constant then none of the energy density and isotropic pressure of the fluid can be a constant.*

Now if the associated scalars a, b, c, d are constants with $d > 0$, then from (6.8) and (6.9) σ, p are constants. Since $\sigma > 0$, $p > 0$ we have from (6.8) and (6.9) we get $\lambda < \frac{2a+3b+c}{2}$ and $\lambda > \frac{6a-b+c-2d}{6}$. And hence

$$\frac{6a - b + c - 2d}{6} < \lambda < \frac{2a + 3b + c}{2}. \tag{6.12}$$

Thus we have the following

Theorem 6.2. *If a viscous fluid $MS(QE)_4$ spacetime admitting heat flux obeys Einstein equation with cosmological constant λ,then λ obeys the above inequality.*

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