Malaya J. Mat. 2(3)(2014) 197–202

Malaya Journal of Matematik

MJM
an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

On fuzzy soft metric spaces

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Abstract

In this paper an idea of fuzzy soft point is introduced and using it fuzzy soft metric space is established .The concepts like fuzzy soft open balls and fuzzy soft closed balls are introduced. Some properties of fuzzy soft metric spaces are developed.

Keywords: Fuzzy set, fuzzy metric space.

2010 MSC: 03E72.

1 Introduction

In daily life, the problems in many fields deal with uncertain data and are not successfully modelled in classical mathematics. There are two types of mathematical tools to deal with uncertainities namely fuzzy set theory introduced by Zadeh [12] and the theory of soft sets initiated by Molodstov [8] which helps to solve problems in all areas. Maji et al. [6] introduced several operations in soft sets and has also coined fuzzy soft sets. Chang [4] has introduced the theory of fuzzy topological spaces and Sanjay Roy et al.[10] has defined open and closed sets on fuzzy topological spaces.

In this paper we have defined fuzzy soft metric space in terms of fuzzy soft points. Fuzzy soft open ball and fuzzy soft closed ball are introduced in fuzzy soft metric space. Fuzzy soft Hausdorff metric is also defined and further some equivalent conditions in a fuzzy soft metric space is developed. Some other properties of fuzzy soft metric spaces are also established.

2 Preliminaries

Definition 2.1. *A fuzzy soft point F_e over (U, t) is a special fuzzy soft set defined by $F_e(a) = \mu F_e$, where $\mu F_e \neq \sigma$ $=\sigma$ if $a \neq e$*

Definition 2.2. *Let F_A be a fuzzy soft set over (U, E) and G_e be a fuzzy soft point over (U, E) then $G_e \in F_A$ if and only if $\mu_{G_e} \subseteq \mu_{F_{A^e}} = F_A(e)$ that is $\mu_{G_e}(x) \leq \mu_{F_{A^e}} \forall x \in U$*

Definition 2.3. *Two fuzzy soft points F_{e1}, F_{e2} are said to be equal if $\mu_{F_{e1}}(a) = \mu_{F_{e2}}(a)$. Thus $F_{e1} \neq F_{e2}$ if and only $\mu_{F_{e1}}(a) \neq \mu_{F_{e2}}(a)$.*

Proposition 2.1. *The union of any collection of fuzzy soft points can be considered as a fuzzy soft set and every fuzzy soft set can be expressed as the union of all fuzzy soft points.*

$$F = \left[\bigcup_{F_e \in F_A} F_e \right]$$

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Proposition 2.2. *Let F_A, G_B be two fuzzy soft sets then $F_A \subseteq G_B$ if and only if $F_e \in F_A$ implies $F_e \in G_B$ and hence $F_A = G_B$ if and only if $F_e \in F_A f$ and only if $F_e \in G_B$.*

Note 2.1. *Let $\mathcal{F}$ be a collection of fuzzy soft points then the fuzzy soft set generated by $\mathcal{F}$ be denoted by FSG($\mathcal{F}$) and the collection of all fuzzy soft points of a fuzzy soft set F_A be denoted by FSC($\mathcal{F}$).*

3 Fuzzy soft metric and fuzzy soft metric space

Let $A \subseteq E$ and let F_E be the absolute fuzzy soft set that is $F_E(e) = \tau$ for all $e \in E$. Let (A)* denote the set of all non negative fuzzy soft real numbers. The fuzzy soft metric using fuzzy soft points is defined as follows:

Definition 3.1. *A mapping $d : FSC(F_E) \times FSC(F_E) \to \mathbb{R}(A)^*$ is said to be a fuzzy soft metric on FE if d satisfies the following conditions.*
$(FSM_1) d(F_{e^1}, F_{e^2}) \geq 0$ *for all* $F_{e^1}, F_{e^2} \in FSM(F_E)$
$(FSM_2) d(F_{e^1}, F_{e^2}) = 0$ *if and only if* $F_{e^1} = F_{e^2}$
$(FSM_3) d(F_{e^1}, F_{e^2}) = d(F_{e^2}, F_{e^1})$ *for all* $F_{e^1}, F_{e^2} \in FSM(F_E)$
$(FSM_4) d(F_{e^1}, F_{e^3}) = d(F_{e^1}, F_{e^2}) + d(F_{e^2}, F_{e^3})$ *for all* $F_{e^1}, F_{e^2}, F_{e^3} \in FSM(F_E)$
The fuzzy soft set F_E with the fuzzy soft metric d is called the fuzzy soft metric space and is denoted by (F_E, d).

Definition 3.2. *Let $A \subseteq R$ and $E \subseteq R$, let F_E be the absolute fuzzy soft set that is $F_E(e) = \tau$ for all $e \in E$.*
Define $d : FSC(F_E) \times FSC(F_E) \to (\mathbb{R}(A))$ by $d(F_{e^1}, F_{e^2}) = \inf\{ |\mu_{F_{e1}}(a) - \mu F_{e^2}(a)| / a \in A\}$ for all $F_{e^1}, F_{e^2} \in FSM(F_E)$ then d is a fuzzy soft metric over F_E, let us verify $(FSM_1) - (FSM_5)$

(i) $d(F_{e^1}, F_{e^2}) \geq 0$ *for all* $F_{e^1}, F_{e^2} \in FSM(F_E)$

(ii) $d(F_{e^1}, F_{e^2}) = \inf\{ |\mu_{F_{e^1}}(a) - \mu_{F_{e^2}}(a)| / a \in A\}$
$=\inf\{ |\mu_{F_{e^2}}(a) - \mu F_{e^1}(a)|\}$
$= d(F_{e^2}, F_{e^1})$

(iii) $d(F_{e^1}, F_{e^3}) = \inf\{ |\mu_{F_{e^1}}(a) - \mu_{F_{e^3}}(a)| / a \in A\}$
$= \inf\{ |\mu_{F_{e^1}}(a) - \mu_{F_{e^2}}(a) + \mu_{F_{e^2}}(a) - \mu_{F_{e^3}}| / a \in A\}$
$\leq \inf\{ |\mu_{F_{e^1}}(a) - \mu_{F_{e^2}}(a) + \mu_{F_{e^2}}(a) - \mu_{F_{e^3}}|\}$
$= d(F_{e^1}, F_{e^2}) + d(F_{e^2}, F_{e^3})$ *Thus d is a fuzzy soft metric on $FSC(F_E)$.*

Definition 3.3. *Let (F_E, d) be a fuzzy soft metric space and G_E, be a fuzzy soft subspace of F_E then distance between a fuzzy soft point F_e and G_E is defined by*
$d(F_e, G_E) = \sup\{ d(F_e, G_{e'}) /$ *for every fuzzy soft point* $G_{e'}$ *in* $G_E\}$

Definition 3.4. *A fuzzy soft subspace G_E is said to be a bounded if there exists a positive number M such that $d(G_{e^1}, G_{e^2}) \leq M$ for all $G_{e^1}, G_{e^2} \in G_E$.*
The diameter of the subspace G_E is defined as
diam $G_E = \sup\{ d(G_{e^1}, G_{e^2}) / G_{e^1}, G_{e^2} \in G_E\}$

Definition 3.5. *(F_E, d) be a fuzzy soft metric space and $\tilde{\tilde{t}}$ be a fuzzy soft real number. An open ball centered at fuzzy soft point $F_e \in F_E$ and radius t is a collection of all fuzzy soft points G_e of F_E such that $d(G_e, F_e) < t$.*
It is denoted by $\tilde{\tilde{B}}(F_e, \tilde{\tilde{t}})$. i.e. $B(F_e, t) = \{ G_e \in F_E / d(G_e, F_e) < t\}$
The fuzzy soft closed ball denoted by $B[F_e, t] = \{ G_e \in F_E / d(G_e, F_e) \leq t\}$

Definition 3.6. *Let (F_E, d) be a fuzzy soft metric space having atleast two fuzzy soft points (F_E, d) is said to be Hausdorff if for any points F_{e^1}, F_{e^2} in F_E such that $d(F_{e^1}, F_{e^2}) > 0$, then there exists two fuzzy soft open ball $B(F_{e^1}, t)$ and $B(F_{e^2}, t)$ with centre F_{e^1}, F_{e^2} and radius $\tilde{\tilde{t}}$ such that $\tilde{\tilde{B}}(F_{e^1}, t) \cap \tilde{\tilde{B}}(F_{e^2}, t) = \phi$.*

Theorem 3.1. *Every fuzzy soft metric space is Hausdorff.*

Proof. Let (F_E, d) be a fuzzy soft metric space having atleast two points. Let F_{e^1}, F_{e^2} be two fuzzy soft points in F_E such that $d(F_{e^1}, F_{e^2}) > 0$. Choose any fuzzy soft real number $\tilde{\tilde{t}}$ such that $0 < t < \frac{1}{2} d(F_{e^1}, F_{e^2})$. Consider two fuzzy soft open balls $\tilde{\tilde{B}}(F_{e_1}, \tilde{\tilde{t}}) = \{ F_e' : d(F_e', F_e) < \tilde{\tilde{t}}\}$ and $\tilde{\tilde{B}}(F_{e_2}, \tilde{\tilde{t}}) = \{ F_e'' : d(F_e, F_e'') < \tilde{\tilde{t}}\}$

Suppose $F_{e_3} \in \tilde{\tilde{B}}(F_{e^1}, \tilde{\tilde{t}}) \cap \tilde{\tilde{B}}(F_{e^2}, \tilde{\tilde{t}})$ then

$F_{e_3} \in \tilde{\tilde{B}}(F_{e^1}, \tilde{\tilde{t}}) \implies d(F_{e_1}, F_{e_3}) < \tilde{\tilde{t}}$
$F_{e_3} \in \tilde{\tilde{B}}(F_{e^2}, \tilde{\tilde{t}}) \implies d(F_{e_2}, F_{e_3}) < \tilde{\tilde{t}}$
By (FSM_4)
$d(F_{e_1}, F_{e_2}) \leq d(F_{e_1}, F_{e_3}) + d(F_{e_3}, F_{e_2})$
$< \tilde{\tilde{t}} + \tilde{\tilde{t}} = 2\tilde{\tilde{t}}$
therefore $\tilde{\tilde{t}} > \frac{1}{2}d(F_{e^1}, F_{e^2})$ which contradicts the hypothesis. So clearly, $\tilde{\tilde{B}}(F_{e^1}, t) \cap \tilde{\tilde{B}}(F_{e^2}, t) = \phi$ and hence (F_E, d) is Hausdorff. □

Definition 3.7. *Let $\{ F_{(e,n)1} \}_n$ be a sequence of fuzzy soft points in a fuzzy soft metric space (F_E, d). The sequence $\{ F_{(e,n)1} \}_n$ is said to converge in (F_E, d) if there is a fuzzy soft point $F^{\sigma}_{e'} \in F_E$ such that . $d(F_{(e,n)}, F^{\sigma}_{e'}) \to \tilde{\tilde{0}}$ as $n \to \infty$*
That is for every $\tilde{\tilde{\varepsilon}} > \tilde{\tilde{0}}$ there exist a positive integer $N = N(\tilde{\tilde{\varepsilon}})$ such that whenever $d(F_{(e,n)}, F^{\sigma}_{e'}) \geq \tilde{\tilde{\varepsilon}}$.
It is denoted as $\lim_{n\to\infty} F_{(e,n)1} = F^{\sigma}_{e'}$

Definition 3.8. *A sequence $\{ F_{(e,n)1} \}_n$ of fuzzy soft points in (F_E, d) is a Cauchy sequence if to every $\tilde{\tilde{\varepsilon}} > \tilde{\tilde{0}}$, there exists N a positive integer such that $d(F_{(e,i)1}, F_{(e,j)1} < \tilde{\tilde{\varepsilon}}$ for all $i, j \geq N$ i.e. $d(F_{(e,i)1}, F_{(e,j)1}) < \tilde{\tilde{\varepsilon}} \to 0$ as $i, j \to \infty$*

Definition 3.9. *A fuzzy soft metric space (F_E, d) is said to complete if every Cauchy sequence in F_E converges to some fuzzy soft point of F_E.*

Definition 3.10. *Let F_{CB} be the soft set of non-empty closed and bounded subspace of the soft metric space (F_E, d). Define a function on $F_{CB} \times F_{CB}$ as*
$H_d(R_A, R_B) = \max\{ \sup_{R^a_e \in R_A} d(R^a_e, R_B), \sup_{R^b_e \in R_B} d(R_A, R^b_e)\}$

Theorem 3.2. *For a fuzzy soft metric space (F_E, d) the following are equivalent.*
a) For each sequence of fuzzy soft real numbers $\{ \tilde{\tilde{t}}_n : n \in N\}$, there a sequence of $\{ F^{a_n}_{e_n}\}$ finite fuzzy soft points of F_E such that each finite fuzzy soft set $G_E \subset F_E$ is contained in $\{ F^{a_n}_e, \tilde{\tilde{t}}_n\}$ for some n.
b) For each sequence of fuzzy soft real numbers $\{ \tilde{\tilde{t}}_n : n \in N\}$ there is a sequence of $\{ F^{a_n}_{e_n}\}$ of finite fuzzy points of F_E such that for each finite fuzzy soft set $G_E \subset F_E$ contained in $\bigcup_{n_k \leq n \leq n_{k+1}} B(F^{a_n}_e, \tilde{\tilde{t}}_n)$ for some k.

Proof. Let us prove $(b) \implies (a)$
Let $\{ \tilde{\tilde{t}}_n : n \in N\}$ be a sequence of fuzzy soft real numbers. For each $n \in N$
Let $\tilde{\tilde{S}}_n = \min\{ \tilde{\tilde{t}}_i = \mu_{\tilde{\tilde{t}}_i}$ for $i \leq n\}$
Applying (b) to $\{ \tilde{\tilde{S}}_n : n \in N\}$. Then there is an increasing sequence $n_1 < n_2 < ...$ in N. Such that each finite fuzzy point set $G_E \subset F_E$ is contained in $\bigcup_{n_k \leq n \leq n_{k+1}} B(F^{a_i}_e, \tilde{\tilde{S}}_i)$ for some k.
Let $H^A_E = \bigcup_{i<n} F^{a_i}_e$ for $n < n_1$
$H^n_E = \bigcup_{n_k \leq i \leq n_{k+1}} F^{a_i}_e$, for each n and $n_k \leq n \leq n_{k+1}$
Let us prove that the sequence $\{ H^n_E : n \in N\}$ satisfies (a). Let S_E be a finite subset of F_E choose $\mathcal{K}$ such that $S_E \subset \bigcup_{n_k \leq i \leq n_{k+1}} B(F_{e_i}, \{ \tilde{\tilde{S}}_n)$
Let $H^n_E = \bigcup_{n_k \leq i \leq n_{k+1}} F_{e_i}$, where $n_k \leq n \leq n_{k+1}$
Then for each $F_e \in S_E$ there is j, $n_k \leq j \leq n_{k+1}$ and F_{e_i} with $F_e \in B(F_{e_i}, \tilde{\tilde{S}}_j)$ we also have $B(F_{e_i}, \tilde{\tilde{S}}_j)$ and since $F_{e_j} \in H^n_E$. We have $F_e \in B(H_{E_j}, \tilde{\tilde{t}}_j)$
Proof for (a) implies (b) is trivial. □

Theorem 3.3. *Cartesian product of two fuzzy soft hausdorff metric spaces is hausdroff.*

Proof. Let (F_E, d) and (G_E, d) be two fuzzy soft hausdorff metric spaces. Let $(F_{e_1}, G^1_{e_1})$ and $(F_{e_2}, G^1_{e_2})$ be points in $F_E \times G_E$, in such a way that $d((F_{e_1}, F_{e_2}) > \tilde{\tilde{0}}$ and $d((G^1_{e_1}, G^1_{e_2}) > \tilde{\tilde{0}}$. So either $F_{e_1} \neq G^1_{e_1}$ or $F_{e_2} \neq G^1_{e_2}$
Suppose $F_{e_1} \neq F_{e_2}$, since (F_E, d) is a fuzzy soft hausdorff metric space, there exists two fuzzy soft open balls $\tilde{\tilde{B}}(F_{e_1}, \tilde{\tilde{t}}_1)$ and $\tilde{\tilde{B}}(F_{e_2}, \tilde{\tilde{t}}_2)$ where $\tilde{\tilde{t}}_1$ and $\tilde{\tilde{t}}_2$ are fuzzy soft real numbers such that $\tilde{\tilde{0}} < \tilde{\tilde{t}}_1 < \frac{1}{2}d(F_{e_1}, F_{e_2})$ and $\tilde{\tilde{0}} < \tilde{\tilde{t}}_2 < \frac{1}{2}d(F_{e_1}, F_{e_2})$ and $\tilde{\tilde{B}}(F_{e_1}, \tilde{\tilde{t}}_1) \cap \tilde{\tilde{B}}(F_{e_2}, \tilde{\tilde{t}}_2)$ is empty. Since every metric space is metrizeable, each F_E and G_E are open and so.
$\tilde{\tilde{B}}(F_{e_1}, \tilde{\tilde{t}}_1) \times F_E$ and $\tilde{\tilde{B}}(F_{e_2}, \tilde{\tilde{t}}_2) \times G_E$ are the fuzzy soft open sets on $F_E \times G_E$.
Hence $(\tilde{\tilde{B}}(F_{e_1}, \tilde{\tilde{t}}_1) \times F_E) \cap (\tilde{\tilde{B}}(F_{e_2}, \tilde{\tilde{t}}_2) \times G_E) = \phi$ □

Theorem 3.4. *Every convergent sequence in a fuzzy soft hausdorff metric space (F_E, d) has a unique limit.*

Proof. Assume $\{ F_{(e,n)m}\}$ a sequence of fuzzy soft points in the fuzzy soft metric space converges to $F_{e'_{\sigma_1}}$ and $F_{e'_{\sigma_2}}$.

Since (F_E, d) is hausdorff there exist $\tilde{\tilde{t_1}}$ and $\tilde{\tilde{t_2}}$ fuzzy soft real numbers such that $\tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{t_1}})$ and $\tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{t_2}})$ are disjoint.

Since $\{ F_{(e,n)m}\}$ converges to $F_{e'_{\sigma_1}}$ there exists a positive integer N_1 such that $d(F_{(e,n)m}, F_{e_{\sigma_1}}) < \tilde{\tilde{\varepsilon_1}}$ where $\tilde{\tilde{\varepsilon_1}} < \tilde{\tilde{t_1}}$ for all $n \leq N_1$, Again since $\{ F_{(e,n)m}\}$ converges to $F_{e_{\sigma_2}}$ there exist a positive integer N_2 such that $d(F_{(e,n)m}, F_{e_{\sigma_2}}) < \tilde{\tilde{\varepsilon_2}}$ where $\tilde{\tilde{\varepsilon_2}} < \tilde{\tilde{t_2}}$ for all $n \leq N_2$,

Let $N = max\{ N_1, N_2\}$ then for all $n \geq N$, $F_{(e,n)m} \in \tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_1}})$ and $F_{(e,n)m} \in \tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_2}})$ which is a contradiction to the fact that $\tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_1}})$ and $\tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_2}})$ are disjoint.

Suppose $\tilde{\tilde{\varepsilon_1}} > \tilde{\tilde{t_1}}$ and $\tilde{\tilde{\varepsilon_2}} > \tilde{\tilde{t_2}}$, then $F_{(e,n)m} \in \tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_1}})$ for all $n \geq N_1$ and for all $F_{(e,n)m} \in \tilde{\tilde{B}}(F_{e_{\sigma_1}}, \tilde{\tilde{\varepsilon_2}})$ for all $n \geq N_2$ cannot happen and so again we arrive at a contradiction. □

Definition 3.11. *F_A is called fuzzy soft open if and only if for every $G_e \in F_A$ there exists $r > 0$. Such that $B(G_e, r) \subseteq F_A$ where $B(G_e, r) = \{ H_e : d(H_e, G_e) < r\}$ and $B(G_e, r)$ is called a sphere with center G_e and radius r.*

Definition 3.12. *Let $\delta = \{ F_A : F_A$is fuzzy soft open$\}$ which satisfies the axioms of A fuzzy soft set F_A is called a neighbourhood of a fuzzy soft point G_e if and only if there exists $H_B \in \delta$ such that $G_e \in H_B \subseteq F_A$.*

Theorem 3.5. *Let $\gamma_{G_e} = \{ F_A : F_A$ is a neighbourhoood of the fuzzy soft point $G_e\}$ then the family γ_{G_e} at any point G_e over (U, E) satisfies the following properties.*
(i) if $F_A \in \gamma_{G_e}$ then $G_e \in F_A$
(ii) if $F_A \in \gamma_{G_e}$ and $F_A \subseteq$ then $H_B \in \gamma_{G_e}$
(iii) if $F_A, H_B \in \gamma_{G_e}$ then $F_A \cap H_B \in \gamma_{G_e}$

Proof. (i) if $F_A \in \gamma_{G_e}$ then F_A is a neighbourhood of the fuzzy soft point G_e. so, there exists a fuzzy soft open set H_B containing G_e and $H_B \subset F_A$. Thus there exists a $r > 0$ such that $B(G_e, r) \subset H_B$.

Hence $B(G_e, r) \subset H_B \subset F_A$ and so $G_e \in F_A$

(ii) Given $F_A \in \gamma_{G_e}$ and $F_A \subseteq H_B$ there exists fuzzy soft open set V_c containing G_e and $V_c \subset F_A$ also there exists $r > 0$, such that $B(G_e, r) \subset V_c \subset F_A$ by the given condition, $B(G_e, r) \subset V_c \subset F_A \subseteq H_B$ and so $B(G_e, r) \subset V_c \subseteq H_B$ implies that $H_B \in \gamma_{G_e}$

(iii) if $F_A, H_B \in \gamma_{G_e}$, then there exists fuzzy soft open sets V_c and W_D such that $V_c \subset F_A$ and $W_D \subset H_B$.

Thus there exists $r_1 > 0$ such that $B(G_e, r_1) \subset V_c \subset F_A$ and there exists $r_2 > 0$ such that $B(G_e, r_2) \subset W_D \subset H_B$ choose $r = min\{ r_1, r_2\}$

then $B(G_e, r_1) \subset V_c \cap W_D \subset F_A \cap H_B$

□

Definition 3.13. *A dual fuzzy soft point is a fuzzy soft point F_{e^d} of F_e over (U, E) where $F_{e^d}(a) = 1 - \mu_{F_e}$ if $a = e$, where $\mu_{F_e}\bar{0}$*
= 1 if $a \neq e$

Definition 3.14. *A fuzzy soft matric is a mapping $d : FSC(F_A) \times FSC(F_A) \to \mathbb{R}(A)^*$ on F_A which is continuous for membership grade and satisfies for all $F_{e_1}, F_{e_2}, F_{e_3} \in FSC(F_A)$ the following axioms*
(i) if $F_{e_2} \subset F_{e_1}$ then $d(F_{e_1}, F_{e_2}) = 0$
(ii) $d(F_{e_1}, F_{e_3}) \leq d(F_{e_1}, F_{e_2}) + d(F_{e_2}, F_{e_3})$
(iii) $d(F_{e_1}, F_{e_2}) = d(F_{d_{e_2}}, F_{d_{e_3}})$
(iv) if $F_{e_2} \subseteq F_{e_1}$, then $\bar{d}(F_{e_1}, F_{e_2}) > 0$
if in the definition mentioned above, if (iv) is omitted then d is called a soft fuzzy pseudo metric. if (iii) and (iv) are omitted then d is called a fuzzy soft quasi metric.

Definition 3.15. *Let d be a fuzzy soft quasi metric on the fuzzy soft set F_A then for any $F_e \in FSC(F_A)$ and $\varepsilon > 0$ then $B_\varepsilon(F_e) = \cup\{ F_{e'} : d(F_e, F_{e'}) < \varepsilon\}$ is a fuzzy soft set which is called an ε - open ball of F_e and $B_\varepsilon(F_e) = \cup\{ F_{e'} : d(F_e, F_{e'}) < \varepsilon\}$ called a fuzzy soft closed ball of F_e.*

Definition 3.16. *The family of all fuzzy soft open balls is known as the base of a fuzzy soft topology τ_F for F_A corresponding to fuzzy soft (quasi, pseudo). This is called as a fuzzy soft metric topology and (F_A, A, τ_F) is a fuzzy soft (quasi, pseudo) metric space.*

Theorem 3.6. *Let (F_A, A, τ_F) be a fuzzy soft quasi metric space then for any fuzzy soft point $F_e \in F_A$ and $\varepsilon > 0$, then the fuzzy soft ε - open ball. $B_\varepsilon(F_E)$ is a fuzzy soft open neighbourhood of F_e.*

Proof. We have to show that $F_e \in B_\varepsilon(F_E)$ for a particular $a \in A$, $F_e(a) = \mu F_e(a)$ if $a = e$, $\mu F_e \neq 0 = 0$ if $a \neq e$ in this case $d(F_{e_1}, F_e) < \varepsilon$ and so $F_e \in B_\varepsilon(F_E)$

For different elements in A, let us show that result is true.

If $a, b \in A$, such that $a < b$ or $b < a$. Then $\mu F_e(a) < \mu F_e(b)$ or $\mu F_e(b) < \mu F_e(a)$

In either cases we conclude from above that $d(F_{e_1}, F_e) < \varepsilon$ and so $F_e \in B_\varepsilon(F_E)$

□

Theorem 3.7. *Let (F_A, A, τ_F) be a fuzzy soft pseudo metric space, if $F_e = \cup_{a \in U} \mu_{F^e}(a)$*

Definition 3.17. *A fuzzy soft point G_e is said to be quasi - coincident with F_A denoted by $G_e q F_A$ if and only if $\mu_{G_e}(x) + \mu^e_{G_e}(x)$ for some $x \in U$*

Definition 3.18. *A fuzzy soft set F_A is said to be a Q - neighbourhood of G_e iff there exists $H_B \in \tau$ such that $G_e q H_B$ and $H_B \subseteq F_A$.*

Theorem 3.8. *A fuzzy soft point $G_e \in F_A$ if and only if each Q - neighbourhood of G_e is quasi - coincident with F_A.*

Definition 3.19. *A family of fuzzy soft sets in (F_A, A, τ_F) is said to be fuzzy soft locally finite if and only if every fuzzy soft point $F_e \in FSC(F_A)$ has a neighbourhood H_A which is quasi - coincident with atmost finite number of $\mathcal{S}$.*

Theorem 3.9. *If $\mathcal{S}$ is a fuzzy soft locally finite family in (F_A, A, τ_F) then*

$$\overline{\bigcup_{G_A \in \mathcal{S}} G_A} = \bigcup_{G_A \in \mathcal{S}} \overline{G_A}$$

Proof. Given any point $F_e \in \overline{\bigcup_{G_A \in \mathcal{S}} G_A}$ then each Q - neighbourhood of F_e is a quasi - coincident with $\bigcup_{G_A \in \mathcal{S}} G_A$. By fuzzy soft locally finite property, there exists a Q - neighbourhood H_A of F_e. which is quasi - coincident with atmost finite number of $\mathcal{S}$. But H_A is quasi - coincident with $\bigcup_{G_A \in \mathcal{S}} G_A$ and hence H_A is quasi - coincident with $\bigcup_{i=1}^n G^i_A$ for all $G^i_A \in \mathcal{S}$

Let us prove that every Q - neighbourhood of F_e is quasi - coincident with $\bigcup_{i=1}^n G^i_A$ for all $G^i_A \in \mathcal{S}$

If for every Q - neighbourhood K_A of F_e which is contained in G_A such that $\mu^e_{K_A}(x) \leq \mu^e_{G_A}(x)$ then we have to show that K_A is quasi - coincident with $\bigcup_{i=1}^n G^i_A$ for all $G^i_A \in \mathcal{S}$. If K_A is contained in G_A then K_A is quasi - coincident with $G^i_A, i = 1, 2, ..., n$.

But K_A and $\bigcup_{G_A \in \mathcal{S}} G^i_A$ are quasi - coincident and thus we have proved that every Q - neighbourhood of F_e is quasi - coincident with $\bigcup_{i=1}^n G^i_A$ and so we have, $F_e \subseteq \overline{\bigcup_{i=1}^n G^i_A} = \bigcup_{i=1}^n \overline{G^i_A} \subset \bigcup_{G_A \in \mathcal{S}} G^i_A$ □

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Received: January 4, 2014; *Accepted*: March 15, 2014

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