Behavior of open sets in bi-Alexandroff topological space

D. Sasikala[1]* and A. Divya [2]

Abstract

The goal of this paper is to establish, the properties of which exhibit the characterization of a j-open set in bi-Alexandroff topological space and some properties of j-open set are analyzed. Also we have studied the notion of j-bi-continuous function in bi-Alexandroff topological space.

Keywords

Alexandroff space, bi-Alex open sets, bi-Alex closed sets, j-bi open set.

AMS Subject Classification

54A10.

[1,2] *Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore-641004, Tamil Nadu, India.*
Corresponding author: [1] dsasikala@psgrkcw.ac.in; [2] divya772248@gmail.com

Contents

1. Introduction

Nowadays the topological structures have many applications in our daily life. Single topology is extended to bitopology, tri topology and quad topology with usual definitions. The concept of bitopological space was first introduced by Kelly [5], and tritopological spaces was implemented by Martin M. Kovar [8]. The quad topological space was invented by Dhanya V. Mukundan [4]. These papers are pushup to study us Alexandroff topological space into bi-Alexandroff topological space.

Alexandroff topological space was first exposed by P. Alexandroff in 1937 [1]. The Alexandroff space (a space with the condition of Alexandroff) is a topological space such that arbitrary intersection of open sets is open (the union of any number of closed sets is closed), equivalently, every point has a minimal neighborhood $V(x)$ and is that the intersection of all open sets containing x, or equivalently it has an unique minimal base [2].

The Alexandroff topological space was a consequence of the important role of finite spaces in digital topology and therefore the undeniable fact that these spaces have all properties of finite spaces [7], [10]. D. Sasikala and I. Arockiarani initiated λ_α-j-closed sets in generalized topological spaces in 2011 [11]. Then they also initiated decomposition of j-closed sets in bigeneralized topological space in 2012 [12].

Furthermore j-sets have motivated us to analyze j-sets in bi-Alexandroff topological space. Our purpose of this paper is to develop the basic concepts and properties of bi-Alexandroff topological space by using j-sets.

2. Preliminaries

Definition 2.1. *[1] Let us consider X be a topological space, then X is an Alexandroff space if arbitrary intersection of open sets is open.*

Theorem 2.2. *[13] Let us consider X be a metric space, then X is an Alexandroff space if and only if X has discrete topology.*

Definition 2.3. *[5] If X is any set, a basis for the topology on the set X is a collection $\mathscr{B}$ of subsets of X called basis elements such that,*
(1) For each $x \in X$, there is atleast one basis element B containing x.

(2) If x belongs to the intersection of two basis element B_1 and B_2, then there is a basis element B_3 containing x such that $B_3 \subset B_1 \cap B_2$.

Theorem 2.4. *[13] If X is an Alexandroff space with topology τ then,*

$$\beta = \{S(x) \quad | \quad x \in X\}$$

is a basis for τ.

Definition 2.5. *[5] Let us consider X be a topological space with τ. If Y is a subset of X, the collection*

$$\tau_Y = \{Y \cap U \quad | \quad U \in \tau\}$$

is a topology on Y, called the subspace topology with this topology, Y is called a subspace of X.

Definition 2.6. *[5] If $\mathscr{B}$ is basis for the topology on a set X then,*

$$\mathscr{B}_Y = \{B \cap Y \quad | \quad B \in \mathscr{B}\}$$

is a basis for the subspace topology on Y.

Definition 2.7. *[9] A subset S of a space X is said to be a pre-open set if $A \subseteq int(cl(S))$.*

Definition 2.8. *[11] A subset S of a space X is said to be a j-open set if $A \subseteq int(precl(S))$.*

3. bi-Alexandroff topological space

Definition 3.1. *Let a non-empty set be X, A_{τ_1} and A_{τ_2} are Alexandroff topologies on X. Then a subset S of X is said to be a bi-Alex open set (briefly $A_{\tau_1 \tau_2}$-open set) if $S \in A_{\tau_1} \cap A_{\tau_2}$ and complement is said to be a bi-Alex closed set (briefly $A_{\tau_1 \tau_2}$-closed set).*

Obviously by the definition of bi-alex open sets it satisfies all the axioms of Alexandroff topological space, in which Alexandroff topology is denoted by $A_{\tau_1} \cap A_{\tau_2}$ and X with two topologies called bi-Alexandroff topological space and denoted by $(X, A_{\tau_1}, A_{\tau_2})$.

Example 3.2. *Let $X = \{a,b,c,d\}$, $A_{\tau_1} = \{X, \emptyset, \{b\}, \{d\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$, $A_{\tau_2} = \{X, \emptyset, \{b\}, \{d\}, \{a,d\}, \{b,d\}, \{a,b,d\}, \{c\}, \{b,c\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$.*
Then bi-Alex open sets are $X, \emptyset, \{b\}, \{d\}, \{a,d\}, \{b,d\}, \{a,b,d\}$ bi-Alex closed sets are $X, \emptyset, \{a,c,d\}, \{a,b,c\}, \{b,c\}, \{a,c\}, \{c\}$.

Definition 3.3. *A subset S of a bi-Alexandroff topological space $(X, A_{\tau_1}, A_{\tau_2})$ is called bi-Alex neighborhood of a point $x \in X$ if and only if there exists a bi-Alex open set U such that $x \subset U \subset S$.*

Definition 3.4. *Let a bi-Alexandroff topological space be $(X, A_{\tau_1}, A_{\tau_2})$ and take $S \subset X$. Then the intersection of bi-Alex closed sets containing S is called a bi-Alex closure of S and denoted by bi-cl(S) and the union of bi-Alex open sets contained in S is called a bi-Alex interior of S and denoted by bi-int(S).*

Theorem 3.5. *X is a bi-Alexandroff topological space if and only if for any $x \in X$ has a minimal bi-Alex open neighborhood .*

Proof. Assume that X is a bi-Alexandroff topology with $y \in X$. Let $O(y) = \{U \subset X : U$ is an open neighborhood of y in A_{τ_1} and $A_{\tau_2}\}$. Let $S(y) = \cap U$ for $U \in O(y)$, then $S(y)$ is an open neighborhood of y in A_{τ_1} and A_{τ_2}, because X is bi-Alexandroff topology. Since $S(y) = \cap U$ it is clear that $S(y)$ is a minimal bi-Alex open neighborhood of y.

Conversely, suppose that for each $y \in X$ has a minimal bi-Alex open neighborhood $S(y)$, then an arbitrary intersection of bi-Alex open sets $V = \cap_{\alpha \in A} U_\alpha$, where U_α is open in A_{τ_1} and A_{τ_2}. If V is empty set then it completes the proof. If not, then pick $y \in V$ and we have $y \in U_\alpha$, $\forall \alpha \in A$. Hence $S(y) \subset U_\alpha$ $\forall \alpha$, because $S(y)$ is the minimal bi-Alex open neighborhood. Therefore $S(y) \subset V$. Hence V is open in A_{τ_1} and A_{τ_2}. Therefore V is bi-Alex open set, since it contains an bi-Alex open set around each of it's point. □

Definition 3.6. *Let $(X, A_{\tau_1}, A_{\tau_2})$ be a bi-Alexandroff topological space. If S is a subset of X, then the collection*

$$\tau_S = \{S \cap G \quad | \quad G \text{ is bi-Alex open in } X\}$$

is a Alexandroff topology on S called the subspace bi-Alexandroff topology and with this topology τ_S, S is called a subspace of X.

Example 3.7. *Let $X = \{1,2,3,4\}$, $A_{\tau_1} = \{X, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}\}$, $A_{\tau_2} = \{X, \emptyset, \{3\}, \{3,4\}, \{1\}, \{1,3\}, \{1,3,4\}, \{1,2,3\}\}$.*
Then bi-Alex open sets are $X, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}$.
Let $S = \{2,3\}$, then τ_S is $\{X, \emptyset, \{2\}, \{3\}, \{2,3\}\}$.

Definition 3.8. *If S is a subspace of bi-Alexandroff topological space X, we say that H is open in S if $H \in \tau_S$.*

Lemma 3.9. *Let S is a subspace of bi-Alexandroff topological space X. If H is open in S and S is bi-Alex open in X, then H is bi-Alex open in X.*

Proof. Since H is open in S, then by the definition of subspace bi-Alexandroff topology, we can write $H = S \cap G$, for some set G is bi-Alex open in X. Since S is bi-Alex open in X which implies that, S and G are both bi-Alex open sets in X. Thus we have $S \cap G$ is bi-Alex open set in X. □

Lemma 3.10. *Let $(X, A_{\tau_1}, A_{\tau_2})$ be a bi-Alexandroff topological space and let $\mathscr{B}$ be base for $A_{\tau_1} \cap A_{\tau_2}$. If (S, τ_S) is a subspace of $(X, A_{\tau_1}, A_{\tau_2})$, then $\mathscr{B}_S = \{B \cap S \quad | \quad B \in \mathscr{B}\}$.*

Proof. $\mathscr{B}$ is a base for $A_{\tau_1} \cap A_{\tau_2}$, and so $\mathscr{B} \subseteq A_{\tau_1} \cap A_{\tau_2}$, which implies that, $\mathscr{B} \subseteq \tau_S$ ———————-(1)
Let $H \in \tau_S$ which implies that $H = G \cap S$, for some bi-Alex open set G in X, and let $y \in H$ from this we get $y = G \cap S$, this means that $y \in G$ and $y \in S$. Since $\mathscr{B}$ is a base for $A_{\tau_1} \cap A_{\tau_2}$, $y \in G$ and $G \in A_{\tau_1} \cap A_{\tau_2}$, which implies that $y \in B \subseteq G$, for some subset B in $\mathscr{B}$. Then $y \in B \cap S \subseteq G \cap S$. We have $H =$

$G \cap S$, so we can write $y \in B \cap S \subseteq G \cap S = H$. Let $\mathscr{B}_S = B \cap S$, then $B_S \in \mathscr{B}_S$. Thus for $H \in \tau_S$ and $y \in H$ there exists $B_S \in \mathscr{B}_S$ such that $y \in \mathscr{B}_S \subseteq H$ ——————————(2)
From (1) and (2), we have $\mathscr{B}_S$ is a base for τ_S. $\qquad\square$

Definition 3.11. *A function $f : X \to Y$ between bi-Alexandroff topological spaces is said to be bi-continuous if for every bi-Alex open set $H \subseteq Y$, the preimage $f^{-1}(H)$ is bi-Alex open in X.*

Example 3.12. *Let $X = \{a,b,c,d\}$, $A_{\tau_1} = \{X, \emptyset, \{a\}, \{a,b\},$ $\{a,b,c\}\}$, $A_{\tau_2} = \{X, \emptyset, \{c\}, \{c,d\}, \{a\}, \{a,c\},$ $\{a,c,d\}, \{a,b\}, \{a,b,c\}\}$*
then bi-Alex open sets are $X, \emptyset, \{a\}, \{a,b\}, \{a,b,c\}$ and bi-Alex closed sets are $X, \emptyset, \{b,c,d\}, \{c,d\}, \{d\}$.
Let $Y = \{1,2,3,4\}$, $A_{\tau_1} = \{Y, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}\}$,
$A_{\tau_2} = \{Y, \emptyset, \{1\}, \{3\}, \{1,3\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\},$ $\{2,3,4\}\}$.
Then bi-Alex open sets are $Y, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}$ and bi-Alex closed sets are $Y, \emptyset, \{2,3,4\}, \{1,3,4\}, \{3,4\}, \{1\}$.
Let us consider the diagrammatic representation of the function $f : X \to Y$ as follows,

Figure 1
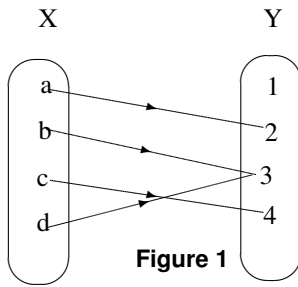
The function $f : X \to Y$ is bi-continuous, since $f^{-1}(H)$ is bi-Alex open in X for every bi-Alex open set H in Y.

Lemma 3.13. *If S is a subspace of a bi-Alexandroff topological space X, then the inclusion function $i : S \to X$ is bi-continuous.*

Proof. Suppose if G is a bi-Alex open in X, then $i^{-1}(G) = G \cap S$ which is bi-Alex open in S by the definiton of subspace bi-Alexandroff topology. $\qquad\square$

Lemma 3.14. *If $f : X \to Y$ is a bi-continuous and if S is a subspace of a bi-Alexandroff topological space X, then $f|S : S \to Y$ (restricted function) is bi-continuous.*

Proof. The function $f|S$ equals the composite of the inclusion function $i : S \to X$ and the function $f : X \to Y$, both of which are bi-continuous function. $\qquad\square$

Lemma 3.15. *If the function $f : X \to Y$ is a bi-continuous and if T is a subspace of bi-Alexandroff topological space Y containing the image set $f(X)$, then $g : X \to T$ obtained by restricting the range of f is bi-continuous. If T is a space having Y as a subspace bi-Alexandroff topology, then $h : X \to T$ obtained by expanding the range of Y is bi-continuous.*

Proof. Let the function $f : X \to Y$ be bi-continuous. If $f(X) \subset T \subset Y$, we prove that the function $g : X \to T$ obtained from f is bi-continuous. Let V be bi-Alex open set in T, and U be bi-Alex open set in Y. Then $V = T \cap U$ for some bi-Alex open set U of Y. Since T contains the entire image set $f(X)$.

$$f^{-1}(U) = g^{-1}(V)$$

by the elementary set theory, since $f^{-1}(U)$ is bi-Alex open set, so is $g^{-1}(V)$.
Now to prove that $h : X \to T$ is bi-continuous. If T has Y as a subspace bi-Alexandroff topology, note that h is the composite of the map $f : X \to Y$ and the inclusion function $i : Y \to T$, it completes the proof. $\qquad\square$

4. pre-bi open set and pre-bi closed set

Definition 4.1. *Let us consider X be a bi-Alexandroff topological space $(X, A_{\tau_1}, A_{\tau_2})$. A subset S of X is said to be a pre-bi open if $S \subseteq bi\text{-}int(bi\text{-}cl(S))$ and S is a pre-bi closed if the complement of S is a pre-bi open.*

Example 4.2. *Let $X = \{a,b,c,d\}$, $A_{\tau_1} = \{X, \emptyset, \{b\}, \{d\},$ $\{a,d\}, \{b,d\}, \{a,b,d\}\}$, $A_{\tau_2} = \{X, \emptyset, \{b\}, \{d\}, \{a,d\},$ $\{b,d\}, \{a,b,d\}, \{c\}, \{b,c\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$.*
Then bi-Alex open sets are $X, \emptyset, \{b\}, \{d\}, \{a,d\}, \{b,d\},$ $\{a,b,d\}$ bi-Alex closed sets are $X, \emptyset, \{a,c,d\}, \{a,b,c\},$ $\{b,c\}, \{a,c\}, \{c\}$. pre-bi open sets are $X, \emptyset, \{b\}, \{d\}, \{a,d\},$ $\{b,d\}, \{a,b,d\}, \{b,c,d\}$, pre-bi closed sets are $X, \emptyset, \{a,c,d\},$ $\{a,b,c\}, \{b,c\}, \{a,c\}, \{c\}, \{a\}$.

Remark 4.3. *It is clear that, every bi-Alex open set is pre-bi open and every bi-Alex closed set is pre-bi closed set in general.*

Theorem 4.4. *Let $(X, A_{\tau_1}, A_{\tau_2})$ be a bi-Alexandroff topological space in which each pre-bi open set a is bi-Alex open then every singleton in X is either a bi-Alex open or a bi-Alex closed.*

Proof. Let $a \in X$, and suppose that the singleton set $\{a\}$ is not bi-Alex open, then clearly $\{a\}$ is not pre-bi open, which implies that $\{a\} \not\subseteq bi - int(bi - cl\{a\})$, so that $bi - int(bi - cl\{a\}) = \emptyset$. We have $bi - int(bi - cl\{X - \{a\}\}) \supseteq bi - int(bi - cl\{a\}) = bi - int(X - bi - int(bi - cl\{a\})) = X \supseteq X - \{a\}$. Thus $X - \{a\}$ is pre-bi open and hence it is bi-Alex open. Therefore $\{a\}$ is bi-Alex closed set. $\qquad\square$

Theorem 4.5. *$(X, A_{\tau_1}, A_{\tau_2})$ is a bi-Alexandroff topological space in which every subset is a pre-bi open if and only if every bi-Alex open set in $(X, A_{\tau_1}, A_{\tau_2})$ is a bi-Alex closed set.*

Proof. Let us consider U be a bi-Alex open set in X, then we have $X - U = bi - cl(X - U)$ which is pre-bi open, which implies that $bi - cl(X - U) \subset bi - int(bi - cl(bi - cl(X - U))) = bi - int(bi - cl(X - U)) = bi - int(X - U)$. Thus $X - U = bi - int(X - U)$. Hence $X - U$ is a bi-Alex open and we get U is a bi-Alex closed.

Conversely, let S be a subset of X, then $S - bi - cl(S)$ is bi-Alex open and hence bi-Alex closed. Thus we have $X - bi - cl(S) = bi - cl(X - bi - cl(S)) = X - bi - int(bi - cl(S))$. So that $S \subset bi - cl(S) = bi - int(bi - cl(S))$, and hence S is pre-bi open. $\square$

Theorem 4.6. *Let $(X, A_{\tau_1}, A_{\tau_2})$ be a bi-Alexandroff topological space, then the following results are equivalent.*
(i) Every pre-bi open set is bi-Alex open.
(ii) Every dense set is bi-Alex open.

Proof. $(i) \Rightarrow (ii)$. Let S be a dense subset of X. Then $bi - int(bi - cl(S)) = X$, so that $S \subset bi - int(bi - cl(S))$ and hence S is pre-bi open and S is bi-Alex open.
$(ii) \Rightarrow (i)$. Let A be a pre-bi open subset of X, so that $A \subset bi - int(bi - cl(A)) = U$ (say). Then $bi - cl(A) = bi - cl(U)$, so that $bi - cl((X - U) \cup A) = bi - cl(X - U) \cup bi - cl(A) = (X - U) \cup bi - cl(U) = X$, and hence $(X - U) \cup A$ is dense in X. Thus we have $(X - U) \cup A$ is bi-Alex open. Now $A = ((X - U) \cup A) \cap U$, the intersection of two bi-Alex open sets, so that A is bi-Alex open. $\square$

5. *j*-open set on bi-Alexandroff topological space

Definition 5.1. *Let us consider X be a bi-Alexandroff topological space $(X, A_{\tau_1}, A_{\tau_2})$. A subset S of X is said to be j-bi open set if $S \subseteq bi - int(pre - bi - cl(S))$ and S is called j-bi closed if the complement of S is j-bi open.*

Example 5.2. *Let $X = \{1, 2, 3, 4\}$, $A_{\tau_1} = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}\}$, $A_{\tau_2} = \{X, \emptyset, \{1\}, \{3\}, \{1, 3\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}, \{2, 3, 4\}\}$.*
Then bi-Alex open sets are $X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}$.
bi-Alex closed sets are, $X, \emptyset, \{2, 3, 4\}, \{1, 3, 4\}, \{3, 4\}, \{1\}$.
j-bi open sets are, $X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{2, 3, 4\}, \{2, 4, 1\}$, j-bi closed sets are, $X, \emptyset, \{2, 3, 4\}, \{1, 3, 4\}, \{3, 4\}, \{1, 4\}, \{1, 3\}, \{4\}, \{1\}, \{3\}$.

Theorem 5.3. *Arbitrary union of j-bi open sets is a j-bi open.*

Proof. Let $\{S_i | i \in I\}$ be a collection of j-bi open sets in X, for each $i \in I$.
$$S_i \subseteq bi - int(pre - bi - cl(S_i))$$
$$\cup_{i \in I} S_i \subseteq \cup_{i \in I}[bi - int(pre - bi - cl(S_i))]$$
$$= [bi - int(\cup_{i \in I} pre - bi - cl(S_i))]$$
$$= [bi - int(pre - bi - cl(\cup_{i \in I} S_i))]$$
$$\cup_{i \in I} S_i \subseteq [bi - int(pre - bi - cl(\cup_{i \in I} S_i))].$$
This implies that $\cup_{i \in I} S_i$ is j-bi open set. $\square$

Theorem 5.4. *Arbitrary intersection of j-bi closed sets is j-bi closed.*

Proof. Let $\{F_i | i \in I\}$ be a collection of j-bi closed sets in a space X, for each $i \in I$, then $bi - int(pre - bi - cl(F_i)) \subset F_i$, since F_i^c is an arbitrary indexed collection of j-bi open sets, from above theorem 5.3 we get $\cup_{i \in I} F_i^c$ is a j-bi open set, since $\cup_{i \in I} F_i^c = (\cap_{i \in I} F_i)^c$, which implies that $(\cap_{i \in I} F_i)^c$ is j-bi open set. Hence $\cap_{i \in I} F_i$ is j-bi closed set. $\square$

Remark 5.5. *We will denote the j-bi interior (resp. j-bi closure) of any subset S of X by j-bi-int(S) (resp. j-bi-cl(S)) where j-bi-int(S) is the union of all j-Bi open sets contained in subset S and j-bi-cl(S) is the intersection of all j-bi closed sets containing subset S.*

Theorem 5.6. *A subset S of X is a j-bi open if and only if $S = j - bi - int(S)$.*

Proof. S is j-bi open and we know that $S \subseteq S$. Therefore $S \in \{P \mid P \subseteq S, P \text{ is } j\text{-bi open}\}$. S is in this collection and other remaining members in this collection is a subset of S and clearly union of this collection is S. That is,
$$\cup\{P \mid P \subseteq S, P \text{ is } j\text{-bi open}\} = S$$
and hence $j - bi - int(S) = S$.
Conversely, since $j - bi - int(S)$ is j-bi open set. That is $S = j - bi - int(S)$, which implies that S is a j-bi open set. $\square$

Theorem 5.7. *A subset S of X is a j-bi closed if and only if $S = j - bi - cl(S)$.*

Proof. From the definition of j-bi closure, we can write $j - bi - cl(S) = \cap\{P \mid P \supset S, P \text{ is } j\text{-bi closed}\}$. If S is a bi-Alex closed then S is a member in $\cap\{P \mid P \supseteq S, P \text{ is } j\text{-bi closed}\}$, and each member contains S. Hence $S = j - bi - cl(S)$.
Conversely, If $S = j - bi - cl(S)$, then S is bi-Alex closed, since j-bi-cl(S) is a j-bi closed set. $\square$

6. *j*-bi-continuous function

Definition 6.1. *Let $(X, A_{\tau_1}, A_{\tau_2})$, $(Y, A_{\tau_1^*}, A_{\tau_2^*})$ be bi-Alexandroff topological spaces. A function from X into Y is called j-bi-continuous if $f^{-1}(H)$ is j-bi open in X for every j-bi open set H in Y.*

Example 6.2. *Let $X = \{w, x, y, z\}$, $A_{\tau_1} = \{X, \emptyset, \{w, x, y\}, \{w, x\}, \{w\}\}$, $A_{\tau_2} = \{X, \emptyset, \{w\}, \{y\}, \{w, y\}, \{y, z\}, \{w, x\}, \{w, y, z\}, \{w, x, y\}\}$. Let $Y = \{v_1, v_2, v_3, v_4\}$, $A_{\tau_1^*} = \{Y, \emptyset, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_1, v_2, v_3\}\}$, $A_{\tau_2^*} = \{Y, \emptyset, \{v_1\}, \{v_3\}, \{v_1, v_3\}, \{v_2\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}\}$.*
Let us consider the diagrammatic representation of the function $f : X \to Y$ as follows,

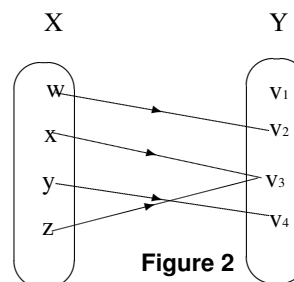**Figure 2**

The bi-Alex open sets of X are $X, \emptyset, \{w\}, \{w, x\}, \{w, x, y\}$
bi-Alex closed sets are $X, \emptyset, \{x, y, z\}, \{y, z\}, \{z\}$

bi-Alex open sets of Y are $Y, \emptyset, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_1, v_2, v_3\}$
bi-Alex closed sets are $Y, \emptyset, \{v_2, v_3, v_4\}, \{v_1, v_3, v_4\}, \{v_3, v_4\}$,
$\{v_1\}$, *and j-bi open sets in* $(X, A_{\tau_1}, A_{\tau_2})$ *are*, $X, \emptyset, \{w\}, \{w, x\}$,
$\{w, z\}, \{w, y\}, \{w, z\}, \{w, x, y\}, \{w, y, z\}, \{z, w, x\}$
j-bi open sets in $(Y, A_{\tau_1^*}, A_{\tau_2^*})$ *are*, $X, \emptyset, \{v_1\}, \{v_2\}, \{v_1, v_2\}$,
$\{v_2, v_3\}, \{v_2, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_2, v_4, v_1\}$.

The function $f : X \to Y$ is *j-bi-continuous, since* $f^{-1}(H)$
is j-bi open in X for every j-bi open set H in Y.

Theorem 6.3. *Let X and Y be bi-Alexandroff topological spaces and let the function from X into Y. Then the following results are equivalent.*
(i) f is j-bi-continous.
(ii) For every subset S of X, one has $f(\bar{S}) \subset \overline{f(S)}$.
(iii) for every j-bi closed set F of Y, the set $f^{-1}(F)$ *is j-bi closed in X.*
(iv) For each $a \in X$ and each neighborhood H of $f(a)$, there is a neighborhood G of a such that $f(G) \subset H$.

Proof. We prove that $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$ and then $(i) \Rightarrow (iv) \Rightarrow (i)$.
$(i) \Rightarrow (ii)$, Assume that f is j-bi-continuous. Let S be a subset of X. We prove that if $a \in \bar{S}$, then $f(a) \in \overline{f(S)}$. Let H be a neighbourhood of $f(a)$, then $f^{-1}(H)$ is a j- bi open set of X containing a, it must intersect S in some point b. Then H intersects $f(S)$ in the point $f(b)$. So that $f(a) \in \overline{f(S)}$ as desired.
$(ii) \Rightarrow (iii)$, Let F be j-bi closed in Y and let $S = f^{-1}(F)$. We have to prove that S is j-bi closed in X, it is enough to prove that, $\bar{S} = S$. By elementary set theory, we have $f(S) = f(f^{-1}(F)) \subset F$. Therefore if $a \in \bar{S}$, then $f(a) \in f(\bar{S}) \subset \overline{f(S)} \subset \bar{F} = F$, so that $a \in f^{-1}(F) = S$, thus $\bar{S} \subset S$, so that $\bar{S} = S$ as desired.
$(iii) \Rightarrow (i)$, Let H be a j-bi open set of Y. Set $F = Y - H$, then $f^{-1}(F) = f^{-1}(Y) - f^{-1}(H) = X - f^{-1}(H)$. Now F be a j-bi closed set of Y. Then $f^{-1}(F)$ is j-bi closed set in X from hypothesis, so that $f^{-1}(H)$ is j-bi open set in X as desired.
$(i) \Rightarrow (iv)$, Let $a \in X$ and let it be a neighbourhood of fa. Then the set $G = f^{-1}(H)$ is a neighbourhood of a such that $f(G) \subset H$.
$(iv) \Rightarrow (i)$, Let H be a j-bi open set of Y and let a be a point of $f^{-1}(H)$. Then $f(a) \in H$; so that by hypothesis, there is a neighbourhood G_a of a such that $f(G_a) \subset H$, then $G_a \subset f^{-1}(H)$. It follows that $f^{-1}(H)$ can be written as the union of all j-bi open sets G_a, so that it is j-bi open set. $\square$

Remark 6.4. *If the condition (iv) in above theorem 6.3 holds for the point a of X, we say that f is j-bi-continuous at the point a.*

7. Rules for constructing j-bi-continuous function

Lemma 7.1. *Let us consider X and Y be bi-Alexandroff topological spaces and $y \in Y$. The function $c_y : X \to Y$ maps all of X in to the single point y is J-bi-continuous, where c_y is called constant function.*

Proof. Let H be a j-bi open set in Y. If $y \in H$, then $c_y^{-1}(H) = X$, which is j-bi open set. On the other hand, if $y \notin H$, then $c_y^{-1}(H) = \emptyset$, so again we get the preimage is a j-bi open. $\square$

Lemma 7.2. *Let X, Y and Z be three bi-Alexandroff topological spaces. If the function $f : X \to Y$ and $g : Y \to Z$ are j-bi-continuous, then their composition map $g \circ f : X \to Z$ is j-bi-continuous.*

Proof. Let $H \subseteq Z$ be a j-bi open, then
$$(g \circ f)^{-1} = \{a \in X | g \circ f(x) \in H\}.$$
$$= \{a \in X | g(f(x)) \in H\}.$$
$$= \{a \in X | f(x) \in g^{-1}(H)\}.$$
$$= \{a \in X | x \in f^{-1}(g^{-1})(H)\}.$$
$$(g \circ f)^{-1} = f^{-1}(g^{-1})(H).$$
Now, g is a j-bi-continuous, so $g^{-1}(H)$ is j-bi open in Y and f is a j-bi-continuous, thus $f^{-1}(g^{-1})(H)$ is a j-bi open in X.

$\square$

References

[1] P. Alexandroff, Diskrete Raume, *Recueil Mathematique*, 2(44)(1937), 501-519.

[2] F. G. Arenas, Alexandroff Spaces, *Acta Mathematica Universitatis Comenianae*, 68(1)(1999), 17-25.

[3] I. Arockiarani, A. A. Nithya, On alexandroff space, *Asian journal of current engineering and maths*, 2(3)(2013), 205-207.

[4] Dhanya V. Mukundan, Introduction to quad topological spaces, *International Journal of Scientific and Engineering Research*, 4(7)(2013), 2483-2485.

[5] James R.Munkres, Topology (second edition), Pearson Education Limited, (2013).

[6] J. C. Kelly, Bitopological Spaces, *London Mathematical Society*, 3(1963), 17-89.

[7] E. H. Kronheimer, The topology of digital images, *Topology and its Applications*, 46(1992), 279-303.

[8] Martin M. Kovar, On 3-topological Version of θ- Regularity, *International Journal of Mathematics and Mathematical Sciences*, 23(6)(2000), 393-398.

[9] A.S. Mashor and M.E.EI.Monsef and S.N.EI.Deeb, On Precontinous and weakprecontinous functions, *Proceedings of the Mathematical and Physical Society of Egypt*, 53(1982), 47-53.

[10] Michael C. McCord, Singular homology groups and homotopy groups of finite topological spaces, *Duke Mathematical Journal*, 33, 465-474, 1966.

[11] D.Sasikala and I.Arockiarani, λ_α-j-closed sets in generalized topological spaces, *International Journal of Applied Science and Technology*, 1(2)(2011), 200-210.

[12] D.Sasikala and I.Arockiarani, Decomposition of j-closed sets in Bi generalized topological spaces, *International Journal of Mathematical Engineering and Science*, 1(7)(2012), 11-18.

[13] Timothy Speer, *A Short Study of Alexandroff Spaces*, New york University, (2007).