Fuzzy inventory model with allowable shortages and backorder

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Abstract
In this paper, we are going to exhibit an inventory model in which shortages are permitted and that can be completely replaced. The model is derived to compute the economic order quantity and to minimize the total cost. To get the optimal solution closer to the reality, the fuzzy techniques are applied. We use the octagonal fuzzy numbers to fuzzify the inventory quantities like ordering cost, holding cost and backorder cost. The ranking function of octagonal fuzzy numbers is used here for defuzzification. The optimal solutions are verified with the help of numerical illustrations.

Keywords
Allowable Shortages, Backorders, Ranking Function, Octagonal Fuzzy Number.

1. Introduction
Inventory means stocking the finished or in process products in a place for some extent of time. Keeping such products for a considerable period of time in business, we have to take decisions over few factors like the quantity of products, length of time etc. so as to minimize the expenses and to maximize the profit. Backorder is the quantity of products that are ordered by the customers and cannot be delivered by the company due to the non-availability. The crisp solutions we derive using mathematical formulation do not fit to the real life situations many times. Because, in reality, many factors are vague and ambiguous which cannot be represented by a single mathematical value. In such cases, the fuzzy concepts have been applied. By taking the fuzzy quantities as Octagonal fuzzy numbers, we represent each vague quantity by eight different possible values. To defuzzify, we use the specific ranking function which is used for intuitionistic octagonal fuzzy numbers.

Rezaei [8] suggested an EOQ model in which shortages are there and are backordered. Hsu and Hsu [2] came up with an inventory model in which screening for imperfect quality items has been done for several number of times and shortages area allowed and are completely backlogged. Jagadeeswari and Chenniappan [3] contributed an inventory model for items that are deteriorating in which shortages are partially backlogged. The inventory model developed by Khanna et al. [4] dealt with imperfect quality items with degeneration in which shortages are backlogged. Kumar and Goswami [5] introduced an EOQ model with imperfect quality products and the shortages are backordered fully. Sujatha and Parvathi [11] developed a fuzzy inventory model for degenerating items where shortages are partially backlogged. Patro et al. [7] introduced a fuzzy inventory model in which shortages are accepted and replaced partially.

This paper comes up with a proposal of a fuzzy EOQ model in which shortages are allowed and are backordered fully. The solution is discussed in both crisp and fuzzy senses. For fuzzy sense, we take quantities as octagonal fuzzy numbers.
paper has a numerical illustration for verification. The conclusion has suggestions and ideas for the further study of the paper.

2. Definitions and Methodologies

Definition 2.1. (Fuzzy set) A fuzzy set $\tilde{A}$ in $X$ is defined as the following set of pair $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$. Here, the membership value of $x \in X$ in a fuzzy set $\tilde{A}$ is the mapping $\mu_{\tilde{A}} : X \rightarrow [0, 1]$.

Definition 2.2. (Octagonal Fuzzy Number) $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is called an octagonal fuzzy number and its membership function is defined as

$$\mu_{\tilde{A}} = \begin{cases} 
0, x < a_1 \\
k \left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\
1, a_2 \leq x \leq a_3 \\
k + (1 - k) \left(\frac{x-a_3}{a_4-a_3}\right), a_3 \leq x \leq a_4 \\
1, a_4 \leq x \leq a_5 \\
k + (1 - k) \left(\frac{x-a_5}{a_6-a_5}\right), a_5 \leq x \leq a_6 \\
k, a_6 \leq x \leq a_7 \\
k \left(\frac{x-a_7}{a_8-a_7}\right), a_7 \leq x \leq a_8 \\
0, x > a_8
\end{cases}$$

2.3 Notations

$D$ — Demand Rate

$A$ — Ordering cost

$H$ — Holding cost

$B$ — Backorder cost

$s$ — Shortage cost

$Q$ — Economic Order Quantity

$T_C$ — Total cost

$\tilde{A}$ — Fuzzy Ordering cost

$\tilde{H}$ — Fuzzy Holding cost

$\tilde{B}$ — Fuzzy Backorder cost

$\tilde{Q}$ — Fuzzy Economic Order Quantity

$T_C$ — Fuzzy Total Cost

2.4 Assumptions

- The demand rate is a known constant.
- There is no limit for time.
- Shortages are permitted and that can be fully backlogged.

3. Mathematical Model in Crisp Sense

For the given mathematical notations and taking the assumptions into account, the total cost is calculated as

$$T_C = \frac{H(Q-s)^2D}{2Q} + \frac{Bs^2D}{2Q} + \frac{AD}{Q}$$  \hspace{1cm} (3.1)

By differentiating partially the equation (3.1) with respect to $Q$ and equating it to zero, we have

$$Q^* = \sqrt{\frac{3^2(H+BD)+2AD}{H}}$$  \hspace{1cm} (3.2)

4. Mathematical Model in Fuzzy sense

The fuzzy mathematical model is arrived by considering the ordering cost, holding cost and the backorder cost as octagonal fuzzy numbers. Let $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$, $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be octagonal fuzzy numbers. Now, the fuzzy total cost can be derived as

$$\tilde{T}_C = \frac{H(Q-s)^2D}{2Q} + \frac{Bs^2D}{2Q} + \frac{AD}{Q}$$  \hspace{1cm} (4.1)

Substituting the octagonal fuzzy numbers, we get

$$\tilde{T}_C = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8) \odot ((Q-s)^2D) + \frac{AD}{Q}$$  \hspace{1cm} (4.2)
This gives the fuzzy EOQ and equation (4.4) gives the fuzzy function for octagonal intuitionistic numbers is used for both shortages and backorder are allowed. The model derived total cost.

By defuzzifying equation (4.3) using ranking function, we get

\[
\tilde{T}_c = \frac{\left( Q \right)^2 D}{6Q} \cdot \left( 2h_1 + 3h_2 + 4h_3 + 5h_4 + 5h_5 + 4h_6 
+ 3h_7 + 2h_8 
+ \frac{\left( (2b_1 + 3b_2 + 4b_3 + 5b_4 + 5b_5 + 4b_6 + 3b_7 + 2b_8) \right)}{28Q} \right)
+ 2(2a_1 + 3a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8) D
\] 

Differentiating the above equation w.r.t \( Q \) and equating to zero, we get By simplifying, we have

\[
\tilde{Q}^* = \sqrt{\frac{\left( 2h_1 + 3h_2 + 4h_3 + 5h_4 + 5h_5 + 4h_6 + 3h_7 + 2h_8 \right)}{2(2h_1 + 3h_2 + 4h_3 + 5h_4 + 5h_5 + 4h_6 + 3h_7 + 2h_8)}}
\]

This gives the fuzzy EOQ and equation (4.4) gives the fuzzy total cost.

5. Numerical Example

Crisp sense:

\( D = 250 \) units per year  
\( A = Rs.10 \) per unit  
\( s = Rs.50 \) per year  
\( H = Rs.60 \) per unit  
\( B = Rs.2 \) per unit, then  
\( Q^* = 153.02 \)  
\( T_c^* = Rs.6181.50 \)

Fuzzy Sense:

\( D = 250 \) units per year  
\( A = (4, 6, 8, 10, 12, 14, 16, 18) \)  
\( s = Rs.50 \) per year  
\( H = (40, 45, 50, 55, 60, 65, 70, 75) \)  
\( B = (0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4) \) then  
\( Q^* = 163.47 \)  
\( T_c^* = Rs.6581.80 \)

6. Conclusion

This paper came up with a fuzzy inventory model in which both shortages and backorder are allowed. The model derived is established in both crisp and fuzzy senses. The ranking function for octagonal intuitionistic numbers is used for defuzzification. Also the inventory costs like cost ordering cost, holding cost and backorder cost are taken as octagonal fuzzy numbers. The arrived solutions are verified using numerical illustrations. This paper can be developed in future for further research work.

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