Malaya Journal of Matematik, Vol. 8, No. 4, 1464-1468, 2020

https://doi.org/10.26637/MJM0804/0021

Eccentricity energy of bistar graph and some of its related graphs

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Abstract
The graphs considered in this article are undirected, finite and simple graphs. In this article we have proved that eccentricity energy of Bistar graph $B_{n,n}$ is $2\sqrt{(3n+1)^2+4(n-1)}$. Also we have investigated eccentricity energy of some graphs related to Bistar graph.

Keywords
Eccentricity Matrix, Eccentricity Eigen values,Eccentricity Energy of a Graph, Bistar Graph.

2010 Mathematics Subject Classification
05C50, 05C76.

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Article History: Received **11** April **2020**; Accepted **19** August **2020**

Contents

1. Introduction

The energy of a graph introduced by Gutman[5] is an important concept of spectral graph theory which links organic chemistry to linear algebra of Mathematics. Generally graph's energy is summation of absolute values of eigen values of the adjacency matrix. Similar energies got from the eigen values of various graphs are considered in recent times. Several other authors also investigated energy of graphs [1,3,8,9]. N. Prabhavathy in [6] introduced the concept of eccentricity energy of graphs.

In a graph G, the distance between any two vertices u and v is denoted as $d(u,v)$ and it is define as the length of the minimum path connecting them, if there is no path between u and v then $d(u,v)$ is defined as ∞. It is useful to note that in a connected graph distance between any two vertices is always finite provided graph is finite. For a vertex v of G, the eccentricity ot a vertex v is denoted as $e(v)$ and is defined as $e(v)=max\{d(u,v); u\in V(G)\}$. Let $D(G)=(d_{uv})$ be the distance matrix of G, where $d_{uv}=d_G(u,v)$. In [11] Wang defined the eccentricity of matrix $\varepsilon(G)$ from distance matrix as $D(G)$ as:

$$[\varepsilon(G)]_{ij}=\begin{cases} (D)_{ij} & if\ (D)_{ij}=min\{e_G(u_i),\ e_G(u_i)\} \\ 0 & if\ (D)_{ij}<min\{e_G(u_i),\ e_G(u_i)\} \end{cases}$$

Eccentricity matrix and adjacency matrix can considered to be in exact opposition by the concept of obtaining them while adjacency matrix is obtained from the distance matrix by selecting only the smallest distance row wise and column wise, the eccentricity matrix takes the largest distance in a similar fashion. Thus the two matrices can be thought of as two extremes of distance – like matrix.

In this article we have considered only finite, simple undirected graphs. It is useful to recall some definition from graph theory.

Definition 1.1 [5]: The *eccentricity energy* of graph G is denoted as $E(G)$ and it is defined by $E(G)=\sum_{i=1}^{n}|\lambda_i|$, where $\lambda_1,\ \lambda_2,\ \lambda_3,\ldots\ldots\lambda_n$ are the eigenvalues of eccentricity matrix of the corresponding graph G.

Definition 1.2 [2]: A graph G is said to be *bipartite* if the vertex set V of G can be partitioned into two disjoint subsets $V_1$and V_2 such that $V_1\cap V_2=\varnothing$ and for each edge has one end vertex is in $V_1$and other is in V_2.

Definition 1.3 [2]: A *complete bipartite* graph is a bipartite graph in which all the vertices in V_1 are adjacent with all the vertices of V_2. If $|V_1| = m$ and $|V_2| = n$ respectively then the corresponding complete bipartite graph is denoted as $K_{m,n}$.

Definition 1.4 [2]: A complete bipartite graph $K_{1,n}$ is known as star graph. Here the vertex of degree n is called the apex vertex.

Definition 1.5 [4]: *Bistar* $B_{n,n}$ is the graph obtained by joining the centre (apex) vertices of two copies of $K_{1,n}$ by an edge. The vertex set of $B_{n,n}$ is
$V(B_{n,n}) = \{v_1, v_2, \ldots, v_n, v, u, u_1, u_2, \ldots u_n\}$, where v,u are apex vertices and $v_1, v_2, ..., v_n$, $u_1, u_2, ...u_n$ are pendent vertices.The edge set of $B_{n,n}$ is
$E(B_{n,n}) = \{vv_1, vv_2, \ldots\ldots, vv_n, vu, uu_1, uu_2, \ldots\ldots uu_n\}$.

Definition 1.6 [10]: For a simple connected graph G the *square of graph* G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in.

Definition 1.7 [4]: The *shadow graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G'' and join each vertex u' in to G' the neighbours of the corresponding vertex v' in G''.

In theorem 2.1, we have shown that Eccentricity energy of Bistar graph $B_{n,n}$ is $2\sqrt{(3n+1)^2 + 4(n-1)}$ *for* $n \in N$, $n \neq 1$.We also provided supportive example in Example 2.2 and in that example we have proven that $E(B_{5,5}) = 2\left(\sqrt{305}\right)$. We also investigate the Eccentricity of square of Bistar graph in Theorem 2.3 and we have shown that its Eccentricity energy is $4n + \sqrt{(4n-1)^2 + 8} - 1$. In Theorem 2.5 we proved that Eccentricity energy of shadow graph of Bistar graph $D_2(B_{n,n})$ is $8n - 2$.

2. Main Result

Theorem 2.1. *Let $n \in N$, $n \neq 1$. Then* $E(B_{n,n}) = 2\sqrt{(3n+1)^2 + 4(n-1)}$, *where $E(B_{n,n})$ is the Eccentricity energy of graph $B_{n,n}$.*

Proof. Let $V(B_{n,n}) = \{v_1, v_2, \ldots\ldots, v_n, v, u, u_1, u_2, \ldots\ldots u_n\}$. Note that $B_{n,n}$ is graph with $2n+2$ vertices and $2n+1$ edges as shown in the following Figure 1.

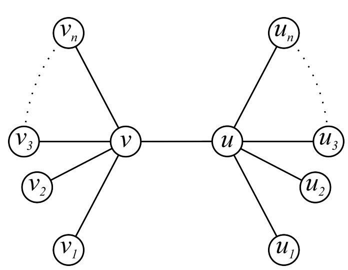

Figure 1

Observe that the distance matrix $D(B_{n,n})$and Eccentricity matrix $\varepsilon(B_{n,n})$ of $B_{n,n}$ are given by

$$D(B_{n,n}) = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \\ v \\ u \\ u_1 \\ u_2 \\ \vdots \\ u_n \end{array} \begin{bmatrix} v_1 & v_2 & \cdots & v_n & v & u & u_1 & u_2 & \cdots & u_n \\ 0 & 2 & \cdots & 2 & 1 & 2 & 3 & 3 & \cdots & 3 \\ 2 & 0 & \cdots & 2 & 1 & 2 & 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 0 & 1 & 2 & 3 & 3 & \cdots & 3 \\ 1 & 1 & \cdots & 1 & 0 & 1 & 2 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 & 1 & 0 & 1 & 1 & \cdots & 1 \\ 3 & 3 & \cdots & 3 & 2 & 1 & 0 & 2 & \cdots & 2 \\ 3 & 3 & \cdots & 3 & 2 & 1 & 2 & 0 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 3 & 3 & \cdots & 3 & 2 & 1 & 2 & 2 & \cdots & 0 \end{bmatrix}$$

and

$$\varepsilon(B_{n,n}) = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \\ v \\ u \\ u_1 \\ u_2 \\ \vdots \\ u_n \end{array} \begin{bmatrix} v_1 & v_2 & \cdots & v_n & v & u & u_1 & u_2 & \cdots & u_n \\ 0 & 0 & \cdots & 0 & 0 & 2 & 3 & 3 & \cdots & 3 \\ 0 & 0 & \cdots & 0 & 0 & 2 & 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 2 & 3 & 3 & \cdots & 3 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 2 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 3 & 3 & \cdots & 3 & 2 & 0 & 0 & 0 & \cdots & 0 \\ 3 & 3 & \cdots & 3 & 2 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 3 & 3 & \cdots & 3 & 2 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Note that the characteristic polynomial of matrix $\varepsilon(B_{n,n})$ is

$$\frac{\lambda^{2(n-1)}}{16}\left(4\lambda^2 - \left(3n - \sqrt{(3n+1)^2 + 4(n-1)}\right)^2\right)$$
$$\left(4\lambda^2 - \left(3n + \sqrt{(3n+1)^2 + 4(n-1)}\right)^2\right)$$

So, eigen values of $\varepsilon(B_{n,n})$= $\varepsilon-$ eigen values are $0, 0, \ldots, 0$ $(2(n-1)\ times)$, $\frac{1}{2}\left(3n + \sqrt{(3n+1)^2 + 4(n-1)}\right)$,
$\frac{1}{2}\left(3n - \sqrt{(3n+1)^2 + 4(n-1)}\right)$,
$\frac{1}{2}\left(-3n + \sqrt{(3n+1)^2 + 4(n-1)}\right)$ and
$\frac{1}{2}\left(-3n - \sqrt{(3n+1)^2 + 4(n-1)}\right)$.

Hence, Eccentricity energy of

$$\begin{aligned} B_{n,n} &= 0 + \tfrac{1}{2}\left|3n + \sqrt{(3n+1)^2 + 4(n-1)}\right| + \\ &\tfrac{1}{2}\left|3n - \sqrt{(3n+1)^2 + 4(n-1)}\right| + \\ &\tfrac{1}{2}\left|-3n + \sqrt{(3n+1)^2 + 4(n-1)}\right| + \\ &\tfrac{1}{2}\left|-3n - \sqrt{(3n+1)^2 + 4(n-1)}\right| \\ &= \tfrac{1}{2}\left(\sqrt{(3n+1)^2 + 4(n-1)} - 3n\right) + \\ &\tfrac{1}{2}\left(3n + \sqrt{(3n+1)^2 + 4(n-1)}\right) + \\ &\tfrac{1}{2}\left(\sqrt{(3n+1)^2 + 4(n-1)} - 3n\right) + \\ &\tfrac{1}{2}\left(3n + \sqrt{(3n+1)^2 + 4(n-1)}\right) \\ &= 2\sqrt{(3n+1)^2 + 4(n-1)}. \end{aligned}$$

□

Example 2.2. *Eccentricity energy of Bistar graph $B_{5,5}$ is $2(\sqrt{305})$.*

Proof.

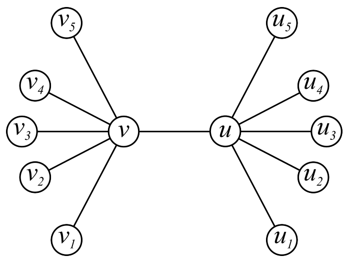

Figure 2

The Eccentricity matrix of $B_{5,5}$ is given by

$$\varepsilon(B_{5,5}) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v \\ u \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that the characteristic polynomial of matrix $\varepsilon(B_{5,5})$ is $\frac{\lambda^8}{16}\left(4\lambda^2-\left(15-\sqrt{305}\right)^2\right)\left(4\lambda^2-\left(15+\sqrt{305}\right)^2\right)$.
So, Eccentricity Eigen values of $B_{5,5}$ are $0,0,\ldots,0(8\ times)$ and $\frac{1}{2}\left(\pm 15\pm\sqrt{305}\right)$.
Hence, Eccentricity energy of
$B_{5,5} = 0+\left|\frac{1}{2}\left(15+\sqrt{305}\right)\right|+\left|\frac{1}{2}\left(15-\sqrt{305}\right)\right|+$
$\left|\frac{1}{2}\left(-15+\sqrt{305}\right)\right|+\left|\frac{1}{2}\left(-15-\sqrt{305}\right)\right|$
$E(B_{5,5}) = 2\sqrt{305}$. □

Theorem 2.3. *Let $n\in N$, $n\neq 1$. Then $E\left(B_{n,n}^2\right) = 4n+\sqrt{(4n-1)^2+8}-1$, where $E\left(B_{n,n}^2\right)$ is the Eccentricity energy of graph $B_{n,n}^2$.*

Proof. Let $V\left(B_{n,n}^2\right) = \{v_1, v_2, \ldots\ldots, v_n, v, u,\ u_1, u_2, \ldots\ldots u_n\}$. Note that $B_{n,n}^2$ is graph with $2n+2$ vertices and $4n+1$ edges as shown in the following Figure 3.

Observe that the distance matrix $D\left(B_{n,n}^2\right)$ and eccentricity matrix $\varepsilon\left(B_{n,n}^2\right)$ of $B_{n,n}^2$ are same and it is given by

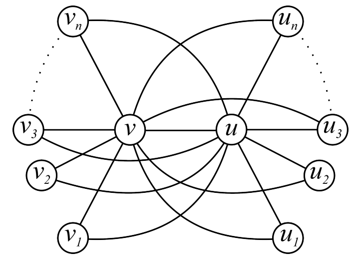

Figure 3

$$D\left(B_{n,n}^2\right) = \varepsilon\left(B_{n,n}^2\right) = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \\ v \\ u \\ u_1 \\ u_2 \\ \vdots \\ u_n \end{array} \begin{bmatrix} v_1 & v_2 & \cdots & v_n & v & u & u_1 & u_2 & \cdots & u_n \\ 0 & 2 & \cdots & 2 & 1 & 1 & 2 & 2 & \cdots & 2 \\ 2 & 0 & \cdots & 2 & 1 & 1 & 2 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 0 & 1 & 1 & 2 & 2 & \cdots & 2 \\ 1 & 1 & \cdots & 1 & 0 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & 1 & 0 & 1 & 1 & \cdots & 1 \\ 2 & 2 & \cdots & 2 & 1 & 1 & 0 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 & 1 & 1 & 2 & 0 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 2 & 1 & 1 & 2 & 2 & \cdots & 0 \end{bmatrix}$$

Note that the characteristic polynomial of $\varepsilon\left(B_{n,n}^2\right)$ is $(\lambda+1)(\lambda+2)^{2n-1}\left(\lambda^2-(4n-1)\lambda-2\right)$.
So, Eccentricity Eigen values are $-1,-2,-2,\ldots\ldots,-2$ $((2n-1)\ \ times)$, $\frac{1}{2}\left((4n-1)+\sqrt{(4n-1)^2+8}\right)$ and $\frac{1}{2}\left((4n-1)-\sqrt{(4n-1)^2+8}\right)$.
Hence, Eccentricity energy of $B_{n,n}^2 = E\left(B_{n,n}^2\right) = |-1|+(2n-1)|-2|+\frac{1}{2}\left|(4n-1)+\sqrt{(4n-1)^2+8}\right|+\frac{1}{2}\left|(4n-1)-\sqrt{(4n-1)^2+8}\right|$
$= 1+4n-2+\frac{1}{2}(4n-1)+\sqrt{(4n-1)^2+8}\ +\frac{1}{2}\sqrt{(4n-1)^2+8-(4n-1)}$
$= 4n+\sqrt{(4n-1)^2+8}-1$ □

Example 2.4. *Eccentricity energy of square of Bistar graph $B_{5,5}^2$ is $19+\sqrt{369}$.*

Proof.

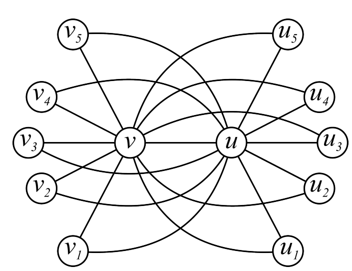

Figure 4

The eccentricity matrix of $B_{5,5}^2$ is given by

Note that the characteristic polynomial of $\varepsilon\left(B_{n,n}^2\right)$ is $(\lambda+1)(\lambda+2)^9\left(\lambda^2-19\lambda-2\right)$.
So, the Eccentricity Eigen values of $B_{5,5}^2$ are

$$E\left(B_{5,5}^2\right)=\begin{array}{c}v_1\\v_2\\v_3\\v_4\\v_5\\v\\u\\u_1\\u_2\\u_3\\u_4\\u_5\end{array}\begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5\\ 0&2&2&2&2&1&1&2&2&2&2&2\\ 2&0&2&2&2&1&1&2&2&2&2&2\\ 2&2&0&2&2&1&1&2&2&2&2&2\\ 2&2&2&0&2&1&1&2&2&2&2&2\\ 2&2&2&2&0&1&1&2&2&2&2&2\\ 1&1&1&1&1&0&1&1&1&1&1&1\\ 1&1&1&1&1&1&0&1&1&1&1&1\\ 2&2&2&2&2&1&1&0&2&2&2&2\\ 2&2&2&2&2&1&1&2&0&2&2&2\\ 2&2&2&2&2&1&1&2&2&0&2&2\\ 2&2&2&2&2&1&1&2&2&2&0&2\\ 2&2&2&2&2&1&1&2&2&2&2&0 \end{bmatrix}$$

$-1,-2,-2,\ldots\ldots,-2$ (9 *times*), $\frac{1}{2}\left(19+\sqrt{369}\right)$ and $\frac{1}{2}\left(19-\sqrt{369}\right)$.

Hence, Eccentricity energy of $B_{n,n}^2 = E\left(B_{n,n}^2\right) = |-1| + 9|-2| + \left|\frac{1}{2}\left(19+\sqrt{369}\right)\right| + \left|\frac{1}{2}\left(19-\sqrt{369}\right)\right|$
$= 19+\sqrt{369}$ □

Theorem 2.5. *Let $n \in N$, $n \neq 1$. Then $E\left(D_2\left(B_{n,n}\right)\right) = 8n-2$, where $E\left(D_2\left(B_{n,n}\right)\right)$is the Eccentricity energy of graph $D_2\left(B_{n,n}\right)$.*

Proof. Let $V\left(D_2\left(B_{n,n}\right)\right) = \{v_1, v_2, \ldots, v_n, v, u,\ u_1, u_2, \ldots u_n\}$. Note that $D_2\left(B_{n,n}\right)$is graph with $4(n+1)$ vertices and $4(2n+1)$ edges as shown in the following Figure 5. Observe that the distance matrix $D\left(D_2\left(B_{n,n}\right)\right)$and Eccentricity matrix $\varepsilon\left(D_2\left(B_{n,n}\right)\right)$ of $D_2\left(B_{n,n}\right)$ are given by

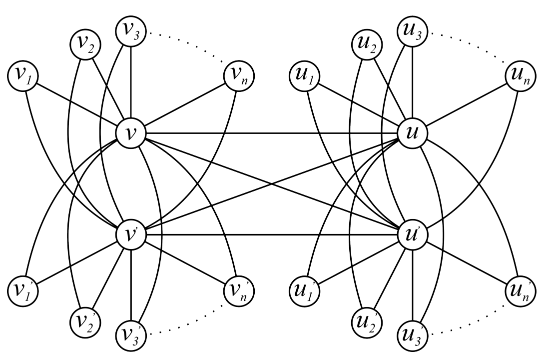

Figure 5

$$D\left(D_2\left(B_{n,n}\right)\right)=\begin{array}{c}v_1\\v_2\\\vdots\\v_n\\v_1'\\v_2'\\\vdots\\v_n'\\v\\v'\\u\\u'\\u_1\\u_2\\\vdots\\u_n\\u_1'\\u_2'\\\vdots\\u_n'\end{array}\begin{bmatrix} v_1 & v_2 & \ldots & v_n & v_1 & u_2 & \ldots & u_n & v & v & u & u & u_1 & u_2 & \ldots & u_n & u_1 & u_2 & \ldots. & u_n\\ 0&2&\ldots&2&2&2&\ldots&2&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 2&0&\ldots&2&2&2&\ldots&2&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 2&2&\ldots&0&2&2&\ldots&2&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 2&2&\ldots&2&0&2&\ldots&2&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 2&2&\ldots&2&2&0&\ldots&2&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 2&2&\ldots&2&2&2&\ldots&0&1&1&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 1&1&\ldots&1&1&1&\ldots&1&0&2&1&1&2&2&\ldots&2&2&2&\ldots&2\\ 1&1&\ldots&1&1&1&\ldots&1&2&0&2&1&2&2&\ldots&2&2&2&\ldots&2\\ 2&2&\ldots&2&2&2&\ldots&2&1&1&0&2&1&1&\ldots&1&1&1&\ldots&1\\ 2&2&\ldots&2&2&2&\ldots&2&1&1&2&0&1&1&\ldots&1&1&1&\ldots&1\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&0&2&\ldots&2&2&2&\ldots&2\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&2&0&\ldots&2&2&2&\ldots&2\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&2&2&\ldots&0&2&2&\ldots&2\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&2&2&\ldots&2&2&2&\ldots&2\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&2&2&\ldots&2&2&0&\ldots&2\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&1&1&2&2&\ldots&2&2&2&\ldots&0 \end{bmatrix}$$

and

$$\varepsilon\left(D_2\left(B_{n,n}\right)\right)=\begin{array}{c}v_1\\v_2\\\vdots\\v_n\\v_1'\\v_2'\\\vdots\\v_n'\\v\\v'\\u\\u'\\u_1\\u_2\\\vdots\\u_n\\u_1'\\u_2'\\\vdots\\u_n'\end{array}\begin{bmatrix} v_1 & v_2 & \ldots & v_n & v_1' & u_2' & \ldots & u_n' & v & v' & u & u' & u_1 & u_2 & \ldots & u_n & u_1' & u_2' & \ldots. & u_n'\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&\ldots&0&0&0&\ldots&0&0&0&2&2&3&3&\ldots&3&3&3&\ldots&3\\ 0&0&\ldots&0&0&0&\ldots&0&0&2&0&0&2&2&\ldots&2&2&2&\ldots&2\\ 0&0&\ldots&0&0&0&\ldots&0&2&0&2&0&2&2&\ldots&2&2&2&\ldots&2\\ 2&2&\ldots&2&2&2&\ldots&2&0&0&0&2&0&0&\ldots&0&0&0&\ldots&0\\ 2&2&\ldots&2&2&2&\ldots&2&0&0&2&0&0&0&\ldots&0&0&0&\ldots&0\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 3&3&\ldots&3&3&3&\ldots&3&2&2&0&0&0&0&\ldots&0&0&0&\ldots&0 \end{bmatrix}$$

Note that the characteristic polynomial of $\varepsilon\left(D_2\left(B_{n,n}\right)\right)$ is

$$\lambda^{2(n-1)}(\lambda+2)^2\left(\lambda^2+2\lambda-2\lambda n-28n\right)\left(\lambda^2-2\lambda-6\lambda n-4n\right).$$

So, Eccentricity Eigen values of $D\left(D_2\left(B_{n,n}\right)\right)$are $0,0,\ldots,0\,(2(2n-1)\ \textit{times}),-2,-2,$
$\left((3n+1)+\sqrt{(3n+2)^2-(2n+3)}\right)$,
$\left((3n+1)-\sqrt{(3n+2)^2-(2n+3)}\right)$,
$\left((1-3n)+\sqrt{(3n+2)^2-(10n-3)}\right)$,
$\left((1-3n)-\sqrt{(3n+2)^2-(10n-3)}\right)$.

Hence, Eccentricity energy of $D_2\left(B_{n,n}\right) = E\left(D_2\left(B_{n,n}\right)\right) = 0+2|-2|+$

$$\left|\left((3n+1)+\sqrt{(3n+2)^2-(2n+3)}\right)\right|+\left|\left((3n+1)-\sqrt{(3n+2)^2-(2n+3)}\right)\right|+\left|\left((1-3n)+\sqrt{(3n+2)^2-(10n3)}\right)\right|+\left|\left((1-3n)-\sqrt{(3n+2)^2-(10n-3)}\right)\right|$$

$$=4+\left((3n+1)+\sqrt{(3n+2)^2-(2n+3)}\right)+\left(\sqrt{(3n+2)^2-(2n+3)-(3n+1)}\right)+\left((1-3n)+\ \sqrt{(3n+2)^2+10n-3}\right)+\left(\sqrt{(3n+2)^2+10n-3}-(1-3n)\right)$$

$$=2\left(2+\sqrt{(3n+2)^2-(2n+3)}+\sqrt{(3n+2)^2+10n-3}\right)$$

□

Example 2.6. *Eccentricity energy of shadow graph of Bistar graph $D_2\left(B_{5,5}\right)$ is $2\left(2+\sqrt{276}+\sqrt{336}\right)$.*

Proof.

The Eccentricity matrix of $D_2\left(B_{5,5}\right)$ is given by

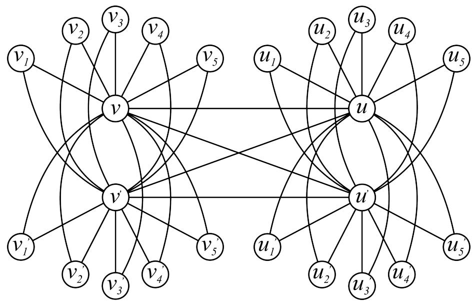

Figure 6

$$\varepsilon\left(D_2\left(B_{5,5}\right)\right)=\begin{array}{c|cccccccccccccccccccccccc|}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v'_1 & v'_2 & v'_3 & v'_4 & v'_5 & v & v' & u & u' & u_1 & u_2 & u_3 & u_4 & u_5 & u'_1 & u'_2 & u'_3 & u'_4 & u'_5 \\
v_1 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v_2 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v_3 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v_4 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v_5 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v'_1 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v'_2 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v'_3 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v'_4 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v'_5 & 0&0&0&0&0&0&0&0&0&0&0&0&2&2&3&3&3&3&3&3&3&3&3&3 \\
v & 0&0&0&0&0&0&0&0&0&0&0&2&0&0&2&2&2&2&2&2&2&2&2&2 \\
v' & 0&0&0&0&0&0&0&0&0&0&2&0&0&0&2&2&2&2&2&2&2&2&2&2 \\
u & 2&2&2&2&2&2&2&2&2&2&0&0&0&2&0&0&0&0&0&0&0&0&0&0 \\
u' & 2&2&2&2&2&2&2&2&2&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0 \\
u_1 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u_2 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u_3 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u_4 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u_5 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u'_1 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u'_2 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u'_3 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u'_4 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0 \\
u'_5 & 3&3&3&3&3&3&3&3&3&3&2&2&0&0&0&0&0&0&0&0&0&0&0&0
\end{array}$$

The characteristics polynomial of $\varepsilon\left(D_2\left(B_{5,5}\right)\right)$ is $\lambda^8(\lambda+2)^2\left(\lambda^2+2\lambda-10\lambda-140\right)\left(\lambda^2-2\lambda-30\lambda-20\right)$.

Hence, Eccentricity Eigen values of $D_2\left(B_{5,5}\right)$ *are* $0,0,\ldots,0$ $(19\ \textit{times})$, $-2,-2,\ \left(16+\sqrt{276}\right),\left(16-\sqrt{276}\right)$, $\left(-14+\sqrt{336}\right)$ and $\left(-14-\sqrt{336}\right)$.

Therefore, Eccentricity energy$=E\left(D_2\left(B_{5,5}\right)\right)=0+2|-2|+\left|16+\sqrt{276}\right|+\left|16-\sqrt{276}\right|+\left|-14+\sqrt{336}\right|+\left|-14-\sqrt{336}\right|$
$=2\left(2+\sqrt{276}+\sqrt{336}\right)$ □

References

[1] C. Adiaga and M. Smitha, On maximum degree energy of a graph, *Int. J. Contemp. Math. Sciences*, 4(8)(2009), 385–396.

[2] R. Balakrishnan and K. Ranganathan, *A Textbook of Graph Theory*, Springer, 2000.

[3] J.A. Bondy and U.R. Murty, *Graph Theory*, Springer, Berlin, 2008.

[4] G.V. Ghodasara and M.J. Patel, Some Bistar related square sum graphs, *IJMTT*, (2017), 172-177.

[5] I. Gutman, The energy of a graph, *Ber. Math-Statist. Sekt. Forschungz. Graz*, 103 (1978), 1-22.

[6] N. Prabhavathy, A new concept of energy from eccentricity matrix of graphs, *Malaya Journal of Matematik*, (2019), 400-402.

[7] Ahmed M. Naji and N.D. Soner, Maximum eccentricity energy of a graph, *IJSER*, 7(5) (2016), 1–12.

[8] S. K. Vaidya and K. M. Popat, Some new results on energy of graphs, *Math. Comput. Chem.*, 77 (2017), 589-594.

[9] S. K. Vaidya and K. M. Popat, Energy of m- Splitting and m-Shadow Graphs, *Far East Journal of Mathematical Sciences*, 102(2017), 1571-1578.

[10] S. K. Vaidya and N.H. Shah, some Star and Bistar Related Divisor Cordial Graphs, *Annals of Pure and Applied Mathematics*, (2013), 67-77.

[11] Jianfeng Wang, Mei Lu, Fransco Belardo and Milan Randic, The anti-adjacency matrix of a graph: Eccentricity matrix, *Discrete Applied Mathematics*, 251(2018), 299-309.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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