Bipolar soft neutrosophic topological region

G. Upender Reddy¹*, T. Siva Nageswara Rao², N. Srinivasa Rao³ and V. Venkateswara Rao⁴

Abstract
In this article deals, different areas with uncertainty data information in bipolar soft neutrosophic topology. In the past time, so many authors are discussed about neutrosophic and bipolar neutrosophic theory. Soft neutrosophic Set theory was derived by Maji. The present article extended to bipolar soft spatial region. Also we obtained definitions of Soft open, soft closed, soft pre-open, soft pre- closed on the bipolar neutrosophic.

Keywords
Soft neutrosophic set; bipolar soft neutrosophic topology; bipolar soft neutrosophic spatial areas.

AMS Subject Classification
11B05.

1. Introduction
Everywhere in the world uncertainty situations are there in each case. In particularly mathematics there are different fields with uncertainty problems. Especially Fuzzy theory [14] and Intuitionist fuzzy theory [1] authors find out different problems deal with uncertainty. By overcome this uncertainty, Smarandache [8] derived neutrosophic theory. Maji [3] collective the two topics soft sets and neutrosophic theory. The author also have the some more research work on neutrosophic theory see the references [2, 4, 5, 6, 7, 9, 10, 11, 12, 13].

Notations:
1. Bipolar Soft Neutrosophic (BSN)
2. Bipolar soft Neutrosophic set (BSNS)

3. Bipolar soft neutrosophic pre-open sets (BSNPOS)
4. Bipolar soft neutrosophic pre-closed sets (BSNPCS).
5. Bipolar soft Neutrosophic topology (BSNTP)
6. Bipolar soft neutrosophic closure (BSNT)
7. Bipolar soft neutrosophic interior (BSNI)
8. Bipolar soft neutrosophic open set (BSNOS)
9. Bipolar soft neutrosophic closed set (BSNCS)
10. Bipolar soft neutrosophic semi-closed (BSNSC)
11. Bipolar Soft neutrosophic semi open (BSNSO)

This article based on the soft neutrosophic topology. Here we start with some basic definitions.

2. Preliminaries
In this section, we recall some definitions and basic results of fractional calculus which will be used throughout the paper.

Definition 2.1. \((W, Z)\) is a soft set in \(\Omega\) where \(W : Z \rightarrow \tilde{P}(\Psi)\) is a mapping where \(\tilde{P}(\Psi)\) is a power set of \(\Psi\).
We express \((W, Z)\) by \(\tilde{W} \tilde{Z} = \{(f, W(f)) : f \in Z\}\).
Definition 2.2. A bipolar neutrosophic set B on Ψ is defined as:

\[ B = \{ \langle \overline{z}, \varepsilon_{BN}(z), \phi_{BN}(z), \varepsilon_{BN}(z), \phi_{BN}(z), \varepsilon_{BP}(z), \phi_{BP}(z), \varepsilon_{BP}(z), \phi_{BP}(z) \rangle : \overline{z} \in \Psi \} \]

where

\[ \varepsilon_{BP}, \phi_{BP}, \varepsilon_{BP} : \Psi \rightarrow [0, 1] \quad \text{and} \]

\[ \varepsilon_{BN}, \phi_{BN}, \varepsilon_{BN} : \Psi \rightarrow [0, 1] \quad \text{and} \]

\[ -3 \leq \varepsilon_{BN}(z) + \phi_{BN}(z) + \varepsilon_{BP}(z) + \phi_{BP}(z) + \phi_{BP}(z) \leq 3 \]

Definition 2.3. Let Ψ be the set and Z be parameter set.

Let \( \overline{P}(\Psi) \) represent the set of all BSNS of \( \Psi \).

Then \( (W, Z) \) is known as BSNS over \( \Psi \) where \( W : Z \rightarrow \overline{P}(\Psi) \) is a mapping.

We express the BSNS \( (W, Z) \) by \( \overline{W}_{Na} \).

That is, \( \overline{W}_{Na} = \{ \langle f, \{ \langle z, \varepsilon_{WN}(z), \phi_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z) \rangle : \overline{z} \in \Psi \} \} \}

Definition 2.4. The complement of the BSNS \( \overline{W}_{Na} \) is represented by \( \overline{W}_{Na}^{C} \) and is defined by \( \overline{W}_{Na}^{C} = \{ \langle f, \{ \langle z, \varepsilon_{WN}(z), \phi_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z), \varepsilon_{WN}(z), \phi_{WN}(z) \rangle : \overline{z} \in \Psi \} \} \}

Definition 2.5. For any two BSNS \( \overline{W}_{Na} \) and \( \overline{S}_{Na} \) over \( \Psi \), \( \overline{W}_{Na} \) is a BSNS subset of \( \overline{S}_{Na} \) if

\[ \varepsilon_{WN}(z) \leq \varepsilon_{SN}(z) \quad \text{and} \quad \phi_{WN}(z) \leq \phi_{SN}(z) \]

\[ \phi_{WN}(z) \leq \phi_{SN}(z) \quad \text{and} \quad \phi_{WN}(z) \leq \phi_{SN}(z) \]

for all \( z \in \Psi \).

It is denoted by \( \overline{W}_{Na} \subseteq \overline{S}_{Na} \).

Definition 2.6. A BSNS \( \overline{W}_{Na} \) over \( \Psi \) is said to be null BSNS if

\[ \varepsilon_{WN}(z) = 0 \quad \text{and} \quad \phi_{WN}(z) = 0 \]

\[ \phi_{WN}(z) = 0 \quad \text{and} \quad \phi_{WN}(z) = 1 \]

\[ \varepsilon_{WN}(z) = 0 \quad \text{and} \quad \phi_{WN}(z) = 0 \]

for all \( z \in \Psi \).

It is represented by \( \overline{W}_{Na} = \emptyset \).

Definition 2.7. A BSNS \( \overline{W}_{Na} \) over \( \Psi \) is said to be absolute BSNS if

\[ \varepsilon_{WN}(z) = 1 \quad \text{and} \quad \phi_{WN}(z) = 1 \]

\[ \phi_{WN}(z) = 1 \quad \text{and} \quad \phi_{WN}(z) = 1 \]

\[ \varepsilon_{WN}(z) = 0 \quad \text{and} \quad \phi_{WN}(z) = 0 \]

for all \( z \in \Psi \).

It is represented by \( \overline{W}_{Na} = \emptyset \).

Definition 2.8. The disjunction of two BSNS \( \overline{W}_{Na} \) and \( \overline{S}_{Na} \) is represented by \( \overline{W}_{Na} \cup \overline{S}_{Na} \) and is defined by \( \overline{U}_{Na} = \overline{W}_{Na} \cup \overline{S}_{Na} \) as follows

\[ \varepsilon_{U_{Na}}(z) = \max \{ \varepsilon_{WN}(z), \varepsilon_{SN}(z) \} \quad \text{if} \quad f \in B \cap C \]

\[ \phi_{U_{Na}}(z) = \max \{ \phi_{WN}(z), \phi_{SN}(z) \} \quad \text{if} \quad f \in B \cap C \]

Definition 2.9. The conjunction of two BSNS \( \overline{W}_{Na} \) and \( \overline{S}_{Na} \) is represented by \( \overline{W}_{Na} \cap \overline{S}_{Na} \) and is defined by \( \overline{V}_{Na} = \overline{W}_{Na} \cap \overline{S}_{Na} \), as follows

\[ \varepsilon_{V_{Na}}(z) = \min \{ \varepsilon_{WN}(z), \varepsilon_{SN}(z) \} \quad \text{if} \quad f \in B \cap C \]

\[ \phi_{V_{Na}}(z) = \max \{ \phi_{WN}(z), \phi_{SN}(z) \} \quad \text{if} \quad f \in B \cap C \]

3. Bipolar soft neutrosophic topological space

Definition 3.1. Let BSNS(\( \Psi, Z \)) be the family of all BNSS over \( Z \) and \( \overline{N}_{BST} \subseteq \text{BSNS}(\Psi, Z) \). Then \( \overline{N}_{BST} \) is known as bipolar soft neutrosophic topology (BSNT) on \( (\Psi, Z) \) if the subsequent circumstances are satisfied

(i) \( \overline{W}_{Na} \subseteq \overline{N}_{BST} \)

(ii) \( \overline{N}_{BST} \) is closed under arbitrary disjunction.

(iii) \( \overline{N}_{BST} \) is closed under infinite conjunction.

Then the triplet \( (\Psi, \overline{N}_{BST}, Z) \) is known as BSNT space.

The elements of \( \overline{N}_{BST} \) are known BSNS in \( (\Psi, \overline{N}_{BST}, Z) \). A BSNS \( \overline{W}_{BNu} \) in \( \text{BSNS}(\Psi, Z) \) is soft closed in \( (\Psi, \overline{N}_{BST}, Z) \) if its complement \( \overline{W}_{BNu}^{C} \) is BSNS in \( (\Psi, \overline{N}_{BST}, Z) \).

The BSN closure of \( \overline{W}_{BNu} \) is the BSNS \( \overline{B}_{Nu} \approx \text{SCL}(\overline{W}_{BNu}) \subseteq \overline{N}_{BST} \).

The BSN interior of \( \overline{W}_{BNu} \) is the BNSS \( \overline{B}_{Nu} \approx \text{SINT}(\overline{W}_{BNu}) \subseteq \overline{N}_{BST} \).

It is easy to see that

\[ \overline{W}_{BNu} \approx \text{SINT}(\overline{W}_{BNu}) \text{ and } \overline{B}_{Nu} \approx \text{SCL}(\overline{W}_{BNu}) \]

Theorem 3.2. Let \( (\Psi, \overline{N}_{BST}, Z) \) be a BSNTS over \( (\Psi, Z) \) and \( \overline{W}_{BNu} \) and \( \overline{S}_{BNu} \in \overline{N}_{BST} \). Then

(i) \( \overline{W}_{BNu} \approx \text{SINT}(\overline{W}_{BNu}) \text{ and } \overline{B}_{Nu} \approx \text{SCL}(\overline{W}_{BNu}) \)

(ii) \( \overline{W}_{BNu} \cap \overline{S}_{BNu} \in \overline{N}_{BST} \approx \text{SINT}(\overline{W}_{BNu}) \)

(iii) \( \overline{W}_{BNu} \cap \overline{S}_{BNu} \approx \text{SINT}(\overline{W}_{BNu}) \) is an BNSS.

That is \( \overline{W}_{BNu} \approx \text{SINT}(\overline{W}_{BNu}) \in \overline{N}_{BST} \).
(iv) $\tilde{W}_{\mathcal{B}N_u}$ is BSNO $\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}) = \tilde{W}_{\mathcal{B}N_u}$
(v) $\mathcal{B}N_u \approx SINT (\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}))$

$$\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u})$$

(vi) $\mathcal{B}N_u \approx SINT (\tilde{\Phi}_{\mathcal{B}N_u}) = \tilde{\Phi}_{\mathcal{B}N_u} \cdot \mathcal{B}N_u \approx SINT (\tilde{\Psi}_{\mathcal{B}N_u})$

$$\mathcal{B}N_u \approx SINT (\tilde{\mathcal{B}N}_u) \cup \mathcal{B}N_u \approx SINT (\tilde{S}_{\mathcal{B}N_u})$$

$$\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u} \cup \tilde{S}_{\mathcal{B}N_u})$$

Theorem 3.3. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS and $\tilde{W}_{\mathcal{B}N_u}$ and $\tilde{S}_{\mathcal{B}N_u} \in (\Psi, Z)$ then

(i) $\tilde{W}_{\mathcal{B}N_u} \subset \mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$ and $\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$

(ii) $\tilde{W}_{\mathcal{B}N_u} \subset \tilde{W}_{\mathcal{B}N_u}$ implies $\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$

(iii) $\tilde{W}_{\mathcal{B}N_u} \approx SCL (\tilde{W}_{\mathcal{B}N_u})$

That is $\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$ is a BSNCS.

4. Bipolar soft neutrosophic nearly open sets

Definition 4.1. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS and $\tilde{W}_{\mathcal{B}N_u}$ be a BSNO in $(\Psi, Z)$, then $\tilde{W}_{\mathcal{B}N_u}$ is known as

(i) Bipolar soft neutrosophic $\alpha$-open $\Leftrightarrow$

$$\tilde{W}_{\mathcal{B}N_u} \subseteq \mathcal{B}N_u \approx SINT (\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u}))$$

(ii) Bipolar soft neutrosophic pre-$\alpha$-open $\Leftrightarrow$

$$\tilde{W}_{\mathcal{B}N_u} \subseteq \mathcal{B}N_u \approx SINT (\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u}))$$

(iii) Bipolar soft neutrosophic semi-$\alpha$-open $\Leftrightarrow$

$$\tilde{W}_{\mathcal{B}N_u} \subseteq \mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$$

(iv) Bipolar soft neutrosophic $\beta$-open $\Leftrightarrow$

$$\tilde{W}_{\mathcal{B}N_u} \subseteq \mathcal{B}N_u \approx SCL (\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u}))$$

(v) Bipolar soft neutrosophic regular-$\alpha$-closed $\Leftrightarrow$

$$\tilde{W}_{\mathcal{B}N_u} = \mathcal{B}N_u \approx SCL (\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}))$$

5. Bipolar soft neutrosophic region

Topology deals with surface area study in that analysis of Geographical information systems (GIS) and Geospatial databases. There is a lot of problems on the uncertainty on the regions. Further, go through the some definitions and proposals for a BSNT region, which supply a hypothetical structure for the modeling of BSNT relations surrounded by uncertain regions.

Definition 5.1. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS over $(\Psi, Z)$ and $\tilde{W}_{\mathcal{B}N_u} \in \text{BSNS}(\Psi, Z)$. Then BSN boundary of $\tilde{W}_{\mathcal{B}N_u}$ is defined by

$$\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}) = \tilde{W}_{\mathcal{B}N_u}$$

Definition 5.2. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS over $(\Psi, Z)$. Then the BSN exterior of $\tilde{W}_{\mathcal{B}N_u} \in \text{BSNS}(\Psi, Z)$ is represented by $(\tilde{W}_{\mathcal{B}N_u})^{ext}$ and is defined by

$$\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u})$$

Theorem 5.3. Let $\tilde{W}_{\mathcal{B}N_u}$ and $\tilde{S}_{\mathcal{B}N_u}$ be two BSNS over $(\Psi, Z)$.

Then

(i) $(\tilde{W}_{\mathcal{B}N_u})^{ext} = \mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u})$

(ii) $(\tilde{W}_{\mathcal{B}N_u} \cup \tilde{S}_{\mathcal{B}N_u})^{ext} = (\tilde{W}_{\mathcal{B}N_u})^{ext} \cap (\tilde{S}_{\mathcal{B}N_u})^{ext}$

(iii) $(\tilde{W}_{\mathcal{B}N_u}^{ext}) \cup (\tilde{S}_{\mathcal{B}N_u}^{ext}) \subset (\tilde{W}_{\mathcal{B}N_u} \cup \tilde{S}_{\mathcal{B}N_u})^{ext}$

Theorem 5.4. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS over $(\Psi, Z)$ and $\tilde{W}_{\mathcal{B}N_u}, \tilde{S}_{\mathcal{B}N_u} \in \text{BSNS}(\Psi, Z)$.

Then

(i) $(\tilde{W}_{\mathcal{B}N_u})^{C} = \mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u})$

(ii) $\mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u} \cup \mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}))$

(iii) $3 \mathcal{W}_{\mathcal{B}N_u} = \mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u}) \cap \mathcal{B}N_u \approx SCL (\tilde{W}_{\mathcal{B}N_u})$

(iv) $3 \mathcal{W}_{\mathcal{B}N_u} \cap \mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}) = \tilde{\Phi}_{\mathcal{B}N_u}$

(v) $3 (\tilde{W}_{\mathcal{B}N_u})^{C} = 3 (\mathcal{W}_{\mathcal{B}N_u})^{C}$

Definition 5.5. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS over $(\Psi, Z)$. Then a couple of non-empty BSNOs are $\tilde{W}_{\mathcal{B}N_u}, \tilde{S}_{\mathcal{B}N_u}$ is known as a BSN separation of $(\Psi, \tilde{N}_{BST+}, Z)$ if

$$\tilde{W}_{\mathcal{B}N_u} = \tilde{W}_{\mathcal{B}N_u} \cup \tilde{S}_{\mathcal{B}N_u} \text{ and }\tilde{W}_{\mathcal{B}N_u} \cap \tilde{S}_{\mathcal{B}N_u} = \tilde{\Phi}_{\mathcal{B}N_u}$

Definition 5.6. A BSNTS $(\Psi, \tilde{N}_{BST+}, Z)$ is known as BSN connected if there does not present a BSN separation of $(\Psi, \tilde{N}_{BST+}, Z)$.

Otherwise $(\Psi, \tilde{N}_{BST+}, Z)$ is known as BSN disconnected.

Next, we go through a model for spatial BSN region based on BSN connectedness.

Definition 5.7. Let $(\Psi, \tilde{N}_{BST+}, Z)$ be a BSNTS. A spatial BSN region in $(\Psi, Z)$ is a non empty BSN subset $\tilde{W}_{\mathcal{B}N_u}$ such that

(i) $\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u})$ is BSN connected.

(ii) $\tilde{W}_{\mathcal{B}N_u} = \mathcal{B}N_u \approx SCL (\mathcal{B}N_u \approx SINT (\tilde{W}_{\mathcal{B}N_u}))$
6. Conclusion

In this article, Bipolar soft neutrosophic topological region explained on soft open, soft closed, soft pre-open and soft pre-closed on the bipolar neutrosophic theory. We discussed about some basic definitions about neutrosophic topological space, bipolar soft neutrosophic set etc... Further we obtained the results based on soft open and soft closed with similar results soft pre-open and soft pre-closed sets on topological region.

References


