On \(K\)-eccentric and \(K\)-hyper eccentric indices of Benzenoid \(H_k\) system

M. Bhanumathi\(^1\), R. Rohini\(^2\) and G. Srividhya\(^3\)

Abstract
Let \(G\) be a connected graph with vertex set \(V(G)\) and edge set \(E(G)\). Bhanumathi and Easu Julia Rani introduced the first \(K\) - Eccentric index \(B_1E(G)\) and the second \(K\) - Eccentric index \(B_2E(G)\) of a graph \(G\) as \(B_1E(G) = \sum_{ue} [e_G(u) + e_{L(G)}(e)]\), \(B_2E(G) = \sum_{ue} [e_G(u)e_{L(G)}(e)]\). They also defined the first \(K\) -Hyper eccentric index \(HB_1E(G)\) and the second \(K\) -Hyper eccentric index \(HB_2E(G)\) of a graph \(G\) as \(HB_1E(G) = \sum_{ue} [e_G(u) + e_{L(G)}(e)]^2\), \(HB_2E(G) = \sum_{ue} [e_G(u)e_{L(G)}(e)]^2\) where in all the cases \(ue\) means that the vertex \(u\) and edge \(e\) are incident in \(G\) and \(e_{L(G)}(e)\) is the eccentricity of \(e\) in the line graph \(L(G)\) of \(G\). They have defined the multiplicative version of these indices also. In this paper, we calculate the first and second \(K\) eccentric, the first and second \(K\)-hyper eccentric indices and their multiplicative versions of benzenoid \(H_k\) system.

Keywords
\(K\)-eccentric index, \(K\)-hyper eccentric index, Multiplicative \(K\)-eccentric index, Multiplicative \(K\)-hyper eccentric index, Circo.

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1. Introduction
A topological index is a real number associated with chemical constitution. It correlates the chemical structure with various physical and chemical properties and biological activity. All graphs in this paper are simple, finite and undirected. A graph \(G\) is a finite nonempty set \(V(G)\) together with a prescribed set \(E(G)\) of unordered pair of distinct elements of \(V\). The cardinality of \(V(G)\) and \(E(G)\) are represented by \(|V(G)|\) and \(|E(G)|\), respectively. Let, \(d_G(v)\) be the degree of a vertex \(v\) of \(G\) and \(N_G(v)\) be the neighborhood of a vertex \(v\) of \(G\). The distance between the vertices \(u\) and \(v\) of a connected graph \(G\) is represented by \(d_G(u, v)\). It is defined as the number of edges in a shortest path connects the vertices \(u\) and \(v\). The eccentricity \(e_G(v)\) of a vertex \(v\) in \(G\) is the largest distance between \(v\) and any other vertices \(u\) of \(G\).

To take an account on contributions of pairs of incident elements, Kulli [5] introduced the first and second Banhatti indices. In [4], Bhanumathi and Easu Julia Rani introduced the first \(K\) - Eccentric index \(B_1E(G)\) and the second \(K\) - Eccentric index \(B_2E(G)\) of a graph \(G\) as

\[B_1E(G) = \sum_{ue} [e_G(u) + e_{L(G)}(e)]\]
\[B_2E(G) = \sum_{ue} [e_G(u)e_{L(G)}(e)]\]

and also defined the first \(K\)-Hyper eccentric index \(HB_1E(G)\) and the second \(K\)-Hyper eccentric index \(HB_2E(G)\) of a graph \(G\) as

\[HB_1E(G) = \sum_{ue} [e_G(u) + e_{L(G)}(e)]^2\]
\[HB_2E(G) = \sum_{ue} [e_G(u)e_{L(G)}(e)]^2\]

where in all the cases \(ue\) means that the vertex \(u\) and edge \(e\) are incident in \(G\) and \(e_{L(G)}(e)\) is the eccentricity of \(e\) in the line graph \(L(G)\) of \(G\) [4].
2. First and second $K$-Eccentric indices, First and Second $K$-Hyper Eccentric indices of Benzenoid $H_k$ system:

The circumcoronene homologous series of benzenoid also belongs to the family of molecular graphs that has several copy of benzene $C_6$ on its circumference. The terms of this series are represented as, $H_1$-benzene, $H_2$-coronene, $H_3$-circumcoronene and $H_4$ circumcircumcoronene etc. A benzenoid system is a connected geometric figure. It is obtained by arranging congruent regular hexagons in a plane. Consequently two hexagons are either disjoint or have a common edge.

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The eccentricities of $u, v \in V(G)$ are denoted by $e(u), e(v)$ respectively and for $e = uv \in E(G)$, denote the eccentricities of the end vertices of the edge $e$ by $(e(u), e(v))$.

Let $V$ be the vertex set of $H_k$ and $E$ be the edge set in $H_k$, then $|V| = 6k^2$ and $|E| = 9k^2 - 3k$ for the structure of $H_k$. First, we shall determine the number of edges $e = uv$ with the eccentricity of the end vertices $e(u), e(v)$ and eccentricity of the edge $e$ in $L(G)$. We give these values in the following Table 1.

**Theorem 2.1.** For any positive integer number $k$, let $H_k$ be the general form of circumcoronene series of benzenoid system, then

\begin{align*}
(i) \quad B_1E(H_k) &= 6 \sum_{i=1}^{k} [8k + 4(2i - 1)] + 6 \sum_{i=1}^{k} [8k + 4(2i - 1) + 1] + 12 \sum_{i=1}^{k-1} [8k + 4(2i + 1)] \\
(ii) \quad B_2E(H_k) &= 6 \sum_{i=1}^{k} \left[ 2(2k + 2i - 1) \right] + 6 \sum_{i=1}^{k-1} [2(2k + 2i - 1) + 2k + 2i - 1] \\
&+ 12 \sum_{i=1}^{k-1} [2(2k + 2i - 1) + 2k + 2i - 1] \\
(iii) \quad HB_1E(H_k) &= 6 \sum_{i=1}^{k} \left[ 2(2k + 2i - 1) \right] + 6 \sum_{i=1}^{k-1} [2(2k + 2i - 1) + (2k + 2i) + (2k + 2i - 1)] \\
&+ 12 \sum_{i=1}^{k-1} [2(2k + 2i) + (2k + 2i) + (2k + 2i)]
\end{align*}

![Figure 1. The Circumcoronene homologous Series of Benzenoid $H_k(k \geq 1)$ with edges](image-url)
(iv) \( HB_2 E (H_k) = 6 \sum_{i=1}^{k} \left[ \left( (2k+2i-1)^2 + (2k+2i-1)^2 \right) \right] \)
+ \( 6 \sum_{i=1}^{k-1} \left[ \left( (2k+2i-1)^2 + ((2k+2i)(2k+2i-1)) \right) \right] \)
+ \( 12 \sum_{i=1}^{k-1} \left[ \left( (2k+2i)^2 + ((2k+2i)(2k+2i-1)) \right) \right] \)

**Proof.** Consider the General form of \( H_k \)-Circumcoronene graph.

(i) \( B_1 E (H_k) = \sum_{uv \in E(G)} \left[ e_{H(u)}(u) + e_{L(H)}(e) \right] = \sum_{uv \in E(G)} \left[ e_{G(u)}e_{L(G)}(e) + e_{G(v)}e_{L(G)}(e) \right] + \ldots \)
+ \( \sum_{uv \in E_{3k-1,1}(G)} \left[ e_{G(u)}e_{L(G)}(e) + e_{G(v)}e_{L(G)}(e) \right] \)

= \( 6 k \left[ (2k+2i-1)^2 \right] \)
+ \( 6 \sum_{i=1}^{k-1} \left[ (2k+2i-1)^2 + ((2k+2i)(2k+2i-1)) \right] \)
+ \( 12 \sum_{i=1}^{k-1} \left[ (2k+2i)^2 + ((2k+2i)(2k+2i-1)) \right] \)

(ii) \( B_2 E (H_k) = \sum_{uv \in E(G)} \left[ e_{H(u)}(u) \times e_{L(H)}(e) \right] = \sum_{uv \in E(G)} \left[ e_{G(u)}e_{L(G)}(e) + e_{G(v)}e_{L(G)}(e) \right] + \ldots \)

(iii) \( HB_1 E (H_k) = \sum_{uv \in E(G)} \left[ e_{H(u)}(u) + e_{L(H)}(e) \right] \)

= \( \sum_{uv \in E(G)} \left[ e_{G(u)}e_{L(G)}(e) + e_{G(v)}e_{L(G)}(e) \right] + \ldots \)
+ \( \sum_{uv \in E_{3k-1,1}(G)} \left[ e_{G(u)}e_{L(G)}(e) + e_{G(v)}e_{L(G)}(e) \right] \)

= \( 6 \sum_{i=1}^{k} \left[ (2k+2i-1)^2 + (2k+2i-1)^2 \right] \)
+ \( 6 \sum_{i=1}^{k-1} \left[ (2k+2i-1)^2 + ((2k+2i)(2k+2i-1)) \right] \)
+ \( 12 \sum_{i=1}^{k-1} \left[ (2k+2i)^2 + ((2k+2i)(2k+2i-1)) \right] \)

Let \( V \) be the vertex set and \( E \) be the edge set in \( H_4 \) = Circumcircumcoronene, then \( |V| = 96 \) and \( |E| = 132 \). Also, the number of edges with eccentricities of end vertices \( e = uv \in E(G) \) and \( e \in L(G) \) are given as follows:

<table>
<thead>
<tr>
<th>Edge set</th>
<th>No. of edges</th>
<th>Eccentricity of end vertices ( (e(u), e(v)) )</th>
<th>Eccentricity of ( e ) in ( L(G)e_{L(G)}(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>6</td>
<td>(9,9)</td>
<td>9</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>6</td>
<td>(9,10)</td>
<td>9</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>12</td>
<td>(10,11)</td>
<td>10</td>
</tr>
<tr>
<td>( E_4 )</td>
<td>6</td>
<td>(11,11)</td>
<td>11</td>
</tr>
<tr>
<td>( E_5 )</td>
<td>12</td>
<td>(11,12)</td>
<td>11</td>
</tr>
<tr>
<td>( E_6 )</td>
<td>24</td>
<td>(12,13)</td>
<td>12</td>
</tr>
<tr>
<td>( E_7 )</td>
<td>6</td>
<td>(13,13)</td>
<td>13</td>
</tr>
<tr>
<td>( E_8 )</td>
<td>18</td>
<td>(13,14)</td>
<td>13</td>
</tr>
<tr>
<td>( E_9 )</td>
<td>36</td>
<td>(14,15)</td>
<td>14</td>
</tr>
<tr>
<td>( E_{10} )</td>
<td>6</td>
<td>(15,15)</td>
<td>15</td>
</tr>
</tbody>
</table>

}\( \square \)
Corollary 2.3. \( B_1E(H_4) = \sum_{u \in E(G)} \left[e_{H}(u) + e_{L}(H_4)(e)\right] \)

\[
= \sum_{u \in E(G)} \left[e_{G}(u) + e_{L}(G)(e) + e_{L}(G)(e) + e_{L}(G)(e)\right] + \ldots
+ \sum_{u \in E(G)} \left[e_{G}(u) + e_{L}(G)(e) + e_{L}(G)(e)\right] = 6588
\]

(ii) \( B_2E(H_4) = \sum_{u \in E(G)} \left[e_{H}(u) \times e_{L}(H_4)(e)\right] \)

\[
= \sum_{e \in E(G)} \left[e_{G}(u)e_{L}(G)(e) + e_{G}(v)e_{L}(G)(e)\right] + \ldots
+ \sum_{e \in E(G)} \left[e_{G}(u)e_{L}(G)(e) + e_{G}(v)e_{L}(G)(e)\right] = 167580
\]

(iii) \( HB_1E(H_4) = \sum_{u \in E(G)} \left[e_{H}(u) + e_{L}(H_4)(e)\right]^2 \)

\[
= \sum_{e \in E(G)} \left[\left(e_{G}(u) + e_{L}(G)(e)\right)^2 + \left(e_{G}(v) + e_{L}(G)(e)\right)^2\right] + \ldots
+ \sum_{e \in E(G)} \left[\left(e_{G}(u) + e_{L}(G)(e)\right)^2 + \left(e_{G}(v) + e_{L}(G)(e)\right)^2\right] = 41868
\]

(iv) \( HB_2E(H_4) = \sum_{u \in E(G)} \left[e_{H}(u) \times e_{L}(H_4)(e)\right]^2 \)

\[
= \sum_{e \in E(G)} \left[\left(e_{G}(u) \times e_{L}(G)(e)\right)^2 + \left(e_{G}(v) \times e_{L}(G)(e)\right)^2\right] + \ldots
+ \sum_{e \in E(G)} \left[\left(e_{G}(u) \times e_{L}(G)(e)\right)^2 + \left(e_{G}(v) \times e_{L}(G)(e)\right)^2\right] = 7105236
\]

Thus we have \( B_1E(H_4) = 6588 \), \( B_2E(H_4) = 41868 \), \( HB_1E(H_4) = 167580 \) and \( HB_2E(H_4) = 7105236 \).

Corollary 2.2. \( H_1 \) be the first terms of this Circumcoronene series of Benzenoid \( H_k \). Then

(i) \( B_1E(H_1) = 72 \)

(ii) \( B_2E(H_1) = 108 \)

(iii) \( HB_1E(H_1) = 432 \)

(iv) \( HB_2E(H_1) = 972 \).

Corollary 2.3. \( H_2 \) be the second terms of this Circumcoronene series of Benzenoid \( H_k \). Then

(i) \( B_1E(H_2) = 714 \)

(ii) \( B_2E(H_2) = 2154 \)

(iii) \( HB_1E(H_2) = 8634 \)

(iv) \( HB_2E(H_2) = 82182 \)

Corollary 2.4. \( H_3 \) be the third terms of this Circumcoronene series of Benzenoid \( H_k \). Then

(i) \( B_1E(H_3) = 2646 \)

(ii) \( B_2E(H_3) = 12366 \)

(iii) \( HB_1E(H_3) = 49770 \)

(iv) \( HB_2E(H_3) = 1134150 \)

3. Multiplicative First and Second \( \kappa \)-Eccentric indices, Multiplicative First and Second \( \kappa \) Hyper Eccentric indices of Benzenoid \( H_k \) system:

Theorem 3.1. For any positive integer number \( k \), let \( H_k \) be the general form of circumcoronene series of benzenoid system, then

(i) \( B\Pi_1E(H_k) = 6 \prod_{i=1}^{k} \left[4(2k + 2i - 1)^2\right] \)

\[
\times 6 \prod_{i=1}^{k-1} \left[2(2k + 2i - 1)(4k + 4i - 1)\right]
\]

(ii) \( B\Pi_2E(H_k) = 6 \prod_{i=1}^{k} \left[(2k + 2i - 1)^4\right] \)

\[
\times 6 \prod_{i=1}^{k-1} \left[(2k + 2i - 1)^3(2k + 2i)\right]
\]

(iii) \( HB\Pi_1(H_k) = 6 \prod_{i=1}^{k} \left[16(2k + 2i - 1)^4\right] \)

\[
\times 6 \prod_{i=1}^{k-1} \left[(4(2k + 2i - 1))^2\right] \]

\[
\times \left[(2k + 2i) + (2k + 2i - 1))^2\right] \]

\[
\times 12 \prod_{i=1}^{k-1} \left[4(2k + 2i)^2 + [(2k + 2i + 1) + (2k + 2i)]^2\right]
\]

(iv) \( HB\Pi_2(H_k) = 6 \prod_{i=1}^{k} \left[(2k + 2i - 1)^8\right] \)

\[
\times 6 \prod_{i=1}^{k-1} \left[(2k + 2i - 1)^6\times[(2k + 2i)]\right]
\]

\[
\times 12 \prod_{i=1}^{k-1} \left[(2k + 2i)^6 \times [(2k + 2i + 1)]\right]
\]

**Proof.** Consider the General form of \( H_k - \) Circumcoronene...
Using MATLAB programme, we have calculated these in-


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Corollary 3.2. $H_1$ be the first terms of this Circumcoronene series of Benzene $H_k$. Then

(i) $BP_1E(H_1) = 2176782336$

(ii) $BP_2E(H_1) = 2.824295365 \times 10^{11}$

(iii) $HBP_1E(H_1) = 4.738381338 \times 10^{18}$

(iv) $HBP_2E(H_1) = 7.976644308 \times 10^{22}$

Corollary 3.3. $H_2$ be the second terms of this Circumcoronene series of Benzene $H_k$. Then

(i) $BP_1E(H_2) = 2.086352657 \times 10^{64}$

(ii) $BP_2E(H_2) = 2.901497086 \times 10^{92}$

(iii) $HBP_1E(H_2) = 3.1023e + 040$

(iv) $HBP_2E(H_2) = 8.4187e + 184$

Corollary 3.4. $H_3$ be the third terms of this Circumcoronene series of Benzene $H_k$. Then

(i) $BP_1E(H_3) = 1.3789e + 093$

(ii) $BP_2E(H_3) = 7.3558e + 234 \times 10^{23}$

(iii) $HBP_1E(H_3) = 1.9013e + 186$

(iv) $HBP_2E(H_3) = 5.4107e + 257 \times 5.7517e + 151$

4. Conclusion

In chemical graph theory a topological index of a molecular graph characterizes its topology. Here, we have computed the first, second K-eccentric indices, K-hyper eccentric indices and multiplicative first, second K-eccentric and K-hyper eccentric indices of benzenoid $H_k$ system.

References


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