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Repeated restricted Bursts error correcting linear codes Over $GF(q); q > 2$

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Abstract

This paper deals with non binary repeated restricted burst errors. In this paper lower and upper bounds on the number of parity check digits needed for a linear code having the capability to correct the repeated restricted bursts are presented. Restricted bursts are introduced by Tyagi and Lata [11] for non binary case over $GF(3)$. By a restricted burst of length l or less we mean a vector whose all the non zero components are confined to some l consecutive positions, the first and the last of which is nonzero with a restriction that all the non zero consecutive positions contain same field element. For example in non binary case for $q = 3, n = 3$ and $l = 2$, we have the following vectors of length 2 or less 110, 220, 011, 022, 100, 010, 001, 200, 020, 002.

Keywords

Restricted burst errors, burst correcting codes, burst error, repeated burst error.

AMS Subject Classification

94B20, 94B65, 94B25.

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1. Introduction

During very early stages in the history of coding theory codes were meant for detecting and correcting only random errors. But at a later stage it was observed that in almost all channels errors were more in adjacent positions and quite less in random manner. Adjacent error correcting codes were introduced and developed by Abramson [1]. The generalization of this idea was put in the category of errors that is now known as burst errors. A burst of length l or less may be defined as follows: "A burst of length l is a vector whose only nonzero components are confined to some l consecutive positions, the first and the last of which is non-zero". This definition is due to P. Fire [6] where he defined such errors as open loop bursts errors. He also defined closed loop bursts errors according to which: "A closed loop bursts of length l is a vector, all of whose nonzero components are confined to some l consecutive positions the first and the last of which is nonzero and the number of its starting positions is n". (i.e. it is possible to come back cyclically at the first position after the last position for enumeration of the length of the burst. There is yet another burst error due to Chien and Tang [3] according to which: "A burst of length l is a vector whose only non-zero components are confined to some l consecutive positions, the first of which is non-zero". Among various generalizations of burst errors, Fire's definition has been found of great importance and a good deal of research has gone into the development of bursts and multiple bursts error correcting codes. See [2, 8, 9, 12, 13] and many more. As there is not any uniform terminology for multiple bursts; repeated busts errors correcting codes are also put in this category. Dass and Verma [4] introduced the idea of repeated bursts error correcting codes and derived both the bounds on the number of parity check digits needed for correcting repeated burst errors over $GF(q)$. It was also pointed out in the end that they have not been able to construct codes for non binary cases and that it's a open problem. We in this paper study non-binary repeated restricted burst error correcting codes over $GF(q); q > 2$. While working on the possibility of the existence of 2-burst correcting non-binary codes, (initially discussed by Tuvi Etzion [5] in binary case)the procedure lead us to the idea of restricted burst errors. Tyagi and Lata [11] have been able to give non-binary optimal restricted 2- burst correcting codes and byte oriented codes over $GF(3)$. We in this paper discuss this new burst defined as 'restricted burst' and develop theorems for the existence of restricted burst errors correcting codes. The paper is organized into three sections. Section 2 gives necessary and sufficient conditions for 2 repeated restricted burst of length 2 or less whereas section 3 presents correction of m- repeated restricted burst of length 2 or less. In section 4, we conclude the paper by presenting an examples of $(8,2)$ codes with an open problem in the end.

Definition 1.1. *"An m-repeated restricted burst of length l whose only non-zero components are confined to m-distinct sets of l-consecutive components, the first and last component of each set being non-zero and all the non-zero components contain same field element".*

In particular a 2-repeated restricted burst may be obtained by putting m=2 in the above definition.
The vector (00222200202200) is an example of a 2-repeated burst of length 4 over $GF(3)$.
To prove our theorems, we use the following results:

Result 1.2. *(Dass and Verma, [4])" A q-ary (n,k) linear code correcting m-repeated burst errors of length l or less must satisfy*

$$q^{n-k} \geq q^{m(l-1)} \left[\binom{n-ml+m}{m} (q-1)^m + \sum_{p=0}^{m-1} \binom{n-ml+p}{p} (q-1)^p q^{m-1-p} \right] \text{''}.$$

Result 1.3. *(Dass and Verma, [4]) "A q-ary (n,k) linear code correcting m-repeated burst errors of length l or less $(n > 2ml)$ will always exist*

$$q^{n-k} > q^{2m(l-1)} \left[\binom{n-2ml+2m-1}{2m-1} (q-1)^{2m-1} + \sum_{p=0}^{2m-2} \binom{n-2ml+p}{p} (q-1)^l q^{2m-2-p} \right] \text{''}.$$

2. Correction of m-Repeated Restricted Bursts

In this section we consider linear codes capable of correcting m-repeated restricted bursts of length l or less and obtain the lower and upper bound for such codes.

Theorem 2.1. *An (n,k) linear code over $GF(q); q > 2$ that corrects all m-repeated restricted bursts of length l or less must satisfies*

$$q^{n-k} \geq (q-1) 2^{m(l-1)} \left[\binom{n-ml+m}{m} + \sum_{p=0}^{m-1} \binom{n-mb+l}{l} 2^{m-1-p} \right] - (q-2). \tag{2.1}$$

Proof. This theorem can be proved simply by enumerating the total number of correctable error vectors which are m-repeated restricted bursts of length l or less.It has been observed that the total number of m-repeated restricted bursts of length l or less for $q = 3$ is equal to one less than the double of the total number m-repeated burst errors of length l or less for binary case in Result 1.2 (Theorem 3.1 [4]). For $q = 4$ the total number of such errors is equal to thrice of the total number of m-repeated burst error of length l or less for binary case in Result 1.2 (Theorem 3.1 [4]) minus two. In this manner we conclude that the total number of m-repeated restricted burst errors of length l or less for $q > 2$ is

$$(q-1) 2^{m(l-1)} \left[\binom{n-ml+m}{m} + \sum_{p=0}^{m-1} \binom{n-mb+l}{l} 2^{m-1-p} \right] - (q-2). \tag{2.2}$$

Since all these error patterns must belong to different cosets for correction and the largest number of cosets available is q^{n-k}, then we have

$$q^{n-k} \geq (q-1) 2^{m(l-1)} \left[\binom{n-ml+m}{m} + \sum_{p=0}^{m-1} \binom{n-ml+p}{p} 2^{m-1-p} \right] - (q-2).$$

This completes the required proof. □

If we put $m = 2$ in Theorem 2.1, we get a corollary, which gives a necessary condition for the codes having the capacity to correct 2-repeated restricted burst errors of length l or less. We give the corollary as follows:

Corollary 2.2. *An (n,k) linear code over $GF(q); q > 2$ that corrects all 2-repeated restricted bursts of length l or less must satisfies*

$$q^{n-k} \geq (q-1) 2^{2(l-1)} \left[\binom{n-2l+2}{2} + \binom{n-2l+1}{1} + 2 \right] - (q-2).$$

We now give a sufficient condition on the number of parity check digits required for the existence of such a code.

Theorem 2.3. *The existence an (n,k) linear code over $GF(q)$; $q > 2$ that corrects all m-repeated restricted bursts of length l or less $(n \geq 2ml)$ provided that*

$$q^{n-k} > \Big((q-1)2^{m(l-1)} - (q-2)\Big) \times \left[(q-1)2^{2m(l-1)}\left[\binom{n-2ml+(2m-1)}{2m-1} + \sum_{p=0}^{2m-2}\binom{n-2ml+p}{p}2^{2m-2-p}\right] - (q-2)\right]. \tag{2.3}$$

Proof. This result can be proved by forming an suitable parity check matrix H by following the method used to prove the Theorem 4.7 [7](also refer Sacks [10] and Theorem 3.2 [4]). Let $H = [c_1 c_2 c_3 \ldots c_n]$ for the desired code. First of all we select $j-1$ columns $c_1, c_2, c_3, \ldots, c_{j-1}$ of H suitably. Now we lay down a restriction to add the j^{th} column c_j to the matrix H as follows: c_j must not be written in the form of linear sum of just preceding $l-1$ or lesser columns $c_{j-l+1}, c_{j-l+2}, \ldots, c_{j-1}$ of H along with any $(2m-1)$ sets of l or fewer consecutive columns that are distinct and each of them is from amongst the first $j-l$ columns $c_1, c_2, c_3, \ldots, c_{j-l}$. In different words with same meaning,

$$\begin{aligned} c_j \neq &(u_1 c_{j-l+1} + u_2 c_{j-l+2} + \cdots + u_{l-1} c_{j-1}) \\ &+ (v_1 c_{i_1} + v_2 c_{i_1+1} + \cdots + v_l c_{i_1+l-1}) \\ &+ (w_1 c_{i_2} + w_2 c_{i_2+1} + \cdots + v_l c_{i_2+l-1}) \\ &+ \\ &\vdots \\ &+ \\ &+ (x_1 c_{i_{2m-1}} + x_2 c_{i_{2m-1}+1} + \cdots + x_l c_{i_{2m-1}+l-1}). \end{aligned} \tag{2.4}$$

where $u_i, v_i, w_i, \ldots x_i \in GF(q)$; $u_i = v_i = w_i = \cdots = x_i \neq 0$ and $i_1 + i_2 + i_3 + \cdots + i_{2m-1} + (2m-1)b - (2m-1) \leq j - b$.
The total number of coefficients u_i's will be equal to $(q-1) \times 2^{l-1} - (q-2)$.The calculation of the number of the coefficients v_i, w_i, x_i will be same as the enumeration of $(2m-1)$-repeated restricted burst errors lying in a vector of length $j-b$ which can be calculated by using the Theorem 3.1 [4] and given as

$$(q-1)2^{(2m-1)(l-1)}\left[\binom{j-2ml+(2m-1)}{2m-1} + \sum_{p=0}^{2m-2}\binom{j-2ml+p}{p}2^{2m-2-p}\right] - (q-2). \tag{2.5}$$

Considering the all coefficients $u_i, v_i, w_i, \ldots x_i$ simultaneously, we get the total number of linear sums that can not be put to be equal to c_j is equal to

$$\Big((q-1)2^{m(l-1)} - (q-2)\Big) \times \left[(q-1)2^{(2m-1)(l-1)}\left[\binom{j-2ml+(2m-1)}{2m-1} + \sum_{p=0}^{2m-2}\binom{j-2ml+p}{p}2^{2m-2-p}\right] - (q-2)\right]. \tag{2.6}$$

Therefore in view of the fact that total number of $(n-k)$ tuples is q^{n-k} ,addition of the $j^t h$ column c_j to H can be done provided q^{n-k} is greater than (2.6).That is

$$q^{n-k} > \Big((q-1)2^{m(l-1)} - (q-2)\Big) \times \left[(q-1)2^{(2m-1)(l-1)}\left[\binom{j-2ml+(2m-1)}{2m-1} + \sum_{p=0}^{2m-2}\binom{j-2ml+p}{p}2^{2m-2-p}\right] - (q-2)\right].$$

The proof of the required theorem is completed by replacing j by n. □

If we put $m = 2$ in Theorem 2.3, we get a corollary, which gives a sufficient condition for the codes having the capacity to correct 2-repeated restricted burst errors of length l or less.We give the corollary as follows:

Corollary 2.4. *The existence of an (n,k) linear code over $GF(q); q > 2$ that corrects all 2-repeated restricted bursts of length l or less $(n \geq 4l)$ is ensured, provided that*

$$q^{n-k} > \Big((q-1)2^{(l-1)} - (q-2)\Big) \times \left[(q-1)2^{3(l-1)}\left[\binom{n-4l+3}{3} + \binom{n-4l+2}{2} + 2\binom{n-4l+1}{1} + 4\right] - (q-2)\right]. \tag{2.7}$$

for the verification of such codes, we are providing an example which is given as follows:

Example 2.5. *For $n = 8, m = 2, l = 2$, Consider a $(8,2)$ code over $GF(3)$ with parity check matrix*

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 \end{bmatrix}$$

This parity check matrix has been formed by the method used in the proof of Theorem 2.3 for repeated restricted burst errors by taking $l = 2$ over $GF(3)$.The error patterns and syndromes table for this parity check matrix is given below as:

Err. Patterns	*Syndromes*	*Err. Patterns*	*Syndromes*
10000000	100000	20000000	200000
01000000	020000	02000000	010000
00100000	001000	00200000	002000
00010000	000200	00020000	000100
00001000	000010	00002000	000020
00000100	000002	00000200	000001
00000010	121212	00000020	212121
00000001	011111	00000002	022222
11000000	120000	22000000	210000
10100000	101000	20200000	202000
10010000	100200	20020000	200200
10001000	100010	20002000	200020
10000100	100002	20000200	200001
10000010	221212	20000020	112121
10000001	111111	20000002	222222
01100000	021000	02200000	012000
01010000	020200	02020000	010100
01001000	020010	02002000	010020
01000100	020002	02000200	010001
01000010	111212	02000020	222121
01000001	001111	02000002	002222
00110000	001200	00220000	002100
00101000	001010	00202000	002020
00100100	001002	00200200	002001
00100010	122212	00200020	211121
00100001	012111	00200002	021222
00011000	000210	00022000	000120
00010100	000202	00020200	000101
00010010	121112	00020020	212221
00010001	011011	00020002	022022
00001100	000012	00002200	000021
00001010	121222	00002020	212111
00001001	011121	00002002	022212
00000110	121211	00000220	212122
00000101	011112	00000202	022221
00000011	102020	00000022	201010
11100000	121000	22200000	212000
11010000	120200	22020000	210100
11001000	120010	22002000	210020
11000100	120002	22000200	210001
11000010	211212	22000020	122121
11000001	101111	22000002	202222
01110000	021200	02220000	012100
01101000	021010	02202000	012020
01100100	021002	02200200	012001
01100010	112212	02200020	221121
01100001	002111	02200002	001222
00111000	001210	00222000	002120
00110100	001202	00220200	002101
00110010	122112	00220020	211221
00110001	012011	00220002	021022
00011100	000212	00022200	000121
00011010	121122	00022020	212211

Err. Patterns	*Syndromes*	*Err. Patterns*	*Syndromes*
00011001	011021	00022002	022012
00001110	121221	00002220	212112
00001101	011120	00002202	022210
00000111	102022	00000222	201011
10110000	101200	20220000	202100
10011000	100210	20022000	200120
10001100	100012	20002200	200021
10000110	221211	20000220	112122
10000011	202020	20000022	101010
01011000	020210	02022000	010120
01001100	020012	02002200	010021
01000110	111211	02000220	222122
01000011	122020	02000022	211010
00101100	001012	00202200	002021
00100110	122211	00200220	211122
00100011	100020	00200022	200010
00010110	121111	00020220	212222
00010011	102220	00020022	201110
00001011	102000	00002022	201000
11110000	121200	22220000	212100
11011000	120210	22022000	210120
11001100	120012	22002200	210021
11000110	211211	22000220	122122
11000011	222020	22000022	111010
01111000	021210	02222000	012120
01101100	021012	02202200	012021
01100110	112211	02200220	221122
01100011	120020	02200022	210010
00111100	001212	00222200	002121
00110110	122111	00220220	211222
00110011	100220	00220022	200110
00011110	121121	00022220	212212
00011011	102200	00022022	201100
00001111	102002	00002222	201001

It can be verified from the above error pattern- syndrome table that the syndromes of different 2-repeated bursts of length 2 or less are distinct. This shows that the code which is the null space of the matrix given above corrects all 2-repeated restricted bursts errors of length 2 or less.

3. Conclusion

We have shown with the help of one example that 2 repeated restricted burst correcting codes over GF(3) exist. It would be interesting to see a general matrix formation for such codes for any given value of q, l, n and k.

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