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Combination labeling of joins of fire cracker graph

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Abstract

Let G be a graph with finite vertices p, finite edges q . An injective function is called a combination labelling if such that each edge has the label $\binom{x}{y}$ or $\binom{y}{x}$ according as $x \leq y$ or $x \geq x$. A graph with a combination labelling is called combination labelling graph . In this paper we study on the joins of Fire Cracker graph by joining one Fire Cracker graph with similar Fire Cracker graph and prove that it is Combination labelling graph. We further study on some properties connecting the Fire Cracker graph. We further analyse on finding the sum of the joins of Fire Cracker graph. We extend our discussion on Permutation labelling and strong k-combination labelling of Fire Cracker graph.

Keywords

Fire Cracker graph, Joins of Fire Cracker graph, Combination Labelling graph, Permutation labelling graph, Strong k-combination labelling.

AMS Subject Classification

05C78.

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Contents

1. Introduction

A graph G consists of finite vertices and finite edges. Gallian[1] has given a extensive survey on labelling. The beginning of labelings can be associated with Rosa. In studying the different labelling techniques in graph theory we have under stood that combination labelling of graphs, permutation labelling of graphs, parity combination cordial labelling [3,4,5,6,7]is one such labelling which has predominant feature in various scientific problems. In order to utilise such labelling techniques we have here taken the Fire Cracker graph and have significantly added one Fire Cracker graph with another Fire Cracker graph and call it as joins of Fire Cracker graph. This technique helps in attaching as many number of joins of Fire Cracker graph so as to enable us to apply combination labelling and prove that they are combination labelling graph. Further we have analysed on the nature of such labelling by comparing it with the permutation labelling. In further motivated towards Strong k-combination labelling techniques we have also analysed on the possibility of Fire Cracker graph being a strong k-combination labelling. Combination Labelling was introduced by Suresh Manjanath Hegde, Sudhakar Shetty[2]. We refer to basic terms and terminology of graphs[8].

2. Preliminaries

In this section, we refer to some definitions which will be useful for our discussion in the course of working on the results for this paper.

Definition 2.1. *A (p,q) graph G=(V,E) is said to be a permutation graph if there exists a bijection f:V(G) to 1,2,3...p such that the induced edge function f:E(G) to N defined for each edge xy as* $f(x)Pf(y)$ *or* $f(y)Pf(x)$ *according as* $x > y$ *or* $y > x$

Definition 2.2. *A (p,q) graph G=(V,E) is said to be a combination labelling graph if there exists a bijection f:V(G) to 1,2,3...p such that the induced edge function f:E(G) to N defined for each edge xy as* $\binom{x}{y}$ *or* $\binom{y}{x}$ *according as* $x > y$ *or* $y > x$

Definition 2.3. *A (p,q) graph G is said to be a strong k-combination graph if there exists a bijection f:V(G) to 1,2,3... p such that the induced edge function g:E(G) to k,k+1,k+2... k+q-1 defined for each edge xy as* $\binom{x}{y}$ *or* $\binom{y}{x}$ *according as* $x > y$ *or* $y > x$

Definition 2.4. *A (n,k) fire cracker is a graph obtained by the concatenation of n, k-stars by linking one leaf from each*

Definition 2.5. *1-Join Fire Cracker graph is defined as join of (n,k) fire cracker graph with another (n,k) fire cracker graph of the same order. Here we discuss the joins of fire cracker graph by fixing n=2 and altering values of k . We also try to attach M-joins of Fire cracker graph to study on combination labelling graph.*

3. Main Results

Theorem 3.1. *1-Join Fire Cracker graph* F_{22} *is combination labelling graph*

Proof. Consider the Fire Cracker graph F_{22} joining by an edge with another Fire Cracker graph F_{22} to form a 1-join of Fire Cracker graph F_{22} . Now the 1 –join Fire Cracker graphF_{22} so formed consists of the vertex set $V = \{u_1^1, u_2^1, u_1^3, u_2^3\}U$ $\{v_1^2, v_2^2, v_1^4, v_2^4\}$ and edge set$E = \{e_1^1, e_2^2, e_3^3, e_4^4\}U$ $\{e_1^1, e_2^3, e_3^4\}$ with total 8 vertices and 7 edges. Now on vertices being labeled for Fire Cracker graph F_{22} as follows
$f(u_i^j) = 4i-3$ for i is odd and for $1 \le i \le 4$, $j = 1,3$
$f(u_i^j) = 4i-2$ for i is even and for $1 \le i \le 4, j = 1,3$
$f(v_i^j) = 4i-1$ for i is odd and for $1 \le i \le 4, j = 1,3$
$f(v_i^j) = 4i$ for i is even and for $1 \le i \le 4, j = 1,3$
Now let us find the edge labelling as follows
$\binom{f(u_{2i}^j)}{f(u_{2i-1}^j)} = \binom{u_{2i}^j}{u_{2i-1}^j}$ if $u_{2i}^j > u_{2i-1}^j$
$\binom{f(u_{2i-1}^j)}{f(u_{2i}^j)} = \binom{u_{2i-1}^j}{u_{2i}^j}$ if $u_{2i-1}^j > u_{2i}^j$
$\binom{f(v_{2i}^j)}{f(v_{2i-1}^j)} = \binom{v_{2i}^j}{v_{2i-1}^j}$ if $v_{2i}^j > v_{2i-1}^j$
$\binom{f(v_{2i-1}^j)}{f(v_{2i}^j)} = \binom{v_{2i-1}^j}{v_{2i}^j}$ if $v_{2i-1}^j > v_{2i}^j$
$\binom{f(u_i^j)}{f(v_i^{j+1})} = \binom{u_i^j}{v_i^{j+1}}$ if $u_i^j > v_i^{j+1}$
$\binom{f(v_i^{j+1})}{f(u_i^j)} = \binom{v_i^{j+1}}{u_i^j}$ if $v_i^{j+1} > u_i^j$
Now on computing the edges the edge labelling follows a combination labelling and hence the 1-join of Fire Cracker graph F_{22} is a combination labelling graph. Hence the proof. □

Definition 3.2. *We call 1-join Fire cracker graph* F_{22} *as basic Fire Cracker graph We add to the basic Fire Cracker graph* F_{22} *increasing the value of k by 1 at each instance to obtain various 1-join Fire Cracker graph*$F_{2,3}$ *, 1-join Fire Cracker graph* $F_{2,4}$ *and so on.*

Theorem 3.3. *1-join Fire Cracker graph* $F_{2,k}$ *fork* ≥ 2 *is combination labelling graph*

Proof. Consider the 1-Join Fire Cracker graph $F_{2,k}$ in which a Fire Cracker graph $F_{2,k}$ joined with another Fire Cracker graph $F_{2,k}$ by an edge. Now the 1-Join Fire Cracker graph $F_{2,k}$ consists of vertex set $V = \{u_1^1, u_2^1, u_3^3, u_4^3\}U$ $\{v_1^1, v_2^2, v_3^3, v_4^4\}U\{w_1^1, w_2^1 ... w_k^1\}U$ $\{w_1^2, w_2^2 ... w_k^2\}U\{w_1^3, w_3^1 ... w_k^3\}U\{w_1^4, w_2^4 ... w_k^4\}$ $E = \{e_1^1, e_2^1, e_3^1, e_4^1\}U\{e_2^2, e_1^3, e_3^4\}U\{u_2^1 w_1^1\}U$ $\{u_2^1 w_2^1\} ... \{u_2^1 w_k^1\}U\{u_2^3 w_1^3\}U\{u_2^3 w_2^4\} ...$ $\{u_2^3 w_k^3\}U\{v_2^2 w_1^2\}U\{v_2^2 w_2^2\} ... \{v_2^2 w_k^2\}U$ $\{v_4^4 w_1^4\}U\{v_4^4 w_2^4\} ... \{v_4^4 w_k^4\}$ The total number of vertices and edges in 1-join Fire Cracker graph $F_{2,k}$ is $8+2k$ and $7+2k$ respectively.
Now let us label the vertices of 1-join Fire Cracker graph $F_{2,k}$
$f(u_i^j) = 2i-1$ for i is odd
$f(u_i^j) = 6i-4$ for i is even
$f(v_i^j) = 4i-1$ for i is odd
$f(v_i^j) = 4i$ for i is even
For labeling the set of vertices
$\{w_1^1, w_2^1, ... w_k^1\}\{w_1^2, w_2^2, ... w_k^2\}\{w_1^3, w_2^3 ... w_k^3\}$
$\{w_1^4, w_2^4 ... w_k^4\}$ we follow the schema given below
We consider the total vertices of 1-join Fire Cracker graph $F_{2,k}$ as $p = 8+2k$ and we find that it is always even and hence we partition the set of vertices equally and write as a combination of $p = p_1 + p_2$ where p_1 represents the set of vertices
$\{u_1^1, u_2^1, u_3^1, u_4^3\}\{w_1^1, w_2^1 ... w_k^1\}$
$\{w_1^3, w_2^3 ... w_k^3\}$ and p_2 represents the set of vertices
$\{v_1^2, v_2^2, v_1^4, v_2^4\}$
$\{w_1^2, w_2^2 ... w_k^2\}$
$\{w_1^4, w_2^4 ... w_k^4\}$ such that $p_1 = p_2$
Further we find that from the structure the vertices u_2^1, u_4^3 are attached with the vertices $\{w_1^1, w_2^1, ... w_k^1\}\{w_1^3, w_2^3 ... w_k^3\}$ respectively. Also the vertices v_2^2, v_4^4 are attached with the vertices $\{w_1^2, w_2^2, ... w_k^2\}\{w_1^4, w_2^4 ... w_k^4\}$ respectively.
We understand that the vertex u_2^1 is labelled as 2 and the vertex u_4^3 is $p_1 + 2$. Suppose the number of vertices of the set $\{w_1^1, w_2^1, ... w_k^1\}+$ number of vertices of the set $\{w_1^2, w_2^2, ... w_k^2\}+$ number of vertices of the set $\{w_1^3, w_2^3 ... w_k^3\}$+number of vertices of the set $\{w_1^4, w_2^4 ... w_k^4\} = k$such that k_1 represents the number of vertices of $\{w_1^1, w_2^1, ... w_k^1\}$ other than u_2^1 , k_3 represents the number of vertices of $\{w_1^3, w_2^3 ... w_k^3\}$ other than u_2^3, k_2 represents the number of vertices of $\{w_1^2, w_2^2, ... w_k^2\}$ other than v_2^2 and k_4 represents the number of vertices of $\{w_1^4, w_2^4, ... w_k^4\}$ other than v_4^4. Now we label the vertices of $\{w_1^1, w_2^1, ... w_k^1\}$ as follows
$f(w_i^1) = i+1$ for $2 \le i \le k_1$
The label for the vertices $\{w_1^2, w_2^2 ... w_k^2\}$
$f(w_i^2) = i+1$ for $k_1 + 2 \le i \le k_2$
The label for the vertices $\{w_1^3, w_2^3 ... w_k^3\}$
$f(w_i^3) = i+1$ for $k_2 + 2 \le i \le k_3$
The label for the vertices $\{w_1^4, w_2^4 ... w_k^4\}$
$f(w_i^4) = i+1$ for $k_3 + 2 \le i \le k_4$
Now let us compute the induced edge labelling as follows
$\binom{f(u_{2i}^j)}{f(u_{2i-1}^j)} = \binom{u_{2i}^j}{u_{2i-1}^j}$ if $u_{2i}^j > u_{2i-1}^j$

$\binom{f(u^j_{2i-1})}{f(u^j_{2i})} = \binom{u^j_{2i-1}}{u^j_{2i}}$ if $u^j_{2i-1} > u^j_{2i}$
$\binom{f(v^j_{2i})}{f(v^j_{2i-1})} = \binom{v^j_{2i}}{v^j_{2i-1}}$ if $v^j_{2i} > v^j_{2i-1}$
$\binom{f(v^j_{2i-1})}{f(v^j_{2i})} = \binom{v^j_{2i-1}}{v^j_{2i}}$ if $v^j_{2i-1} > v^j_{2i}$
$\binom{f(u^j_i)}{f(v^{j+1}_i)} = \binom{u^j_i}{v^{j+1}_i}$ if $u^j_i > v^{j+1}_i$
$\binom{f(v^{j+1}_i)}{f(u^j_i)} = \binom{v^{j+1}_i}{u^j_i}$ if $v^{j+1}_i > u^j_i$
$\binom{f(u_2 i^{2j-1})}{f(w^{2j-1}_k)} = \binom{u_2 i^{2j-1}}{w^{2j-1}_k}$ if $u_{2i} > w^{2j-1}_k$ for $j = 1,2$
$\binom{f(w^{2j-1}_k)}{f(u_2 i^{2j-1})} = \binom{w^{2j-1}_k}{u_2 i^{2j-1}}$ if $w^{2j-1}_k > u_{2i}$ for $j = 1,2$
$\binom{f(v_2 i^{2j})}{f(w^{2j}_k)} = \binom{v_2 i^{2j}}{w^{2j}_k}$ if $v_{2i} > w^{2j}_k$ for $j = 1,2$

$\binom{f(w^{2j}_k)}{f(v_2 i^{2j})} = \binom{w^{2j}_k}{v_2 i^{2j}}$ if $w^{2j}_k > v_{2i}$ for $j = 1,2$
Hence on computing the induced edge labelling we find that the edges are distnct and hence the Fire Cracker graph $F_{2,k}$ is combination labelling graph. Hence the proof. □

We construct from 1- join Fire Cracker graph $F_{2,k}$ for $k \geq 2$ by adding another Fire Cracker graph $F_{2,k}$ to form a 2- join Fire Cracker graph and the adopting the same procedure we can obtain M-join Fire Cracker graph $F_{2,k}$ for $k \geq 2$.

Theorem 3.4. *M-join Fire Cracker graph $F_{2,k}$ for $k \geq 2$ is combination labelling graph*

Proof. Consider the M-Join Fire Cracker graph $F_{2,k}$ for $k \geq 2$. Let us prove that it is combination labelling graph by adopting the method of mathematical induction. In the above theorem we have proved that 1-join Fire Cracker graph $F_{2,k}$ is combination labelling graph. Now let us assume that M-1 Join Fire Cracker graph $F_{2,k}$ is combination labelling graph. Now to prove that the theorem is true for M-Join Fire Cracker graph . Let us combine the M-1 join Fire Cracker graph $F_{2,k}$ with 1-join Fire Cracker graph which are both already a combination labelling graph and hence on combining we find that the M-join of Fire Cracker graph$F_{2,k}$ is combination labelling graph. Hence the proof. □

Remark 3.5. *The same understanding can be made with reference to Permutation labelling graph for M-join Fire Cracker graph $F_{2,k}$ for $k \geq 2$. But understanding the computation difficulty and the edge labels being a large number we restrict our discussion with Combination labelling graph of M-Join Fire Cracker graph $F_{2,k}$ for $k \geq 2$.*

Remark 3.6. *Further we try to analyse the M-Join Fire Cracker graph $F_{2,k}$ for $k \geq 2$ so obtained is whether strong k-combination graph. We try to find the sum of the edges which plays a vital role in identifying whether the Fire Cracker$F_{2,k}$ for $k \geq 2$ and its joins are strong k-combination graph.*

Let us now define the sum for the Fire Cracker graph

Definition 3.7. *The Sum of the Fire Cracker $F_{2,k}$ for $k \geq 2$ is defined as $S_1, S_2S_3...$ where S_1 represents the sum of the edges of the first hand $(u^1_1u^1_2),(u^1_2w^1_1),(u^1_2w^1_2)...(u^1_2w^1_k)$ and S_2 represents the sum of the second hand $(v^2_1v^2_2),(v^2_2w^2_1),(v^1_2w^2_2)...$ $(v^2_2w^2_k)$ and along with the edges $(u^1_1v^1_1)(u^2_2v^2_2)...$ with the joins $(v^2_2u^3_3)(v^4_4u^5_5)...$ where $(v^2_2u^3_3)$ represents the 1-join edge between the two Fire Cracker graph and $(v^4_4u^5_5)$ represents the 2-join edge between the Fire Cracker graph.*

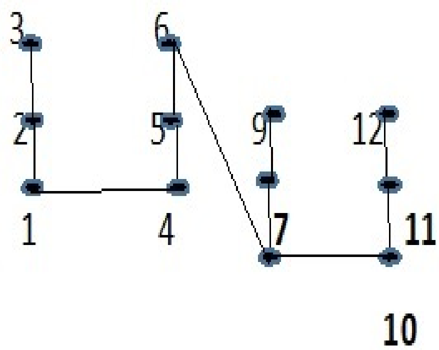
To put forward the theory on finding whether the Fire Cracker graph $F_{2,k}$and its joins are strong k-combination graph we give a general condition for which the Fire Cracker graph $F_{2,k}$is strong k –combination graph in the following theorem.

Theorem 3.8. *Proving that M-join Fire Cracker graph $F_{2,k}$ is not a strong k-combination graph since the following condtion holds good*
1. The sum of the edges so labelled is greater than to $\frac{(k+q-1)(k+q)}{2} - 1$
That is $S_n = \frac{(k+q-1)(k+q)}{2} - 1$
2. Some of the adjacent vertices of M-join of Fire Cracker graph labelled have a difference more than 1.

Proof. Consider Fire Cracker graph $F_{2,k}$by labelling the vertices and joining the Fire Cracker graph with another Fire Cracker graph such that the induced edges forms a combination labelling graph. Continuing the process we obtain M-join Fire Cracker graph . For suppose that the M-Join Fire Cracker graph $F_{2,k}$ is a combination labelling graph then we find according to the labelling techniques adopted in labelling given in the theorem for the vertices we find that the vertices u^i_i and w^i_i, the vertices v^i_i and w^i_i are labelled with more than 2 difference and hence the combination between those vertices will increase the induced edge labelling and hence exceeding the otal sum such that $S_n = \frac{(k+q-1)(k+q)}{2} - 1$ and hence both condtion satisfy for the required theorem. Hence the Fire Cracker graph $F_{2,k}$ and its joins for $k > 2$ are not k-combination graph. Hence the theorem. □

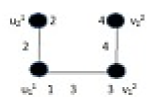
4. Example

In this section we give some examples illustrating the labelling techniques used for Fire Cracker Graph $F_{2,k}$ and its joins to prove that it is combination labelling and further we give example where we find the sum of the edges of Fire Cracker Graph and its joins to prove that it is not k-combination labeling . As a first one we illustrate 1-join of Fire Cracker graph $F_{2,3}$ below where we label the vertices and edges so as to prove that it is combination labeling graph



Now let us consider the Fire Cracker graph $F_{2,2}$ and let us obtain the sum of the edges as explained

We have the Sum $S = S_1 + S_2 + \binom{f(u_1^1)}{f(v_1^2)}$



Where $S_1 = \binom{f(u_1^1)}{f(u_2^1)}$ Hence $S_1 = \binom{2}{1}$

$S_1 = 2$ In a similar computation we have $S_2 = 4$ and $\binom{f(u_1^1)}{f(v_1^2)}$ is 3

Hence the sum of the Fire Cracker graph $F_{2,2}$ as 9 and hence we can prove that Fire Cracker graph $F_{2,2}$ is strong k-combination labeling graph.

But we can compute that for the joins of Fire Cracker graph $F_{2,k}$ the combination labelling of graph is not a strong k-combination graph. On computing the sum of the 1- join of Fire Cracker graph $F_{2,3}$we find that $S = 2+3+5+6+8+10+11+4+120+7 = 186$ Hence we can prove that 1-join Fire Cracker $F_{2,3}$is not a strong k-combination labelling graph. In general from the result obtained we find that M-join Fire Cracker $F_{2,2}$is not a strong k-combination labelling graph.

5. Conclusion

We have identified in this paper Fire Cracker graph $F_{2,k}$ and have joined with set of Fire Cracker graph $F_{2,k}$ to form a M-Join Fire Cracker graph $F_{2,k}$ and have proved that it is combination labelling graph and studied on some characteristics of labelling them. We in future we like to identify some more graphs for which we can prove that it is combination labelling and try to find that whether they a strong k-combination graph or not.

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