Malaya Journal of Matematik, Vol. S, No. 1, 431-434, 2020
https://doi.org/10.26637/MJM0S20/0080

Comparative study of fuzzy assignment problem with various ranking

R. Srinivasan[1]*, G. Saveetha[2] and T. Nakkeeran[3]

Abstract
In this study, we compare to the methodology for assignment problem with fuzzy optimal solutions. The comparisons of the fuzzy optimal solutions are considered as triangular fuzzy numbers. Ranking techniques are used for the triangular fuzzy numbers and so fuzzy numbers are converted to crisp values. Then we also proceeded to Hungarian methodology is to resolve this type of fuzzy assignment problem. The Numerical examples are effectively during this study.

Keywords
Triangular fuzzy numbers; Assignment problem; Fuzzy assignment problem; Ranking Techniques; Hungarian method.

AMS Subject Classification
03E72.

[1,2,3]*Department of Ancient Science, Tamil University, Thanjavur-613010, Tamil Nadu, India.*
***Corresponding author**: [1]srinivasanmaths@gmail.com; [2]saveemaths@gmail.com
Article History: Received **01** February **2020**; Accepted **29** March **2020**

Contents

1. Introduction

Assignment problem is a specific form of linear programming optimization problem that deals with one-to-one allocation of the different resources to the different jobs. The total cost of optimizing the full benefit of the allocation. Hungarian approach is Kuhn's most traditional problem solving method. The problem with the assignment occurs because of an individual or a computer's varying ability to perform a given task of work. The question of assignment arises due to the varying capacity of an individual or computer to perform a given task or work. In this article, we tend to compare the fuzzy assignment problem to position various ranking methods.

2. Preliminaries

2.1 Fuzzy Set

Let X be non–empty set. A fuzzy set $\hat{A}$ in X, mapping a membership function in the range $\mu_{\hat{A}}(x) : X \to [0,1]$, for each $x \in X$. Then the fuzzy set is defined as $\hat{A} = \{x, \mu_{\hat{A}}(x) \backslash x \in X\}$.

2.2 Fuzzy Number

A fuzzy number $\hat{A}$ on $\hat{R}$ in the real line set $\hat{A} : \hat{R} \to I$ where $\hat{A}(x)$ is the membership function of set$\hat{A}$. Then the fuzzy number $\hat{A}$ is defined by

(i) $\hat{A}$ is normal.

(ii) $\hat{A}$ is convex.
(iii) $\hat{A}$ is upper semi continuous $X \in \hat{R}$.

2.3 α-cut

An α-cut of a fuzzy set P is a Crisp set P_α comprising all the elements of the universal set X with a membership grade P greater than or equal to a given value thus
$P = \{x \in X / \mu_A(x) \geqslant \alpha\}, 0 \leqslant \alpha \leqslant 1$

2.4 Triangular Fuzzy Number

A triangular fuzzy number $P = (a_1, a_2, a_3)$ specified as membership function

$$P(x) = \begin{cases} 0, & \text{if } x \leqslant a \\ \dfrac{(x-a)}{(b-a)}, & \text{if } a \leqslant x \leqslant b \\ \dfrac{(x-c)}{(b-c)}, & \text{if } b \leqslant x \leqslant c \\ 0, & \text{if } x \geqslant c \end{cases}$$

3. Assignment Problem

The problem of assignment can be represented as a $n \times n$ cost matrix $[C_{ij}]$ of real numbers as shown in Table 3.1

Table 3.1

	1	2	3	...	j	...	M
1	B_{11}	B_{12}	B_{13}	...	B_{1j}	...	B_{1n}
2	B_{21}	B_{22}	B_{23}	...	B_{2j}	...	B_{2n}
...	...	...	...	...	...	...	...
i	B_{i1}	B_{i2}	B_{i3}	...	B_{ij}	...	B_{in}
...	...	...	...	...	...	...	...
N	B_{n1}	B_{n2}	B_{n3}	...	B_{nj}	...	B_{nn}

Mathematically assignment problem can be expressed as

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} x_{ij}$$

$$\text{Subject to } \sum_{i=1}^{n} x_{ij} = 1,\ i = 1,2,3,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ j = 1,2,3,\ldots,n \tag{3.1}$$

$$x_{ij} \in \{0,1\}$$

$$x_{ij} = \begin{cases} 1, & \text{if the } i^{\text{th}} \text{ person is assigned the } j^{\text{th}} \text{ job} \\ 0, & \text{otherwise} \end{cases}$$

4. Fuzzy Assignment Problem

The generalized fuzzy assignment problem can be represented with in the form of $n \times n$ fuzzy cost matrix A_{ij} as given in Table 4.1.

The generalized trapezoidal fuzzy numbers cost or time $[A_{ij}]$ $A_{ij} = \left(A_{ij}^{(1)}, A_{ij}^{(2)} A_{ij}^{(3)}, A_{ij}^{(4)}; w_{ij}\right)$.

Table 4.1

	1	2	3	...	j	...	M
1	B_{11}	B_{12}	B_{13}	...	B_{1j}	...	B_{1n}
2	B_{21}	B_{22}	B_{23}	...	B_{2j}	...	B_{2n}
...	...	...	...	...	...	...	...
i	B_{i1}	B_{i2}	B_{i3}	...	B_{ij}	...	B_{in}
...	...	...	...	...	...	...	...
N	B_{n1}	B_{n2}	B_{n3}	...	B_{nj}	...	B_{nn}

5. Ranking Methods for Fuzzy Assignment Problem

5.1 Centroid of a Generalized Triangular

The Centroid of a triangle fuzzy number $\tilde{A} = (a,b,c;w)$ as $G_{\tilde{A}} = \left(\dfrac{a+b+c}{3}, \dfrac{w}{3}\right)$. The ranking function is known as the generalized triangle fuzzy number $\tilde{A} = (a,b,c;w)$ that maps the set of all fuzzy numbers to a set of real numbers is defined as [1]

$$R(\tilde{A}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right).$$

5.2 Magnitude Ranking Method

Triangular fuzzy number is defuzzified by ranking method for an arbitrary triangular fuzzy number $(a,b,c) = (a_1,b_1,c_1)$ magnitude of triangular fuzzy number is given by equation [2].

$$\text{Mag}(a,b,c) = \frac{1}{2}\left(\int_0^1 (c_1 3a_1 - b_1) f(r) dr\right).$$

5.3 Robust's ranking technique

Given a convex triangluar fuzzy number S,

$$R(s) = \left(\int_0^1 0.5\left(s_\alpha^L, s_\alpha^U\right) d\alpha\right)$$

where $\left(s_\alpha^L, s_\alpha^U\right)$ is the α-level cut of the fuzzy number S determines the Robust's Ranking Index. The ranking index of the Robust provides the representive value of the S fuzzy number [3].

5.4 Sub Interval Average Method

Let $\hat{A} = (a,b,c)$ be a triangular fuzzy number, where a,b,c is a real numbers. The ranking function is generally defined by [4].

$$R\left(A^i\right) = \frac{(i+1)}{2x_i} \sum_{i=1}^{n} a_i = \frac{(i+1)}{2i(i+1)/2} \sum_{i=1}^{n} a_i.$$

5.5 Pascal Triangular Approach

We recall the triangular Pascal method for the triangular fuzzy numbers derived from [5]

$$P\left(\hat{A}\right) = \frac{a+2b+c}{4}.$$

Table 5.1

S. No.	Ranking Methods	Ranking Formula	Matrices Formulation	Optimal Solution (By Hungarian Method)
1	Robust's Ranking	$R(\tilde{a}) = \int_0^1 0.5\left(a_\alpha^L, a_\alpha^U\right) d\alpha$	$\begin{pmatrix} 13 & 10 & 11 & 10 \\ 12 & 13 & 11 & 12 \\ 8 & 10 & 10 & 8 \\ 11 & 9 & 11 & 8 \end{pmatrix}$	46
2	Pascal Triangular Approach	$P(A) = \frac{a+2b+c}{4}$	$\begin{pmatrix} 13 & 10 & 11 & 10 \\ 12 & 13 & 11 & 12 \\ 8 & 10 & 10 & 8 \\ 11 & 9 & 11 & 8 \end{pmatrix}$	46
3	Sub Interval Average Method	$R\left(A^i\right) = \frac{(i+1)}{2x_i}\sum_{i=1}^{n} a_i = \frac{(i+1)}{2i(i+1)/2}\sum_{i=1}^{n} a_i$	$\begin{pmatrix} 13 & 10 & 11 & 10 \\ 12 & 13 & 11 & 12 \\ 8 & 10 & 10 & 8 \\ 11 & 9 & 11 & 8 \end{pmatrix}$	46
4	Magnitude Ranking	$Mag(a,b,c) = \frac{1}{2}\left(\int_0^1 c+3a-b\right) f(r)dr$	$\begin{pmatrix} 8.25 & 6 & 6.75 & 6 \\ 7.5 & 8.25 & 6.75 & 7.5 \\ 4.5 & 6 & 6 & 4.5 \\ 6.75 & 5.25 & 6.75 & 4.5 \end{pmatrix}$	28.5
5	Centroid Ranking	$R\left(\tilde{A}\right) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$	$\begin{pmatrix} 4.33 & 3.33 & 3.67 & 3.33 \\ 4.00 & 4.33 & 3.67 & 4.00 \\ 2.67 & 3.33 & 3.33 & 2.67 \\ 3.67 & 3.00 & 3.67 & 2.67 \end{pmatrix}$	15.33

6. Numerical Example

Here we are going to solve fuzzy assignment problem to allocate 4 jobs to different 4 machines, the fuzzy assignment cost C_{ij} is given below

$$\begin{bmatrix} (10,13,16) & (7,10,13) & (8,11,14) & (7,10,13) \\ (9,12,15) & (10,13,16) & (8,11,14) & (9,12,15) \\ (5,8,11) & (7,10,13) & (7,10,13) & (5,8,11) \\ (8,11,14) & (6,9,12) & (8,11,14) & (5,8,11) \end{bmatrix}$$

Solution: Given Triangular fuzzy cost matrix is balanced one. Above problem can be solved by using Centroid Ranking Method, Magnitude Ranking Method, Robust's Ranking Technique, Sub Interval Average Method and Pascal Triangular Approach (See Table 5.1).

7. Conclusion

In this paper, the fuzzy assignment problem for the placement of four candidates for four totally different posts is successfully resolved. The comparisons of the completely different ranking methods are successfully comparable. The numerical examples are strengthening to the comparisons.

References

[1] R.Q. Mary and D. Selvi, Solving Fuzzy Assignment Problem Using Centroid Ranking Method. *International Journal of Mathematics And its Applications*, 55(2018) 7–13.

[2] D. Selvi and G. Velammal, Method for Solving Fuzzy Assignment Problem Using Magnitude Ranking Technique, *International Journal of Applied and Advanced Scientific Research*, (2017), 16–20.

[3] K. Kalaiarasi, S. Sindhu and M. Arunadevi, Optimization of fuzzy assignment model with triangular fuzzy numbers using Robust Ranking technique, *International Journal of Innovative Science, Engg. Technology*, 1(3)(2014), 10–15.

[4] J.P. Singh and N.I. Thakur, A novel method to solve assignment problem in fuzzy environment, *Industrial Engineering Letters*, 5(2)(2015), 31–35.

[5] S.K. Khaddar Babu and K. Karthikeyen, Statistical Optimization for Generalized Fuzzy Number, *International Journal of Modern Engineering Research (IJMER)*, 3(2)(2013), 647–651.

[6] V. Ramadas, and P. Shanmugam, An Effective Fuzzy Ranking Method In Convex Triangular Fuzzy Soft Numbers, *International Journal of Applied and Advanced Scientific Research*. 3(1)(2018), 2456–3080.

[7] Stephen Dinakar, Kamalanathan and Rameshan, Sub interval average method for ranking of linear fuzzy num-

bers, *International Journal of Pure and Applied Mathematics*, 4(6)(2017), 119–130.

[8] T.A. Thakra, O.K. Chaudhri and N.R. Dhawade, Placement of Staff in LIC using Fuzzy Assignment Problem, *International Journal of Mathematics Trends and Technology*, 53(4)(2018), 259–266.

[9] X. Wang, Fuzzy optimal assignment problem, *Fuzzy Math.*, 3(1987), 101–108.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆