Malaya Journal of Matematik, Vol. 9, No. 1, 256-258, 2021
https://doi.org/10.26637/MJM0901/0042

$\Gamma-$Synchronization of single valued neutrosophic automata

V. Karthikeyan[1]

Abstract
In this paper $\Gamma-$ Synchronization of single valued neutrosophic automata(SVNA) are introduced, algorithm is given for finding $\Gamma-$ Synchronized words of single valued neutrosophic automata.

Keywords
Single Valued neutrosophic set, Single Valued neutrosophic automaton, S$\Gamma-$Synchronization.

AMS Subject Classification
03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

[1] *Department of Mathematics, Government College of Engineering, Dharmapuri-636704, Tamil Nadu, India.*
***Corresponding author**: [1] vkarthikau@gmail.com
Article History: Received **11** November **2020**; Accepted **16** January **2021**

Contents

1 Introduction 256
2 Preliminaries 256
3 $\Gamma-$Synchronization of Single Valued Neutrosophic Automata 257
4 Algorithm for Finding $\Gamma-$ Synchronized Word of Single Valued Neutrosophic Automata 257
5 Procedure for Finding $\Gamma-$ Synchronized Word of Single Valued Neutrosophic Automata 258
6 Conclusion 258
References 258

1. Introduction

The theory of neutrosophy and neutrosophic set was introduced by Florentin Smarandache in 1999 [6]. The neutrosophic set is the generalization of classical sets, fuzzy set[9], intuitionstic fuzzy set[1], interval valued intuitionistic fuzzy sets [2] and so on. The concept of fuzzy set and intuitionstic fuzzy set unsuccessful when the relation is indeterminate.

The theory of fuzzy sets was introduced by Zadeh in 1965[9] as a generalizations of crisp sets. Since then the fuzzy sets and fuzzy logic are used widely in many applications involving uncertainty. Attanasov introduced the concept of intuitionistic fuzzy sets in 1986 [1] which is an extension of fuzzy set. In intuitionistic fuzzy set, each element of the set representing by a membership grade and non-membership grade. Some other generalizations of fuzzy sets are bipolar valued fuzzy set [4], vague set [3] and so on.

A neutrosophic set N is classified by a Truth membership function T_N, Indeterminacy membership function I_N, and Falsity membership function F_N, where T_N, I_N, andF_N are real standard and non-standard subsets of $]0^-, 1^+[$.
Wang $etal$. [7] introduced the notion of single valued neutrosophic sets.

In this paper we consider deterministic,strongly connected and aperiodic SVNA then it is called a $\Gamma-$synchronized SVNA. The synchronizing word does not always exist in any SVNA. But in a deterministic,strongly connected and aperiodic SVNA, synchronizing word is always exist. Initially, it may not exist but changing some labeling we get synchroning word.

The notion of the automaton was first fuzzified by Wee [8]. The concept of single valued neutrosophic finite state machine was introduced by Tahir Mahmood [5]. In this paper, the concept of $\Gamma-$synchronization of single valued neutrosophic automata are introduced. Also, algorithm is given to find $\Gamma-$synchronized words. Finally, Procedure to find Synchronization word in single valued neutrosophic automata is given.

2. Preliminaries

In this section, we recall some definitions and basic results of fractional calculus which will be used throughout the paper.

Definition 2.1. *[5] A fuzzy automata is triple $F = (S, A, \alpha)$ where S, A are finite non empty sets called set of states and* *set of input alphabets and α is fuzzy transition function in $S \times A \times S \to [0,1]$.*

Definition 2.2. *[6] Let U be the universe of discourse. A neutrosophic set (NS) N in U is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N, where T_N, I_N, and F_N are real standard or non-standard subsets of $]0^-, 1^+[$. That is*
$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_N, I_N, F_N \in]0^-, 1^+[\ \}$
and with the condition
$0^- \leq sup\ T_N(x) + sup\ I_N(x) + sup\ F_N(x) \leq 3^+$.
we need to take the interval $[0,1]$ for technical applications instead of $]0^-, 1^+[$.

Definition 2.3. *[6] Let U be the universe of discourse. A single valued neutrosophic set (NS) N in U is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N.*
$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_N, I_N, F_N \in\ [0,1].\}$

Definition 2.4. *[5] $F = (S, A, N)$ is called single valued neutrosophic automaton (SVNA for short), where S and A are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \eta_N(x), \zeta_N(x), \rho_N(x)\rangle\}$ is an SVNS in $S \times A \times S$. The set of all words of finite length of A is denoted by A^*. The empty word is denoted by ε, and the length of each $x \in A^*$ is denoted by $|x|$.*

Definition 2.5. *[5] $F = (S,\ A,\ N)$ be an SVNA. Define an SVNS $N^* = \{\langle \eta_{N^*}(x), \zeta_{N^*}(x), \rho_{N^*}(x)\rangle\}$ in $S \times A^* \times S$ by*

$$\eta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 1 & if\ q_i = q_j \\ 0 & if\ q_i \neq q_j \end{cases}$$

$$\zeta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & if\ q_i = q_j \\ 1 & if\ q_i \neq q_j \end{cases}$$

$$\rho_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & if\ q_i = q_j \\ 1 & if\ q_i \neq q_j \end{cases}$$

$\eta_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\eta_{N^*}(q_i, x, q_r) \wedge \eta_{N^*}(q_r, y, q_j)]$,
$\zeta_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\zeta_{N^*}(q_i, x, q_r) \vee \zeta_{N^*}(q_r, y, q_j)]$,
$\rho_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\rho_{N^*}(q_i, x, q_r) \vee \rho_{N^*}(q_r, y, q_j)]$,
$\forall q_i, q_j \in S,\ x \in A^*$ *and* $y \in A$.

3. $\Gamma-$Synchronization of Single Valued Neutrosophic Automata

Definition 3.1. *Let $F = (S, A, N)$ be an SVNA. If F is said to be deterministic SVNA then for each $q_i \in Q$ and $x \in A$ there exists unique state q_j such that $\eta_{N^*}(q_i, x, q_j) > 0$. $\zeta_{N^*}(q_i, x, q_j) < 1$, $\rho_{N^*}(q_i, x, q_j) < 1$.*

Definition 3.2. *Let $\Theta = p_1, p_2, ..., p_z$ be a partition of the states set S such that if $\eta_{N^*}(q_i, x, q_j) > 0$. $\zeta_{N^*}(q_i, x, q_j) < 1$, $\rho_{N^*}(q_i, x, q_j) < 1$. for some $x \in A$ then $q_i \in p_s$and $q_j \in p_{s+1}$. Then Θ will be called periodic partition of order $z \geq 2$. An SVNA F is periodic of period $z \geq 2$ if and only if $z = Maxcard(\Theta)$ where this maximum is taken over all periodic partitions Θ of F. If F has no periodic partition, then F is called aperiodic. Throughout this paper we consider aperiodic SVNA.*

Definition 3.3. *Let $F = (S, A, N)$ be an SVNA. If two states q_i and q_j are said to be stability related and it is denoted by $q_i \Omega\ q_j$ if for any word $w_1 \in A^*$ there exists a word $w_2 \in A^*$, $q_k \in S$ such that*
$\eta_{N^*}(q_i, w_1w_2, q_k) > 0 \Leftrightarrow \eta_{N^*}(q_j, w_1w_2, q_k) > 0$
$\zeta_{N^*}(q_i, w_1w_2, q_k) < 1 \Leftrightarrow \zeta_{N^*}(q_j, w_1w_2, q_k) < 1$
$\rho_{N^*}(q_i, w_1w_2, q_k) < 1 \Leftrightarrow \rho_{N^*}(q_j, w_1w_2, q_k) < 1$

Example 3.4. *Let $F = (S, A, N)$ be an single valued neutrosophic automaton, where*
$S = \{q_1, q_2, q_3, q_4\}$, $A = \{x, y\}$, *and N are defined as below.*
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, x, q_4) = [0.3, 0.4, 0.5]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, y, q_2) = [0.5, 0.2, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, x, q_3) = [0.7, 0.1, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, y, q_4) = [0.1, 0.6, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, x, q_2) = [0.2, 0.5, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, y, q_4) = [0.5, 0.2, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, x, q_1) = [0.6, 0.3, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, y, q_3) = [0.7, 0.3, 0.2]$
For anyword $w \in A^*$, there exists a word $xyy \in A^*$ such that
$\eta_{N^*}(q_1, wxyy, q_k) > 0 \Leftrightarrow \eta_{N^*}(q_4, wxyy, q_k) > 0$
$\zeta_{N^*}(q_1, wxyy, q_k) < 1 \Leftrightarrow \zeta_{N^*}(q_4, wxyy, q_k) < 1$
$(\rho_{N^*}(q_1, wxyy, q_k) < 1 \Leftrightarrow \rho_{N^*}(q_4, wxyy, q_k) < 1 and$
$\eta_{N^*}(q_2, wxyy, q_l) > 0 \Leftrightarrow \eta_{N^*}(q_3, wxyy, q_l) > 0.$
$\zeta_{N^*}(q_2, wxyy, q_l) < 1 \Leftrightarrow \zeta_{N^*}, (q_3, wxyy, q_l) < 1.$
$\rho_{N^*}(q_2, wxyy, q_l) > 0 \Leftrightarrow \rho_{N^*}(q_3, wxyy, q_l) < 1.$
The states q_1, q_4 and q_2, q_3 are stability related.

Definition 3.5. *Let $F = (S, A, N)$ be an SVNA. We say that F is said to be $\Gamma-$ synchronization if there exists a word $w \in A^*$, $q_j \in S$ and a real number Γ with $\Gamma \in (0,1]$such that $\eta_{N^*}(q_i, w, q_j) \geq \Gamma > 0$, $\zeta_{N^*}(q_i, w, q_j) \leq \Gamma < 1$, $\rho_{N^*}(q_i, w, q_j) \leq \Gamma < 1\ \ \forall q_i \in S$.*

Definition 3.6. *Let $F = (S, A, N)$ be an SVNA. Let $P \subseteq Q$ and the $\Gamma-$ synchronization degree is defined as,*
$\theta_F = \wedge_{w \in A^*}\{card(P)\ |\ \wedge\{\eta_{F^*}(S, w, P) > 0, \zeta_{F^*}(S, w, P) < 1, \rho_{F^*}(S, w, P) < 1\}\}$.
A SVNA is $\Gamma-$synchronization if and only if θ_F is equal to 1.

4. Algorithm for Finding $\Gamma-$ Synchronized Word of Single Valued Neutrosophic Automata

Let $F = (S, A, N)$ be an Single valued neutrosophic automata.
1) Obtain the equivalence classes of the states S using stability

relation.
2) Construct the quotient single valued neutrosophic automata G by considering each equivalence class as a state.
3) Relabel the quotient single valued neutrosophic automata along with neutrosophic values G into G' preserving the stability class.
4) Construct New single valued neutrosophic automata F' from G'.
5) New single valued neutrosophic automata G' gives the synchronized word.

Example 4.1. *Consider the Example 3.4 and the quotient single valued neutrosophic automata G is as follows.*
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1q_4, x, q_1q_4) = [0.3, 0.4, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1q_4, y, q_2q_3) = [0.5, 0.3, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2q_3, x, q_2q_3) = [0.2, 0.5, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2q_3, y, q_1q_4) = [0.1, 0.6, 0.3]$
Relabled quotient single valued neutrosophic automata G' is as follows
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1q_4, y, q_1q_4) = [0.5, 0.3, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1q_4, x, q_2q_3) = [0.3, 0.4, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2q_3, x, q_2q_3) = [0.2, 0.5, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2q_3, y, q_1q_4) = [0.1, 0.6, 0.3]$
Relabled single valued neutrosophic automata F' from G' is as follows
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, x, q_2) = [0.5, 0.2, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, y, q_4) = [0.3, 0.4, 0.5]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, x, q_3) = [0.7, 0.1, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, y, q_4) = [0.1, 0.6, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, x, q_2) = [0.2, 0.5, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, y, q_4) = [0.5, 0.2, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, x, q_3) = [0.7, 0.3, 0.2]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, y, q_1) = [0.6, 0.3, 0.3]$
In the relabeled single valued neutrosophic automata there exists a word $xy \in A^*$ in F' such that
$\eta_{N^*}(q_i, xy, q_4) > 0$, $\zeta_{N^*}(q_i, xy, q_4) < 1$, $\rho_{N^*}(q_i, xy, q_4) < 1 \ \forall q_i \in S$.

5. Procedure for Finding $\Gamma-$ Synchronized Word of Single Valued Neutrosophic Automata

Let $F = (S, A, N)$ be a Single valued neutrosophic automata. We define another SVNA F_N as follows:
$F_N = (2^S, A, M_N, S, D \subseteq S)$ where,
$S-$ initial state on F_N,
$D-$ set of all final states on F_N
M_N-single valued neutrosophic transition function on F_N. The SVN transition function M_N is defined by $\eta_{M_N}(Q_i, x, Q_j) = \wedge\{\eta_N(q_i, x, q_j)\}$, $\zeta_{M_N}(Q_i, x, Q_j) = \vee\{\eta_N(q_i, x, q_j)\}$ and $\rho_{M_N}(Q_i, x, Q_j) = \vee\{\rho_N(q_i, x, q_j), q_i \in Q_i, q_j \in Q_j\}, Q_i, Q_j \in 2^Q$ for $x \in A$.
M_N is a deterministic SVNA and further a word W is $\Gamma-$ synchronized in F if and only if there exists a singleton subsets $Q_s \in 2^Q$ such that $\eta_{M_{N^*}}(Q_i, w, Q_s) = \Gamma > 0$, $\zeta_{M_{N^*}}(Q_i, w, Q_s) = \Gamma_1 < 1$, and $\rho_{M_{N^*}}(Q_i, w, Q_s) = \Gamma_2 < 1$.

6. Conclusion

In this paper $\Gamma-$synchronization of single valued neutrosophic automata(SVNA) are introduced, algorithm is given for finding $\Gamma-$synchronized words of single valued neutrosophic automata. Finally procedure is given to find $\Gamma-$ Synchronized word of single valued neutrosophic automata(SVNA).

References

[1] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1986), 87–96.

[2] K. Atanassov, Interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 31(3)(1989), 343–349.

[3] W. L. Gau and D. J. Buehrer, Vague sets, *IEEE Trans. Syst. Man Cybern.* 23(2)(1993), 610–614.

[4] K. M. Lee, Bipolar-valued fuzzy sets and their operations, *Proc.Int. Conf. on Intelligent Technologies,* (2000), 307–312.

[5] T. Mahmood, and Q. Khan, Interval neutrosophic finite switchboard state machine, *Afr. Mat.* 20(2)(2016),191-210.

[6] F. Smarandache, A Unifying Field in Logics, Neutrosophy: *Neutrosophic Probability, set and Logic, Rehoboth: American Research Press,* (1999).

[7] H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman *Single Valued Neutrosophic Sets* , *Multispace and Multistructure,* 4(2010), 410–413.

[8] W. G. Wee, On generalizations of adaptive algorithms and application of the fuzzy sets concepts to pattern classification Ph.D. Thesis, *Purdue University,* (1967).

[9] L. A. Zadeh, Fuzzy sets, *Information and Control,* 8(3)(1965),338–353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆