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# New generalized $\hat{\hat{g}}$-closed sets in intuitionistic fuzzy topological spaces

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**Abstract**

In this paper, we introduce the concepts of intuitionistic fuzzy $\hat{\hat{g}}$-closed sets and intuitionistic fuzzy $\hat{\hat{g}}$-open sets. Further, we study some of their properties.

**Keywords**

Intuitionistic fuzzy topology, Intuitionistic fuzzy $\hat{\hat{g}}$-closed set, Intuitionistic fuzzy $\hat{\hat{g}}$-open set.

**AMS Subject Classification**

54A05, 54A40, 54C08, 54C10.

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## Contents



## 1. Introduction

The notions of fuzzy sets was introduced by Zadeh [17] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of generalized closed sets in topological spaces was introduced by Levine [9]. In this paper we introduce intuitionistic fuzzy $\hat{\hat{g}}$-closed sets and intuitionistic fuzzy $\hat{\hat{g}}$-open sets. The relations between intuitionistic fuzzy $\hat{\hat{g}}$-closed sets and other generalizations of intuitionistic fuzzy closed sets are given.

**Definition 1.1.** *[1] Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$$

*where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.*

*Denote IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout the paper, X denotes a non empty set.*

**Definition 1.2.** *[4] A subset of A topological space $(X, \tau)$ is called $\hat{\hat{g}}$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open.*

**Definition 1.3.** *[1] Let A and B be any two IFSs of the form $A=\{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$ and $B=\{\langle x,\ \mu_B(x),\ \nu_B(x)\rangle \mid x \in X\}$. Then*

1. $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,
2. $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,
3. $A^c = \{\langle x,\ \nu_A(x),\ \mu_A(x)\rangle \mid x \in X\}$,
4. $A \cap B = \{\langle x,\ \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,
5. $A \cup B = \{\langle x,\ \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

**Definition 1.4.** *[1] The intuitionistic fuzzy sets $0_\sim = \{\langle x,\ 0,\ 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x,\ 1,\ 0\rangle \mid x \in X\}$ are called the empty set and the whole set of X respectively.*

**Definition 1.5.** *[1] Let A and B be any two IFSs of the form $A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x,\ \mu_B(x),\ \nu_B(x)\rangle \mid x \in X\}$. Then*

1. $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,
2. $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

3. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$,
4. $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$,
5. $((A)^c)^c = A$,
6. $(1_\sim)^c = 0_\sim$ and $(0_\sim)^c = 1_\sim$.

**Definition 1.6.** *[3] An intuitionistic fuzzy topology (IFT in short) on X is a family $\tau$ of IFSs in X satisfying the following axioms :*

1. $0_\sim, 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
3. $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

*In this case the pair (X, $\tau$) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set(IFOS in short) in X.*
*The complement $A^c$ of an IFOS A in an IFTS (X, $\tau$) is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

**Definition 1.7.** *[3] Let (X, $\tau$) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup\{G \mid G \text{ is an IFOS in X and } G \subseteq A\},$$
$$cl(A) = \cap\{K \mid K \text{ is an IFCS in X and } A \subseteq K\}.$$

**Proposition 1.8.** *[3] For any IFSs A and B in (X, $\tau$), we have*

1. $int(A) \subseteq A$,
2. $A \subseteq cl(A)$,
3. A is an IFCS in X $\Leftrightarrow cl(A) = A$,
4. A is an IFOS in X $\Leftrightarrow int(A) = A$,
5. $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$,
6. $int(int(A)) = int(A)$,
7. $cl(cl(A)) = cl(A)$,
8. $cl(A \cup B) = cl(A) \cup cl(B)$,
9. $int(A \cap B) = int(A) \cap int(B)$.

**Proposition 1.9.** *[3] For any IFS A in (X, $\tau$), we have*

1. $int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,
2. $int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,
3. $(int(A))^c = cl(A^c)$,
4. $(cl(A))^c = int(A^c)$.

**Proposition 1.10.** *[3] If A is an IFCS in (X, $\tau$) then cl(A) = A and if A is an IFOS in (X, $\tau$) then int(A) = A. The arbitrary union of IFCSs is an IFCS in (X, $\tau$).*

**Definition 1.11.** *An IFS A in an IFTS (X, $\tau$) is said to be an*

1. *intuitionistic fuzzy $\alpha$-closed set (IF$\alpha$CS in short) if $cl(int(cl(A))) \subseteq A$, [6]*
2. *intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [2]*
3. *intuitionistic fuzzy semi pre closed set (IFSPCS in short) if $int(cl(int(A))) \subseteq A$. [16]*

**Definition 1.12.** *An IFS A in an IFTS (X, $\tau$) is said to be an*

1. *intuitionistic fuzzy $\alpha$-open set (IF$\alpha$OS in short) if $A \subseteq int(cl(int(A)))$, [6]*
2. *intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$, [5]*
3. *intuitionistic fuzzy semi pre open set (IFSPOS in short) if $A \subseteq cl(int(cl(A)))$. [16]*

**Remark 1.13.** *[8] We have the following implications.*

$$IFCS \rightarrow IF\alpha CS \rightarrow IFSCS \rightarrow IFSPCS$$

*None of the above implications are reversible.*

**Definition 1.14.** *[10] Let A be an IFS in an IFTS (X, $\tau$). Then the $\alpha$-interior of A ($\alpha int(A)$ in short) and the $\alpha$-closure of A ($\alpha cl(A)$ in short) are defined as*

$$\alpha int(A) = \cup\{G \mid G \text{ is an IF}\alpha\text{OS in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap\{K \mid K \text{ is an IF}\alpha\text{CS in } (X, \tau) \text{ and } A \subseteq K\}.$$

*sint(A), scl(A), spint(A) and spcl(A) are similarly defined.*
*For any IFS A in (X, $\tau$), we have $\alpha cl(A^c) = (\alpha int(A))^c$ and $\alpha int(A^c) = (\alpha cl(A))^c$.*

**Remark 1.15.** *[10] Let A be an IFS in an IFTS (X, $\tau$). Then*

1. $\alpha cl(A) = A \cup cl(int(cl(A)))$,
2. $\alpha int(A) = A \cap int(cl(int(A)))$.

**Definition 1.16.** *An IFS A in (X, $\tau$) is said to be an*

1. *intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, $\tau$), [14]*
2. *intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, $\tau$), [12]*
3. *intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, $\tau$), [13]*
4. *intuitionistic fuzzy $\alpha$ generalized closed set (IF$\alpha$GCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, $\tau$), [10]*

5. *intuitionistic fuzzy $\alpha$ generalized semi closed set (IF$\alpha$GSCS in short) if $\alpha$cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, $\tau$), [7]*
6. *intuitionistic fuzzy $\omega$ closed set (IF$\omega$CS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, $\tau$), [13]*
7. *intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, $\tau$). [11]*

*The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.*

**Remark 1.17.** *[13]*

1. *Every IFOS is an IFSGOS,*
2. *Every IFSOS is an IFSGOS.*

**Definition 1.18.** *[15] Two IFSs A and B are said to be q-coincident (AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFS A and B of (X, $\tau$), A$\bar{q}$B if and only if A $\subseteq$ $B^c$.*

## 2. Intuitionistic fuzzy $\hat{\hat{g}}$-closed sets

In this section we introduce intuitionistic fuzzy $\hat{\hat{g}}$-closed sets and study some of their propositionerties.

**Definition 2.1.** *An IFS A in an IFTS (X, $\tau$) is said to be an intuitionistic fuzzy $\hat{\hat{g}}$-closed set (IF$\hat{\hat{G}}$OS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$).*

*The collection of all intuituionistic fuzzy $\hat{\hat{g}}$-closed sets in X is denoted by IF$\hat{\hat{G}}$C(X).*

**Example 2.2.** *Let X = $\{a, b\}$. Let $\tau$ = $\{0_\sim$, G, $1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. We have $\mu_G(a)$ = 0.3, $\mu_G(b)$=0.4, $\nu_G(a)$ = 0.6 and $\nu_G(b)$ = 0.5. Consider an IFS A = $\langle$x, (0.6, 0.5), (0.3, 0.4)$\rangle$. Then A is an IF$\hat{\hat{G}}$OS.*

**Example 2.3.** *Let X = $\{a, b\}$. Let $\tau$ = $\{0_\sim$, G, $1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. We have $\mu_G(a)$ = 0.3, $\mu_G(b)$=0.4, $\nu_G(a)$ = 0.6 and $\nu_G(b)$ = 0.5. Consider an IFS A = $\langle$x, (0.3, 0.4), (0.6, 0.5)$\rangle$. Then A is not an IF$\hat{\hat{G}}$OS.*

**Theorem 2.4.** *Every IFCS in an IFTS (X, $\tau$) is an IF$\hat{\hat{G}}$OS, but not conversely.*

*Proof.* Let A be an IFCS in (X, $\tau$). Let U be an IFSGOS such that A $\subseteq$ U. Since A is IFCS, cl(A) = A. Thus we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Therefore A is an IF$\hat{\hat{G}}$OS in (X, $\tau$). Hence every IFCS is an IF$\hat{\hat{G}}$OS.

**Theorem 2.5.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IFGSPCS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Since spcl(A) $\subseteq$ cl(A), we have spcl(A) $\subseteq$ U. Therefore A is an IFGSPCS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IFGSPCS.

**Theorem 2.6.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IF$\omega$CS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFSOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Therefore A is an IF$\omega$CS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IF$\omega$CS.

**Theorem 2.7.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IFGCS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Therefore A is an IFGCS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IFGCS.

**Theorem 2.8.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IF$\alpha$GCS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Since $\alpha$cl(A) $\subseteq$ cl(A), we have $\alpha$cl(A) $\subseteq$ U. Therefore A is an IF$\alpha$GCS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IF$\alpha$GCS.

**Theorem 2.9.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IFGSCS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Since scl(A) $\subseteq$ cl(A), we have scl(A) $\subseteq$ U. Therefore A is an IFGSCS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IFGSCS.

**Definition 2.10.** *An IFS A in an IFTS (X, $\tau$) is said to be an intuitionistic fuzzy $\psi$-closed set (IF$\psi$CS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). The complement of intuitionistic fuzzy $\psi$-closed set is called intuitionistic fuzzy $\psi$-open set (IF$\psi$OS in short).*

**Theorem 2.11.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IF$\psi$CS, but not conversely.*

*Proof.* Let A $\subseteq$ U where U is an IFSGOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSGOS in (X, $\tau$). Since scl(A) $\subseteq$ cl(A), we have scl(A) $\subseteq$ U. Therefore A is an IF$\psi$CS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IF$\psi$CS.

**Example 2.12.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨x, (0.3, 0.2), (0.6, 0.7)⟩. Consider an IFS A = ⟨x, (0.4, 0.3), (0.5, 0.6)⟩. Then A is an IF$\psi$CS but not an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

**Theorem 2.13.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IF$\alpha$GSCS, but not conversely.*

*Proof.* Let A ⊆ U where U is an IFSOS in (X, $\tau$). Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFSGOS in (X, $\tau$). Since $\alpha$cl(A) ⊆ cl(A), we have $\alpha$cl(A) ⊆ U. Therefore A is an IF$\alpha$GSCS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IF$\alpha$GSCS.

**Definition 2.14.** *An IFS A in an IFTS (X, $\tau$) is said to be an intuitionistic fuzzy $\hat{\hat{g}}_\alpha$-closed set (IF$\hat{\hat{g}}_\alpha$CS in short) if $\alpha$cl(A) ⊆ U whenever A ⊆ U and U is an IFSGOS in (X, $\tau$). The complement of intuitionistic fuzzy $\hat{\hat{g}}_\alpha$-closed set is called an intuitionistic fuzzy $\hat{\hat{g}}_\alpha$-open set (IF$\hat{\hat{g}}_\alpha$OS in short).*

**Theorem 2.15.** *Every IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) is an IF$\hat{\hat{g}}_\alpha$CS, but not conversely.*

*Proof.* Let A be an IF$\hat{\hat{G}}$OS in (X, $\tau$). Then we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFSGOS in (X, $\tau$). Since $\alpha$cl(A) ⊆ cl(A), we have $\alpha$cl(A) ⊆ U, whenever A ⊆ U and U is an IFSGOS in (X, $\tau$). Therefore A is an IF$\hat{\hat{g}}_\alpha$CS in (X, $\tau$). Hence every IF$\hat{\hat{G}}$OS is an IF$\hat{\hat{g}}_\alpha$CS.

**Remark 2.16.** *IF$\alpha$CS and IF$\hat{\hat{G}}$OS are independent.*

**Example 2.17.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨x, (0.3, 0.2), (0.7, 0.8)⟩. Consider an IFS A = ⟨x, (0.2, 0.4), (0.8, 0.6)⟩. Then A is an IF$\alpha$CS but not an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

**Example 2.18.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, $G_1$, $G_2$, $1_\sim$} be an IFT on X, where $G_1$ = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and $G_2$ = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Consider an IFS A = ⟨x, (0.75, 0.65), (0.15, 0.25)⟩. Then A is an IF$\hat{\hat{G}}$OS but not an IF$\alpha$CS in (X, $\tau$).*

**Remark 2.19.** *IFSCS and IF$\hat{\hat{G}}$OS are independent.*

**Example 2.20.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩. Consider an IFS A = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Then A is an IFSCS but not an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

**Example 2.21.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, $G_1$, $G_2$ $1_\sim$} be an IFT on X, where $G_1$ = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and $G_2$ = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Consider an IFS A = ⟨x, (0.8, 0.7), (0.1, 0.2)⟩. Then A is an IF$\hat{\hat{G}}$OS but not an IFSCS in (X, $\tau$).*

**Theorem 2.22.** *If A and B are IF$\hat{\hat{G}}$OSs in an IFTS (X, $\tau$), then A ∪ B is also an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

*Proof.* If A ∪ B ⊆ G and G is IFSGOS, then A ⊆ G and B ⊆ G. Since A and B are IF$\hat{\hat{G}}$OSs, cl(A) ⊆ G and cl(B) ⊆ G and hence cl(A) ∪ cl(B) = cl(A ∪ B) ⊆ G. Thus A ∪ B is IF$\hat{\hat{G}}$OS in (X, $\tau$).

**Remark 2.23.** *The intersection of two IF$\hat{\hat{G}}$OSs in an IFTS (X, $\tau$) need not be an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

**Example 2.24.** *Let X = {a, b}. Let $\tau$ = {$0_\sim$, $G_1$, $G_2$ $1_\sim$} be an IFT on X, where $G_1$ = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and $G_2$ = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Consider the two IFSs A = ⟨x, (0.1, 0.7), (0.8, 0.2)⟩ and B = ⟨x, (0.8, 0.2), (0.1, 0.7)⟩. Then A and B are IF$\hat{\hat{G}}$OSs. But A ∩ B = ⟨x, (0.1, 0.2), (0.8, 0.7)⟩ is not an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

**Theorem 2.25.** *If A is an IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$) and A ⊆ B ⊆ cl(A), then B is an IF$\hat{\hat{G}}$OS in (X, $\tau$).*

*Proof.* Let B ⊆ U where U is an IFSGOS in (X, $\tau$). Since A ⊆ B, A ⊆ U. Since A is an IF$\hat{\hat{G}}$OS in (X, $\tau$), cl(A) ⊆ U. Since B ⊆ cl(A), cl(B) ⊆ cl(A) ⊆ U. Therefore B is an IF$\hat{\hat{G}}$OS in (X, $\tau$).

**Theorem 2.26.** *Let A be an IFS in an IFTS (X, $\tau$). Then A is an IF$\hat{\hat{G}}$OS if and only if A$\bar{q}$F implies cl(A)$\bar{q}$F for every IFSGCS F in (X, $\tau$).*

*Proof.* Necessary Part: Let F be an IFSGCS in (X, $\tau$) and let A$\bar{q}$F. Then A ⊆ $F^c$, where $F^c$ is an IFSGOS in (X, $\tau$). Therefore by hypothesis cl(A) ⊆ $F^c$. Hence cl(A)$\bar{q}$F.
Sufficient Part: Let F be an IFSGCS in (X, $\tau$) and let A be an IFS in (X, $\tau$). By hypothesis, A$\bar{q}$F implies cl(A)$\bar{q}$F. Then cl(A) ⊆ $F^c$ whenever A ⊆ $F^c$ and $F^c$ is an IFSGOS in (X, $\tau$). Hence A is an IF$\hat{\hat{G}}$OS in (X, $\tau$).

**Theorem 2.27.** *Let (X, $\tau$) be an IFTS. Then IFC(X) = IF$\hat{\hat{g}}$C(X) if every IFS in (X, $\tau$) is an IFSGOS in X, where IFC(X) denotes the collection of IFCSs of an IFTS (X, $\tau$).*

*Proof.* Suppose that every IFS in (X, $\tau$) is an IFSGOS in X. Let A ∈ IF$\hat{\hat{g}}$C(X). Then cl(A) ⊆ U whenever A ⊆ U and U is an IFSGOS in X. Since every IFS is an IFSGOS, A is also an IFSGOS and A ⊆ A. Therefore cl(A) ⊆ A. Hence cl(A) = A. Therefore A ∈ IFC(X). Hence IF$\hat{\hat{g}}$C(X) ⊆ IFC(X) → (i). Let A ∈ IFC(X). Then by Theorem 2.4, A ∈ IF$\hat{\hat{g}}$C(X). Hence IFC(X) ⊆ IF$\hat{\hat{g}}$C(X) → (ii). From (i) and (ii), we have IFC(X) = IF$\hat{\hat{g}}$C(X).

**Proposition 2.28.** *If A is an IFSGOS and IF$\hat{\hat{G}}$OS in an IFTS (X, $\tau$), then A is an IFCS in (X, $\tau$).*

*Proof.* Since A is an IFSGOS and IF$\hat{\hat{G}}$OS, cl(A) ⊆ A. Hence A is an IFCS in (X, $\tau$).

## 3. Conclusion

One must be in "love" with Mathematics is the intrinsic nature and beauty of Mathematics. As a result, the nature of inquisitiveness in a person gets always enkindled and triggered by new theorems, axioms, even if it is mighty small in its nature or incredibly big.

Intuitionistic fuzzy topology is applied to many fields such as Mathematics, Physics, Chemistry, Biology, Engineering and so on. This theory is definitely an eye opener for new research works. We can apply these findings into other research areas of general topology such as Fuzzy topology, Digital Topology, Nano Topology and so on.

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