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Different operations of Sunlet graph S_n

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Abstract

In this paper, we clarify that the Sunlet diagram Sn has concedes Vertex Prime labeling. At the point when the graph has Vertex Prime labeling that diagram is called Vertex Prime graph. A Graph $G = (V,E)$ is said to have a Vertex Prime labeling if its edges can be marked with unmistakable whole numbers from $\{1,2,3,\ldots,|E|\}$ with the end goal that for every vertex of degree at least two, the Greatest Common Divisor of the names on its occurrence edges is 1. We also prove that Vertex Prime labeling in the context of the operations of namely duplication, fusion and switching of a vertex.

Keywords

Sunlet graph, Prime labeling, Vertex Prime labeling, Duplication of a vertex, Fusion and Switching of a vertex.

AMS Subject Classification

05C78.

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Contents

1. Introduction

The learning of authority in diagram hypothesis is quickest developing zone and it comes because of learn of games, for example, round of chess where the objective is to rule different squares of a chess board by certain chess pieces. In this paper let us consider a finite connected, undirected and simple graph. The Sunlet graph G with $V(G) = \{v_1, v_2, \ldots, v_n; v'_1, v'_2, \ldots, v'_n\}$ and $E(G) = \{v_1v_2,\ v_2v_3, \ldots, v_nv_1; v_1v'_1, v_2v'_2, \ldots, v_nv'_n\}$ is gotten from the cycle C_n with adding swinging vertices to the every one of the vertices v_i of C_n for $i = 1,2,\ldots,n$. Sunlet diagram is meant by S_n. The arrangement of vertices neighboring a vertex v of G is meant by $N(v)$. In the event that the vertices or edges or both of the graph are marked qualities dependent upon certain condition(s) at that point it is known as graph labeling. One of the marvels, Rosa has presented the investigation of graph labeling in 1967[1] . Labeling of graph has huge application in numerous down to earth issues engaged with circuit planning, correspondence organization, cosmology and so forth the idea of Vertex Prime labeling has presented by T. Deretsky, S. M. Lee and J. Milton [2]. A graph G has Vertex Prime labeling if its edges can be marked with particular numbers $1,2,\ldots|E|$. A bijection $f : E \to \{1,2,\ldots,|E|\}$ characterized to such an extent that of every vertex with degree at least 2 than G.C.D of marks on its episode edges is 1. L. W. Beineke and $S.M.$ Hedge [3] has continued graph labeling is portrayed as an outskirts between number hypothesis and structure of graph. A current review of different graph labeling issue can be found in Joseph A. Gallian [4].

In graph labeling issue we are utilize coming up next are the normal highlights.

- ✠ Labels all the vertices utilizing a bunch of numbers.
- ✠ A guideline that marks esteems to each edge.
- ✠ A express that these qualities should fulfill.

We use and allude J.A.Bondy and U.S.R. Murthy [5] for documentations and phrasing. Additionally the idea of a Prime labeling was presented by Roger Entringer and was examined in a paper by A.Tout [6]. In [7] Dr. P. Kavitha and Dr. S. Meena has demonstrated Vertex Prime Labeling for some Helm related graph. Likewise S.Lavanya and Dr. V. Ganesan has demonstrated Vertex Prime labeling for Umbrella graph $U(m,n)$ [8]. At last Dr.R.Ganesan et.al [9] has demonstrated prime marking for some New Classes of graph. For definitions, I referred some published paper [7], [8].

We will make accessible brief diagram of definition and other data which are fundamental for the current examinations.

2. Main Results

Theorem 2.1. *The Sunlet graph S_n is a Vertex Prime graph for all $n \geq 3$.*

Proof. Let G be the Sunlet graph S_n Obliviously, $|V(S_n)| = 2n$ and $|E(S_n)| = 2n$. The vertex set is

$$V(S_n) = \{v_1, v_2, \ldots, v_n; v'_1, v'_2, \ldots, v_n\}$$

and edge set is

$$E(S_n) = \{v_i v'_i / 1 \leq i \leq n\} U \{v_i v_{i+1} / 1 \leq i \leq n-1\} U \{v_1 v_n\}.$$

Where n is both even and odd. "Characterize a bijection $f : E(S_n) \to \{1, 2, \ldots, 2n\}$ such that

$$\begin{aligned} f(v_i v_{i+1}) &= 2i - 1 \text{ for } 1 \leq i \leq n-1 \\ f(v_n v_1) &= 2n - 1 \\ f(y_i v'_i) &= 2i \text{ for } 1 \leq i \leq n\text{"} \end{aligned}$$

Clearly every edge of S_n receives distinct labeling. Now let $f^*(v)$ denotes the gcd of labeling of edges of S_n where $d(v) \geq 2$ Consider,

$$\begin{aligned} f^*(v_1) = &\text{ gcd of labels of all edges incident at } v_1 \\ &= \gcd\{f(v_1 v_2), f(v_1 v'_1), f(v_1 v_n)\} \\ &= \gcd\{1, 2, 2n-1\} \\ &= 1 \\ f^*(v_i) = &\text{ gcd of labels of all edges incident at } v_i \\ &= \gcd\{f(v_i v_{i-1}), f(v_i v_{i+1}), f(y_i v'_i)\} \\ &= \gcd\{2i-3, 2i-1, 2i\} \quad \text{for} \quad 1 \leq i \leq n-1 \\ &= 1 \text{ since each } y_i \text{ has degree 3 and two edges} \\ &\quad \text{have consecutive labels.} \\ f^*(v_n) = &\text{ gcd of labels of all edges incident at } v_n \\ &= \gcd\{f(v_n v_{n-1}), f(v_n v_1), f(v_n v'_n)\} \\ &= \gcd\{2n-3, 2n-1, 2n\} \\ &= 1 \end{aligned}$$

For example we consider the Sunlet graph for $n = 8$. See Fig. 1.

The Vertex Prime labeling of the Sunlet graph S_8 is given below. See Fig. 2. Thus the labeling for every edge has gcd 1 (i.e.,) $f^*(v_i) = f^*(v_n) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence S_n is a Vertex Prime graph. □

Theorem 2.2. *The graph acquired by duplicating a vertex v_k in the rim of the Sunlet graph S_n is a Vertex Prime graph.*

Proof. Let S_n be the Sunlet graph with $2n$ vertices and $2n$ edges. The vertex set is $V(S_n) = \{v_1, v_2, \ldots, v_n; v'_1, v'_2, \ldots, v'_n\}$ and edge set is

$$E(S_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} U \{v_i v'_i / 1 \leq i \leq n\} U \{v_n v_1\}.$$

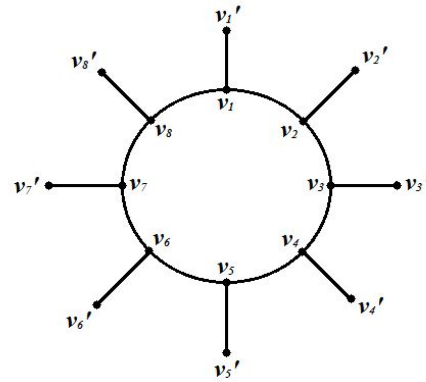

Figure 1. Sunlet graph S_8

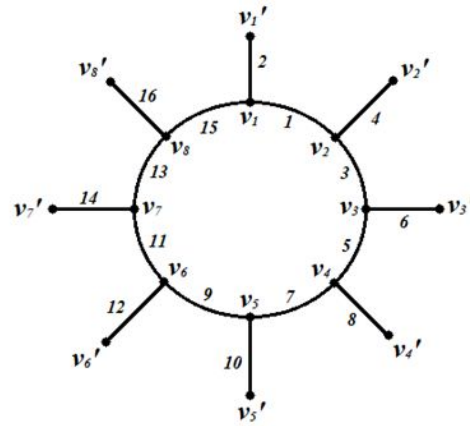

Figure 2. Vertex Prime labeling of the Sunlet graph S_8

Let G_k be the graph acquired by duplicating the vertex v_k in S_n and u_k be the new vertex. Then $|V(G_k)| = 2n+1$ and $|E(G_k)| = 2n+3$.

"Characterize a bijection $f : E(G_k) \to \{1, 2, \ldots, (2n+3)\}$ as follows,

$$\begin{aligned} f(v_i v_{i+1}) &= 2i - 1 \text{ for } 1 \leq i \leq n-1 \\ f(v_n v_1) &= 2n - 1 \\ f(v_i v'_i) &= 2i \text{ for } 1 \leq i \leq n \\ f(v'_k u_k) &= 2n + 1 \\ f(v'_{k-1} u_k) &= 2n + 2 \\ f(v_{k+1}{}' u_k) &= 2n + 3 \text{ where } k \text{ is a positive integer} \end{aligned}$$

For example we consider the Sunlet graph for $n = 8$ (Refer Figure 1) Also the Vertex Prime labeling of the duplication of v_3 in S_8 is given below. See Fig. 3.

Obviously the edge labels are dissimilar. Now consider.

$$\begin{aligned} f^*(v_1) &= \text{gcd of labels of all edges incident at } v_1 \\ &= \gcd\{f(v_1 v_2), f(v_1 v'_1), f(v_1 v_n)\} \\ &= \gcd\{1, 2, 2n-1\} \\ &= 1 \end{aligned}$$

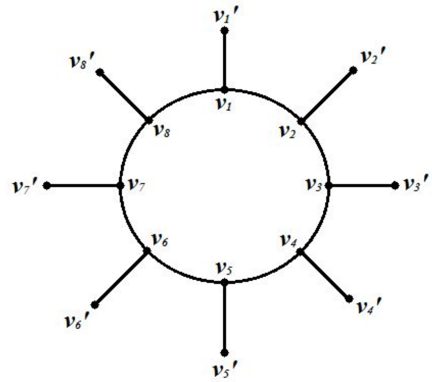

Figure 3. Vertex Prime labeling of duplication of v_3 in the Sunlet graph S_8

$$
\begin{aligned}
f^*(v_2) &= \text{gcd of labels of all edges incident at } v_2 \\
&= \gcd\left\{f(v_2v_1), f(v_2v_3), f\left(v_2v_2'\right), f(v_2u_3)\right\} \\
&= \gcd\{1,3,4,18\} \\
&= 1 \\
f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i \\
&= \gcd\left\{f(v_iv_{i-1}), f(v_iv_{i+1}), f\left(y_iv_i'\right)\right\} \\
&= \gcd\{2i-3, 2i-1, 2i\} \text{ for } i = 3 \ \& \ 5 \le i \le n-1 \\
&= 1 \\
f^*(v_4) &= \text{gcd of labels of all edges incident at } v_4 \\
&= \gcd\left\{f(v_4v_3), f(v_4v_5), f\left(v_4v_4'\right), f(v_4u_3)\right\} \\
&= \gcd\{5,7,8,19\} \\
&= 1 \\
f^*(v_n) &= \text{gcd of labels of all edges incident at } v_n \\
&= \gcd\left\{f(v_{n-1}v_n), f(v_nv_1), f\left(v_nv_n'\right)\right\} \\
&= \gcd\{2n-3, 2n-1, 2n\} \\
&= 1 \\
f^*\left(v_k'\right) &= \text{gcd of labels of all edges incident at } v_k' \\
&= \gcd\left\{f\left(v_k'v_k\right), f\left(v_k'u_k\right)\right\} \\
&= \gcd\{2k, 2n+1\} \\
&= 1 \\
f^*(u_k) &= \text{gcd of labels of all edges incident at } u_k \\
&= \gcd\left\{f\left(u_kv_k'\right), f(u_kv_{k-1}), f(u_kv_{k+1})\right\} \\
&= \gcd\{2n+1, 2n+2, 2n+3\} \\
&= 1
\end{aligned}
$$

Thus $f^*(v_i) = f^*(v_n) = f^*(u_k) = f^*\left(v_k'\right) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence the duplication of any vertex in Sunlet graph is a Vertex Prime graph. □

Theorem 2.3. *The graph acquired by duplicating a pendent vertex v_k' of the Sunlet graph S_n is a Vertex Prime graph.*

Proof. Let G be the Sunlet graph S_n with

$$V(G) = \left\{v_1, v_2, \dots, v_n; v_1', v_2', \dots, v_n'\right\}$$

and

$$E(G) = \{v_iv_{i+1}/1 \le i \le n-1\} U \left\{v_iv_i'/1 \le i \le n\right\} U \{v_nv_1\}.$$

Let G_k be the graph acquired by duplicating any pendent vertex v_k' in S_n. Then $|V(G_k)| = 2n+1$ and $|E(G_k)| = 2n+1$. Let the new vertex be u_k'.

"Characterize a bijection $f : E(G_k) \to \{1, 2, \dots, (2n+1)\}$ as follows,

$$
\begin{aligned}
f(v_iv_{i+1}) &= 2i-1 \text{ for } 1 \le i \le n-1 \\
f(v_nv_1) &= 2n-1 \\
f\left(v_iv_i'\right) &= 2i \text{ for } 1 \le i \le n \\
f\left(v_ku_k'\right) &= 2n+1''
\end{aligned}
$$

Obviously the edge labels are dissimilar. If the duplication vertex $v_k = v_1$ then $v_{k-1} = v_n$ and if the duplication vertex $v_k = v_n$ then $v_{k+1} = v_1$.
Now consider the Sunlet graph for $n = 8$ (Refer Figure 1). The Vertex Prime labeling of duplication of a pendent vertex v_3' of the Sunlet graph S_8 is given below.

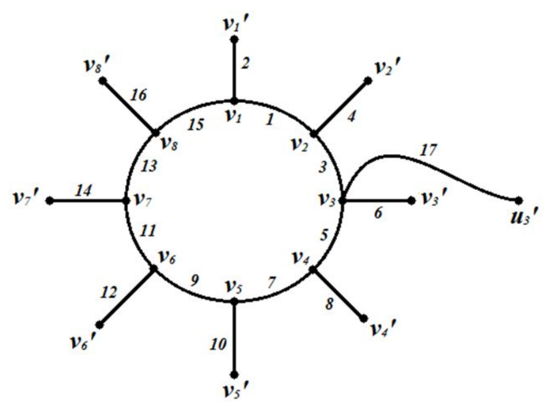

Figure 4. Vertex Prime labeling of duplication of a pendent vertex v_3' in Sunlet graph S_8

Now consider,

$$
\begin{aligned}
f^*(v_1) &= \text{gcd of labels of all edges incident at } v_1 \\
&= \gcd\left\{f(v_1v_2), f\left(v_1v_1'\right), f(v_1v_n)\right\} \\
&= \gcd\{1, 2, 2n-1\} \\
&= 1 \\
f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i \\
&= \gcd\left\{f(v_iv_{i-1}), f(v_iv_{i+1}), f\left(v_iv_i'\right)\right\} \\
&= \gcd\{2i-3, 2i-1, 2i\} \ for \ i = 2 \ \& \ 4 \le i \le n-1 \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
f^*(v_k) &= \text{gcd of labels of all edges incident at } v_k \\
&= \gcd\left\{f\left(v_k u_k'\right), f\left(v_k v_k'\right), f\left(v_k v_{k-1}\right), f\left(v_k v_{k+1}\right)\right\} \\
&= \gcd\{2n+1, 2k, 2k-3, 2k-1\} \quad \text{for} \quad i=k=3 \\
&= 1 \\
f^*(v_n) &= \text{gcd of labels of all edges incident at } v_n \\
&= \gcd\left\{f\left(v_n v_{n-1}\right), f\left(v_n v_1\right), f\left(v_n v_n'\right)\right\} \\
&= \gcd\{2n-3, 2n-1, 2n\} \\
&= 1
\end{aligned}
$$

Thus the labeling for every edges has gcd one. (i.e.,) $f^*(v_i) = f^*(v_n) = f^*(v_k) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence the duplication of any pendent vertex in Sunlet graph is a Vertex Prime graph. □

Theorem 2.4. *The Sunlet graph S_n acquired by duplication of all the rim vertices v_k in S_n is Vertex Prime graph, for all the integers $n \geq 3$.*

Proof. Let S_n be a Sunlet graph with $2n$ vertices and $2n$ edges. Let $V(S_n) = \{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots, v_n'\}$ and

$$E(S_n) = \{v_i v_{i+1}/1 \leq i \leq n-1\} U \left\{v_i v_i'/1 \leq i \leq n\right\} U \{v_n v_1\}.$$

Let G_k be the graph acquired by duplicating of all rim vertices in S_n and $w_1, w_2, \ldots, w_n$ be the new vertices of G_k by duplicating $v_1, v_2, \ldots, v_n$ then

$$V(G_k) = \left\{v_i, v_i', w_i/1 \leq i \leq n\right\}$$

and

$$
\begin{aligned}
E(G_k) = &\left\{y_i v_i', v_i w_i/1 \leq i \leq n\right\} U \{v_i v_{i+1}/1 \leq i \leq n-1\} \\
&U\{v_1 v_n\} U \{v_1 w_n, v_1 w_2\} \\
&U\{v_i w_{i-1}, v_i w_{i+1}/2 \leq i \leq n-1\} U \{v_n w_{n-1}, v_n w_1\}
\end{aligned}
$$

Then $|V(G_k)| = 3n$ and $|E(G_k)| = 5n$.

"Characterize a bijection $f: E(G_k) \to \{1, 2, \ldots, 5n\}$ as follows,

$$
\begin{aligned}
f(v_n v_1) &= 2n-1 \\
f(v_i v_{i+1}) &= 2i-1 \text{ for } 1 \leq i \leq n-1 \\
f\left(v_i v_i'\right) &= 2i \text{ for } 1 \leq i \leq n \\
f\left(v_i' w_i\right) &= 2n+2i-1 \text{ for } 1 \leq i \leq n \\
f(w_i v_{i+1}) &= 2n+2i \text{ for } 1 \leq i \leq n-1 \\
f(w_n v_1) &= 4n \\
f(v_i w_{i+1}) &= 4n+i \text{ for } 1 \leq i \leq n-1 \\
f(v_n w_1) &= 5n\text{"}
\end{aligned}
$$

Consider the Sunlet graph for $n = 8$ (Refer Figure 1). Now, the Vertex Prime labeling of Sunlet graph S_8 and duplication of all rim vertices in S_8 is given below.

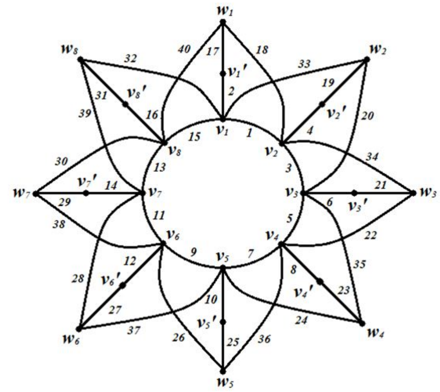

Figure 5. Vertex Prime labeling of duplication of all rim vertices in S_8

Clearly,

$$
\begin{aligned}
f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i \\
&= \gcd\{f(v_{i-1}v_i), f(v_i v_{i+1}), f(w_{i-1}v_i), f(v_i w_{i+1}), \\
&\qquad f\left(v_i v_i'\right)\}; 2 \leq i \leq n-1 \\
&= 1 \text{ since each} v_i \text{ has degree five and two edges} \\
&\qquad \text{have following numbers. (consecutive numbers).} \\
f^*(v_n) &= \text{gcd of labels of all edges incident at } v_n \\
&= \gcd\{f(v_n v_{n-1}), f(v_n v_1), f(w_{n-1}v_n), f\left(v_n v_n'\right), \\
&\qquad f(v_n w_1)\} \\
&= \gcd\{(2n-3), (2n-1), (4n-2), 2n, 4n\} \\
&= 1 \\
f^*(v_1) &= \text{gcd of labels of all edges incident at } v_1 \\
&= \gcd\{f(v_n v_1), f(v_1 v_2), f\left(v_1 v_1'\right), f(w_n v_1), \\
&\qquad f(v_1 w_2)\} \\
&= \gcd\{2n-1, 1, 2, 4n, 4n+1\} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
f^*(w_i) &= \text{gcd of labels of all edges incident at } w_i \\
&= \gcd\left\{f(w_i v_{i-1}), f(w_i v_{i+1}), f\left(w_i v_i'\right)\right\} \\
&\qquad \text{for } 2 \leq i \leq n-1 \\
&= \gcd\{(4n-1)+i, 2n+2i, 2n+2i-1\} \\
&= 1 \\
f^*(w_n) &= \text{gcd of labels of all edges incident at } w_n \\
&= \gcd\left\{f(v_{n-1}w_n), f\left(v_n' w_n\right), f(w_n v_1)\right\} \\
&= \gcd\{(5n-1), (4n-1), 4n\} \\
&= 1 \\
f^*(w_1) &= \text{gcd of labels of all edges incident at } w_1 \\
&= \gcd\left\{f(v_n w_1), f\left(v_1' w_1\right), f(v_2 w_1)\right\} \\
&= \gcd\{5n, 2n+1, 2n+2\} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
f^*\left(v_i'\right) &= \text{gcd of labels of all edges incident at} v_i' \\
&= \gcd\left\{f\left(y_i'v_i\right), f\left(v_i'w_i\right)\right\} \text{ for } 1 \le i \le n \\
&= 1
\end{aligned}
$$

Thus

$$
\begin{aligned}
f^*(v_i) = f^*(v_1) = f^*(v_n) &= 1 \\
f^*(w_i) = f^*(w_1) = f^*(w_n) &= 1 \\
f^*\left(v_i'\right) &= 1 \quad \text{for all } i
\end{aligned}
$$

Therefore f admits Vertex Prime labeling. Hence duplication of all rim vertices in Sunlet graph is a Vertex Prime graph. □

Theorem 2.5. *The Sunlet graph S_n acquired by duplicating of all the pendent edges at each vertex of n-cycle is admits Vertex Prime graph.*

Proof. Let Graph G be a Sunlet graph S_n has $2n$ vertices and $2n$ edges. Let

$$V(G) = \left\{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots, v_n'\right\}$$

and

$$E(G) = \left\{v_iv_{i+1}/1 \le i \le n-1\right\} U \left\{v_iv_i'/1 \le i \le n\right\} U \left\{v_nv_1\right\}$$

Where, $V(G) = V(S_n)$ and $E(G) = E(S_n)$ Let G_k be the graph acquired by duplicating of all the pendent edges at each vertex of the n-cycle. Consider the duplicating vertices is $w_1, w_2, \ldots\ldots w_n$ in S_n. Then $|V(G_k)| = 3n+1$ and $|E(G_k)| = 3n+1$.

"Characterize a bijection $f: E(G_k) \to \{1, 2, \ldots, 3n+1\}$ as follows,

$$
\begin{aligned}
f(v_iv_{i+1}) &= 2i-1 \text{ for } 1 \le i \le n-1 \\
f(y_nv_1) &= 2n-1 \\
f\left(v_iv_i'\right) &= 2i \text{ for } 1 \le i \le n \\
f(v_iw_i) &= 2n+i \text{ for } 1 \le i \le n\text{"}
\end{aligned}
$$

Obviously the edge labels are dissimilar. Consider the Sunlet graph for $n = 8$ (Refer Figure 1). Duplicating of all the pendent edges at each vertex of the n-cycle (8-cycle) is given below. Also its Vertex Prime labels are distinct.
Now consider

$$
\begin{aligned}
f^*(v_1) &= \text{ gcd of labels of all edges incident at } v_1 \\
&= \gcd\left\{f(v_nv_1), f(v_1v_2), f\left(v_1v_1'\right), f(v_1w_1)\right\} \\
&= \gcd\{2n-1, 1, 2, 2n+1\} \\
&= 1
\end{aligned}
$$

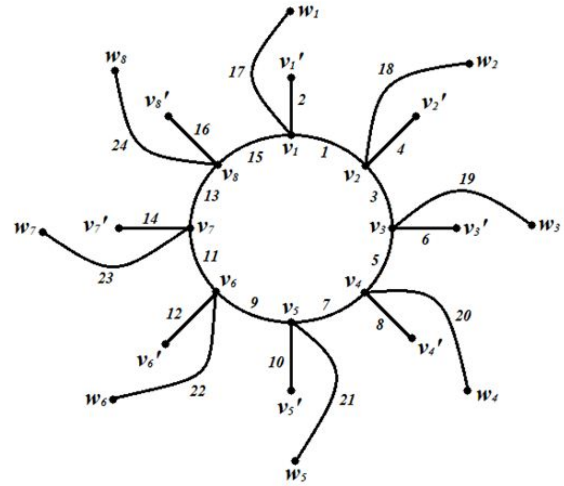

Figure 6. Vertex Prime labeling of duplication of all the pendent edges in S_8

$$
\begin{aligned}
f*(v_i) &= \text{ gcd of labels of all edges incident at } v_i \\
&= \gcd\left\{f(v_{i-1}v_i), f(v_iv_{i+1}), f\left(v_iv_i'\right), f(v_iw_i)\right\} \\
&\quad \text{for } 2 \le i \le n-1 \\
&= 1 \text{ since each } v_i \text{ has degree 4 and 2 edges} \\
&\qquad \text{have consecutive labels.} \\
f^*(v_n) &= \text{ gcd of labels of all edges incident at } v_n \\
&= \gcd\left\{f(v_{n-1}v_n), f(v_nv_1), f\left(v_nv_n'\right), f(v_aw_n)\right\} \\
&= \gcd\{2n-3, 2n-1, 2n, 3n\} \\
&= 1
\end{aligned}
$$

Thus $f^*(v_1) = f^*(v_i) = f^*(v_n) = 1$ for all i. Therefore f admits Vertex Prime labeling. Hence duplication of all the pendent edges in Sunlet graph is a Vertex Prime graph. □

Theorem 2.6. *The graph S_n acquired by fusing any two consecutive vertices in a Sunlet graph S_n is a Vertex Prime labeling.*

Proof. Let S_n be the Sunlet graph with

$$V(S_n) = \left\{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots, v_n'\right\}$$

and

$$E(S_n) = \left\{v_iv_{i+1}/1 \le i \le n-1\right\} U \left\{v_iv_i'/1 \le i \le n\right\} U \left\{v_nv_1\right\}$$

Where, $V(G) = V(S_n)$ and $E(G) = E(S_n)$. Clearly $|V(S_n)| = 2n$ and $|E(S_n)| = 2n$. Let G_k be the graph acquired from S_n by fusing any two consecutive vertices y_i of S_n. In common, we may take the consecutive vertices v_1 and v_2 are fused to the new vertex v (i.e) $v = v_1v_2$. Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$.

Characterize a bijection $f: E(G_k) \to \{1, 2, \ldots, 2n\}$ as follows

$$
\begin{aligned}
f(v) = f(v_1v_2) &= 1\\
f(vv_1') &= 2\\
f(vv_2') &= 4\\
f(vv_3) &= 3\\
f(v_iv_{i+1}) &= 2i-1 \text{ for } 3 \le i \le n-1\\
f(v_nv) &= 2n-1\\
f(v_iv_i') &= 2i \text{ for } 3 \le i \le n
\end{aligned}
$$

For example we consider the Sunlet graph of $n = 8$ (Refer Figure 1). This is clearly Vertex Prime labeling graph. Let G_k be a new graph acquired by fusion v_1 with v_2 is given below, Now consider,

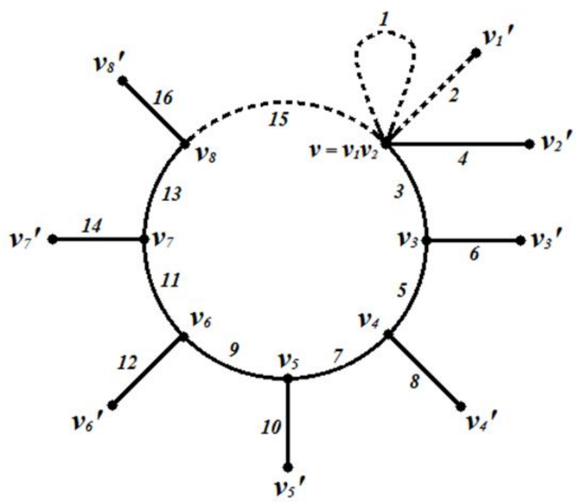

Figure 7. Fusion of consecutive vertices v_1 with v_2 of S_8 and its Vertex Prime labeling

$$
\begin{aligned}
f^*(v) &= \text{gcd of labels of all edges incident at } v\\
&= \gcd\{f(v_xv), f(vv), f(vv_1'), f(vv_2'), f(vv_3)\}\\
&= \gcd\{2n-1, 1, 2, 3, 4\}\\
&= 1 \text{ since } v \text{ has degree 6 and 4 edges have consecutive numbers.}\\
f^*(v_3) &= \text{gcd of labels of all edges incident at } v_3\\
&= \gcd\{f(vv_3), f(v_3v_3'), f(v_3v_4)\}\\
&= \gcd\{3, 6, 5\}\\
&=\\
f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i\\
&= \gcd\{f(v_iv_{i-1}), f(y_iv_i'), f(v_iv_{i+1})\}\\
&= \gcd\{2i-3, 2i, 2i-1\} \quad for 3 \le i \le n-1\\
&= 1\\
f^*(v_n) &= \text{gcd of labels of all edges incident at } v_n\\
&= \gcd\{f(v_{n-1}v_n), f(v_nv_n'), f(v_nv)\}\\
&= \gcd\{2n-3, 2n, 2n-1\}\\
&= 1
\end{aligned}
$$

Since all the labels are distinct. Thus $f^*(v) = f^*(v_3) = f^*(v_i) = f^*(v_n) = 1$ for all i. Therefore f admits Vertex Prime labeling. Hence fusing of any two consecutive vertices in a Sunlet graph is a Vertex Prime graph. □

Theorem 2.7. *The graph S_n acquired by fusing two non adjacency vertices v_i in a Sunlet graph S_n admits Vertex Prime labeling.*

Proof. Let S_n be the Sunlet graph with

$$V(S_n) = \{v_1, v_2, \dots, v_n; v_1', v_2', \dots, v_n'\}$$

and

$$E(S_n) = \{v_iv_{i+1}/1 \le i \le n-1\} U \{y_iv_i'/1 \le i \le n\} U \{v_nv_1\}$$

. Clearly $|V(S_n)| = 2n$ and $|E(S_n)| = 2n$. Let G_k be the graph acquired from S_n by fusion of any two non adjacency vertices v_i of S_n. Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$. In common, we may take the vertices v_1 and v_5 are fusing into new vertex v(i.e)$v = v_1v_5$.

Characterize a bijection $f : E(G_k) \to \{1, 2, \dots, 2n\}$ as follows,

$$
\begin{aligned}
f(v_iv_{i+1}) &= 2i-1 \text{ for } 2 \le i \le n-1\\
f(v_4v_5) &= f(v_4v) \text{ and } f(v_5v_6) = f(vv_6)\\
f(v_nv_1) &= f(v_nv) = 2n-1\\
f(v_iv_i') &= 2i \text{ for } 2 \le i \le n\\
f(vv_1') &= 2\\
\text{and } f(vv_2) &= 1
\end{aligned}
$$

Now consider the Sunlet graph of $n = 8$ (Refer Figure 1) v_1 is fusing with v_5

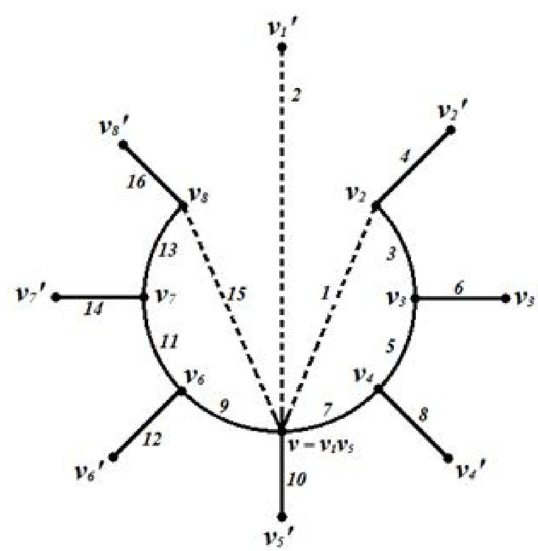

Figure 8. Vertex Prime labeling of fusion of two non adjacency vertices v_1 and v_5 of S_8

Now consider

$$\begin{aligned} f^*(v) &= \text{gcd of labels of all edges incident at } v \\ &= \gcd\{f(vv_2), f(vv_1'), f(vv_4), f(vv_6), f(vv_5'), \\ &\quad f(vv_8)\} \\ &= \gcd\{1,2,7,9,10,15\} \\ &= 1 \\ f^*(v_2) &= \text{gcd of labels of all edges incident at} v_2 \\ &= \gcd\{f(vv_2), f(v_2v_3), f(v_2v_2')\} \\ &= 1 \\ f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i \\ &= \gcd\{f(v_{i-1}v_i), f(v_iv_i'), f(v_iv_{i-1})\} \\ &= \gcd\{2i-3, 2i, 2i-1\} \text{ for } i = 3\&4 \\ &= 1 \\ f^*(v_6) &= \text{gcd of labels of all edges incident at } v_6 \\ &= \gcd\{f(vv_6), f(v_6v_7), f(v_6v_6')\} \\ &= 1 \\ f^*(v_7) &= \text{gcd of labels of all edges incident at } v_7 \\ &= \gcd\{f(v_6v_7), f(v_7v_8), f(v_7v_7')\} \\ &= 1 \\ f^*(v_8) &= \text{gcd of labels of all edges incident at } v_8 \\ &= \gcd\{f(v_7v_8), f(v_8v_8'), f(v_8v)\} \\ &= 1 \end{aligned}$$

Thus $f^*(v_i) = 1$ for all i and $f^*(v) = 1$ Therefore f admits Vertex Prime labeling. Hence fusing of two non adjacency vertices in Sunlet graph is Vertex Prime graph. □

Theorem 2.8. *The graph S_n acquired by fusing any two consecutive vertices v_i' in a Sunlet graph S_n is admits a Vertex Prime graph.*

Proof. Let G be the Sunlet graph with

$$V(S_n) = \{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots, v_n'\}$$

and

$$E(S_n) = \{v_iv_{i+1}/1 \le i \le n-1\} U \{v_iv_i'/1 \le i \le n\} U \{v_nv_1\}$$

Clearly $|V(S_n)| = 2n$ and $|E(S_n)| = 2n$. Let G_k be the graph obtained from S_n by fusion of any two consecutive vertices $y_i{}'$ of S_n. In common, we may take the consecutive vertices v_1' and v_2' are fused to the new vertex v' (i.e) $v' = v_1'v_2'$. Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$.

We consider the Sunlet graph of $n = 8$ (Refer Figure 1). This is clearly Vertex Prime labeling graph. Let G_k be a new graph obtained by fusion $v_1{}'$ with $v_2{}'$ is given below,

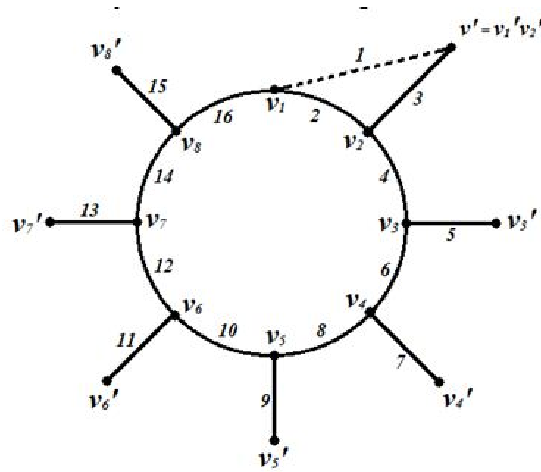

Figure 9. Fusion of consecutive vertices v_1' with v_2' of S_8 and its Vertex Prime labeling

Characterize a bijection $f : E(G_k) \to \{1, 2, \ldots, 2n\}$ as follows,

$$\begin{aligned} f(v_iv_{i+1}) &= 2i \text{ for } 1 \le i \le n-1 \\ f(v_nv_1) &= 2n \\ f(v_1v') &= 1 \\ f(v_2v') &= 3 \\ f(v_iv_i') &= 2i-1 \text{ for } 3 \le i \le n \end{aligned}$$

Obviously the edge labels are dissimilar. Now consider.

$$\begin{aligned} f^*(v_i) &= \text{ gcd of labels of all edges incident at } v_i \\ &= \gcd\{f(v_{i-1}v_i), f(v_iv_i'), f(v_iv_{i+1})\} \text{ for } \\ &\quad 3 \le i \le n-1 \\ &= 1 \\ f^*(v_n) &= \text{ gcd of labels of all edges incident at } v_n \\ &= \gcd\{f(v_{n-1}v_n), f(v_nv_n'), f(v_nv_1)\} \\ &= \gcd\{2n-2, 2n-1, 2n\} \\ &= 1 \\ f^*(v_1) &= \text{ gcd of labels of all edges incident at } v_1 \\ &= \gcd\{f(v_nv_1), f(v_1v_2), f(v_1v')\} \\ &= 1 \\ f^*(v_2) &= \text{ gcd of labels of all edges incident at } v_2 \\ &= \gcd\{f(v_1v_2), f(v'v_2), f(v_2v_3)\} \\ &= 1 \\ f^*(v') &= \text{ gcd of labels of all edges incident at } v' \\ &= \gcd\{f(v'v_1), f(v'v_2)\} \\ &= 1 \end{aligned}$$

Thus $f^*(v_i) = 1$ for all i and $f^*(v') = 1$ Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph. □

Theorem 2.9. *The graph S_n acquired by fusing any two non adjacency vertices v_i' in a Sunlet graph S_n is admits a Vertex Prime labeling.*

Proof. Let G be the Sunlet graph S_n with

$$V(G)=V(S_n)=\{v_1,v_2,\ldots,v_n;v'_1,v'_2,\ldots,v_n{}'\}$$

and

$$E(G)=E(S_n)=\{v_iv_{i+1}/1\leq i\leq n-1\}U\{v_iv'_i/1\leq i\leq n\}U\{v_nv_1\}.$$

Clearly $|V(S_n)|=2n$ and $|E(S_n)|=2n$. Let G_k be the graph obtained from S_n by fusion of any non adjacency vertices v'_i of S_n. In common, we may take the non adjacency vertices v'_1 and v'_3 are fused to the new vertex v'(i.e)$v'=v'_1v'_3$ Then $|V(G_k)|=2n-1$ and $|E(G_k)|=2n$.

Characterize a bijection $f:E(G_k)\to\{1,2,\ldots,2n\}$ as follows,

$$\begin{aligned} f(v_iv_{i+1}) &= 2i \text{ for } 1\leq i\leq n-1\\ f(v_nv_1) &= 2n\\ f(v_1v') &= 1\\ f(v_3v') &= 5\\ f(v_iv'_i) &= 2i-1 \text{ for } i=2 \text{ and } i=4,5,\ldots,n \end{aligned}$$

and Obviously the edge labels are dissimilar. We consider the Sunlet graph of $n=8$ (Refer Figure 1). This is clearly Vertex Prime labeling graph. Let G_k be a new graph obtained by fusion v'_1 with v'_3 is given below,

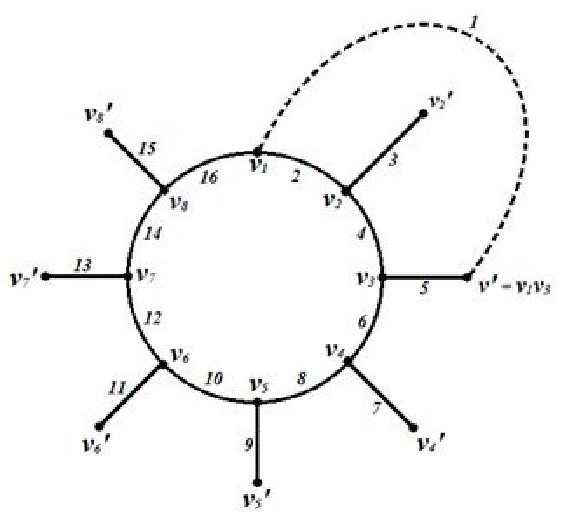

Figure 10. Fusion of two non adjacency vertices v'_1 with v'_3 of S_8 and its Vertex Prime labeling

Now consider,

$$\begin{aligned} f^*(v_1) &= \text{gcd of labels of all edges incident at } v_1\\ &= \gcd\{f(v_nv_1),f(v_1v_2),f(v_1v')\}\\ &= \gcd\{2n,2,1\}\\ &= 1\\ f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i\\ &= \gcd\{f(v_{i-1}v_i),f(y_iv'_i),f(v_iv_{i+1})\}\\ &\qquad \text{for } i=2 \text{ and } 4\leq i\leq n-1\\ &= \gcd\{2i-2,2i-1,2i\}\\ &= 1 \end{aligned}$$

$$\begin{aligned} f^*(v_3) &= \text{gcd of labels of all edges incident at } v_3\\ &= \gcd\{f(v_2v_3),f(v_3v'),f(v_3v_4)\}\\ &= \gcd\{4,6,5\}\\ &= 1\\ f^*(v') &= \text{gcd of labels of all edges incident at } v'\\ &= \gcd\{f(v'v_1),f(v'v_3)\}\\ &= 1 \end{aligned}$$

Thus $f^*(v_i)=1$ for all i and $f^*(v')=1$. Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph. □

Theorem 2.10. *Fusion of any two adjacency and any two non adjacency vertices v_i and v'_i of S_n is admits a Vertex Prime labeling.*

Proof. Let S_n be the Sunlet graph with

$$V(S_n)=\{v_1,v_2,\ldots,v_n;v'_1,v'_2,\ldots,v'_n\}$$

and

$$E(S_n)=\{v_iv_{i+1}/1\leq i\leq n-1\}U\{v_iv'_i/1\leq i\leq n\}U\{v_nv_1\}.$$

Then $|V(S_n)|=2n$ and $|E(S_n)|=2n$.

Now consider the Sunlet graph of $n=8$ (Refer Figure 1). Here we establish the following four cases:

Case (i): v_1 is fusing with v'_1(**i.e**)$v'=v'_1v_1$ (adjacency vertices)

Where v' is new vertex. Let G_k be the graph acquired from S_n by fusing of two vertices $v_1{}'$ and v_1. Then $|V(G_k)|=2n-1$ and $|E(G_k)|=2n$.

Characterize a bijection $f:E(G_k)\to\{1,2,\ldots,2n\}$ as follows,

$$\begin{aligned} f(v')=f(v'_1v_1) &= 1\\ f(v'v_2) &= 2\\ f(v_nv') &= 2n\\ f(v_iy'_i) &= 2i-1 \text{ for } 2\leq i\leq n\\ f(v_iv'_{i+1}) &= 2i \text{ for } \quad 2\leq i\leq n-1 \end{aligned}$$

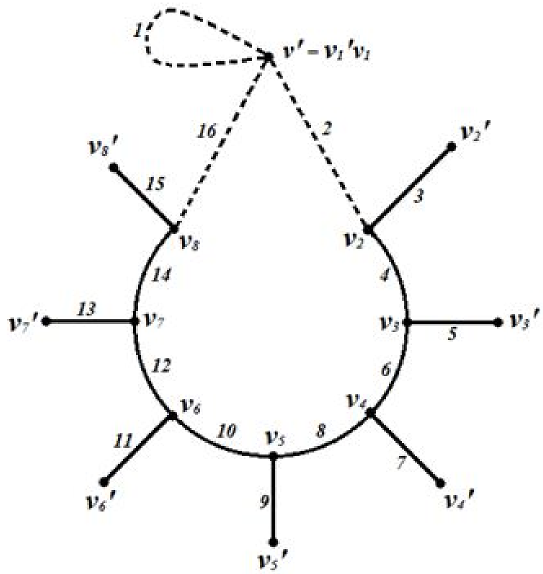

Figure 11. Vertex Prime labeling of fusing of two vertices v_1' and v_1 of S_8

Obviously the edge labels are dissimilar. Also $f^*(v_i) = f^*(v') = 1$ Therefore f admits Vertex Prime labeling. Hence S_8 is admits Vertex Prime graph. (i.e) Fusion of two vertices v_1' and v_1 of S_8 is admits a Vertex Prime labeling.

Case (ii) v_1' is fusing with v_1 (i.e) $v = v_1v_1'$ (adjacency vertices).

Where v is new vertex. Let G_k be the graph acquired from S_n by fusing of two vertices v_1 and $v_1{}'$. Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$.

Characterize a bijection $f : E(G_k) \to \{1, 2, \ldots, 2n\}$ as follows,

$$
\begin{aligned}
f(v) = f\left(v_1v_1{}'\right) &= 2 \\
f(vv_2) &= 1 \\
f(v_nv) &= 2n-1 \\
f(v_iv_{i+1}) &= 2i-1 \text{ for } 2 \le i \le n-1 \\
f\left(v_iv_i'\right) &= 2i \text{ for } \quad 2 \le i \le n
\end{aligned}
$$

Obviously the edge labels are dissimilar. Also $f^*(v_i) = f^*(v) = 1$ Therefore f admits Vertex Prime labeling. Hence S_8 is admits Vertex Prime graph. (i.e) Fusion of two vertices v_1 and v_1 'of S_8 is admits a Vertex Prime labeling.

Case (iii) v_1 is fusing with v_3'(**i.e**) $\cdot v' = v_3'v_1$ (non adjacency vertices)

Let G_k be the graph acquired from S_n by fusion of two non adjacency vertices v_1 and $v_3{}'$ of S_8. Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$.

Characterize a bijection $f : E(G_k) \to \{1, 2, \ldots, 2n\}$ as follows,

$$
\begin{aligned}
f\left(v_1'v'\right) &= 1 f\left(v_2v'\right) = \qquad 2 \\
f\left(v_nv'\right) &= 2n \\
f(v_iv_{i+1}) &= 2i \text{ for } \quad 2 \le i \le n-1 \\
f\left(v_iv_i'\right) &= 2i-1 \text{ for } 4 \le i \le n \\
f\left(v_2'v_2'\right) &= 3 \\
f\left(v_3v'\right) &= 5''
\end{aligned}
$$

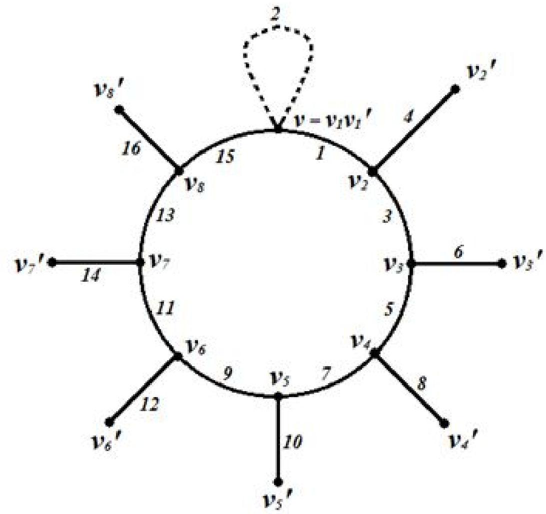

Figure 12. Vertex Prime labeling of fusing of two vertices v_1 and v_1' in S_8

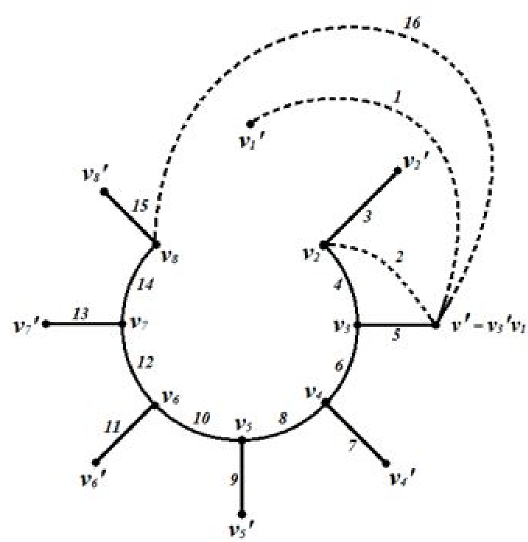

Figure 13. Vertex Prime labeling of fusing of two non adjacency vertices v_1 and v_3' of S_8

Obviously the edge labels are dissimilar. Also $f^*(v_i) = f^*(v) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph.

Case (iv) v_3' is fusing with v_1 (i.e) $v = v_1v_3'$ Let G_k be the graph acquired from S_n by fusion of two non adjacency vertices $v_3{}'$ and v_1 of S_8 Then $|V(G_k)| = 2n-1$ and $|E(G_k)| = 2n$.

Characterize a bijection $f : E(G_k) \rightarrow \{1,2,\ldots,2n\}$ as follows,

$$
\begin{aligned}
f(vv_2) &= 1\\
f(v_n v) &= 2n-1\\
f(vv_1') &= 2\\
f(vv_3) &= 6\\
f(v_2 v_2') &= 4\\
f(v_i v_{i+1}) &= 2i-1 \text{ for } 2 \le i \le n-1\\
f(v_i v_i') &= 2i \text{ for } 4 \le i \le n
\end{aligned}
$$

Obviously the edge labels are dissimilar. Also $f^*(v_i) =$

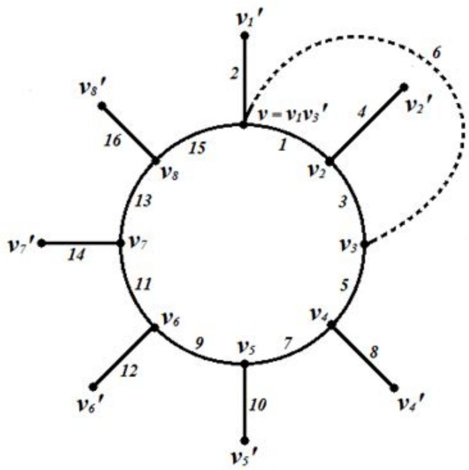

Figure 14. Vertex Prime labeling of fusing of two non adjacency vertices v_3' and v_1 of S_8

$f^*(v) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph. □

Theorem 2.11. *Let G_k be the graph acquired from the Sunlet graph $S_n(n=7)$ by switching any arbitrary vertex v_k of S_n is admits a Vertex Prime labeling.*

Proof. Let S_n be the Sunlet graph with $2n$ vertices and $2n$ edges. Let

$$V(S_n) = \{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots, v_n'\}$$

and

$$E(S_n) = \{v_i v_{i+1} / 1 \le i \le n-1\} U \{v_i v_i' / 1 \le i \le n\} U \{v_n v_1\}.$$

Let G_k be the graph acquired from S_n by switching any arbitrary vertex v_k of S_n. Then $|V(G_k)| = 2n$ and $|E(G_k)| = 4n-7$. In common, we may take the vertices v_1 as the switching vertex.

Characterize a bijection $f : E(G_k) \rightarrow \{1,2,\ldots,4n-7\}$ as follows.

$$
\begin{aligned}
f(v_1 v_{i+1}') &= i \text{ for } 1 \le i \le n-1\\
f(v_i v_{i+1}) &= (n-2)+i \text{ for } 2 \le i \le n-1\\
f(v_i v_i') &= (2n-4)+i \text{ for } 2 \le i \le n\\
f(v_l v_i) &= (3n-6)+i \text{ for } 3 \le i \le n-1\text{”}
\end{aligned}
$$

Consider the Sunlet graph for $n=7$. Now take the vertex v_1 as the switching vertex.

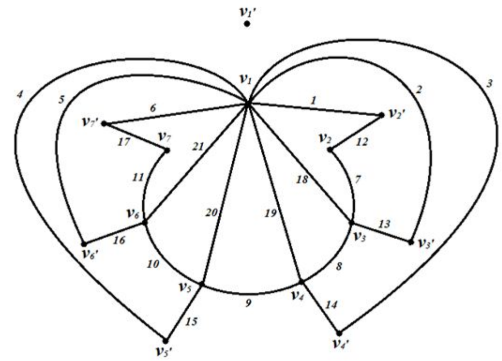

Figure 15. Vertex Prime labeling for switching of v_1 in S_7

Obviously the edge labels are dissimilar. Now consider,

$$
\begin{aligned}
f^*(v_1) &= \text{gcd of labels of all edges incident at } v_1\\
&= \gcd\{f(v_1 v_{i+1}'), f(v_1 v_j)\} \text{ for } \quad 1 \le i \le n-1\\
&\qquad \text{and } 3 \le j \le n-1\\
&= 1 \text{ since } v_1 \text{ has degree 10 and six of the}\\
&\qquad \text{edges have consecutive labels.}\\
f^*(v_2) &= \text{gcd of labels of all edges incident at } v_2\\
&= \gcd\{f(v_2 v_2'), f(v_2 v_3)\}\\
&= 1\\
f^*(v_i) &= \text{gcd of labels of all edges incident at } v_i\\
&= \gcd\{f(v_i v_{i-1}), f(v_i v_{i+1}), f(v_i v_1), f(v_i v_i')\}\\
&\qquad \text{for } 3 \le i \le n-1\\
&= 1\\
f^*(y_n) &= \text{gcd of labels of all edges incident at } v_n\\
&= \gcd\{f(v_{n-1} v_n), f(y_n v_n')\}\\
&= 1\\
f^*(v_i') &= \text{gcd of labels of all edges incident at } v_i'\\
&= \gcd\{f(y_i v_i'), f(v_i' v_1)\}\\
&= 1
\end{aligned}
$$

Thus $f^*(v_i) = f^*(v_i') = 1$ for all i. Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph. □

Theorem 2.12. *Let G_k be the graph acquired from the Sunlet graph S_n by switching any arbitrary vertex v_k ' of S_n is admits a Vertex Prime labeling.*

Proof. Let S_n be the Sunlet graph with $2n$ vertices and $2n$ edges. Let

$$V(S_n) = \{v_1, v_2, \ldots, v_n; v_1', v_2', \ldots v_n'\}$$

and

$$E(S_n) = \{v_i v_{i+1} / 1 \le i \le n-1\} U \{v_i v_i' / 1 \le i \le n\} U \{v_n v_1\}.$$

Let G_k be the graph acquired from S_n by switching any arbitrary vertex $v_k{}'$ of S_n. Then $|V(G_k)| = 2n$ and $|E(G_k)| = 4n-3$. In common, we may take the vertices v'_1 as the switching vertex.

Characterize a bijection $f : E(G_k) \to \{1,2,\ldots,4n-3\}$ as follows,

$$f\left(v'_1 v'_{1+i}\right) = 2i-1 \text{ for } 1 \le i \le n-1$$
$$f\left(v_i v'_i\right) = 2i-1 \text{ for } 2 \le i \le n$$
$$f\left(v_n v_1\right) = 4n-3$$
$$f\left(v_i v_{i+1}\right) = 2(n+i)-3 \text{ for } 1 \le i \le n-1$$
$$f\left(v'_1 v_i\right) = 2(n+i)-4 \text{ for } 2 \le i \le n$$

We consider the Sunlet graph of $n = 8$ (Refer Figure 1). Now take the vertex v_1 'as the switching vertex.

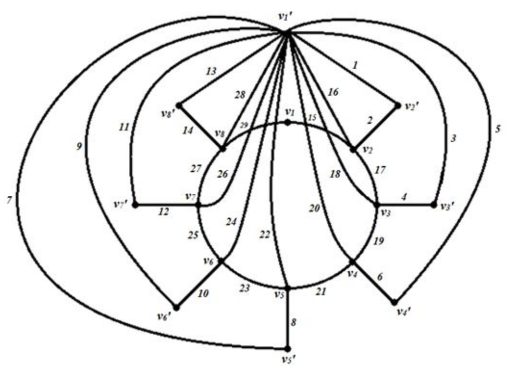

Figure 16. Vertex Prime labeling for Switching of v'_1 in S_8

Obviously the edge labels are dissimilar. Now consider,

$$f^*\left(v'_1\right) = \text{gcd of labels of all edges incident at} v'_1$$
$$= \text{gcd } \left\{f\left(v'_1 v_i\right), f\left(v'_1 v'_i\right)\right\} \quad \text{for} \quad 2 \le i \le n$$
$$=1$$
$$f^*\left(v_1\right) = \text{gcd of labels of all edges incident at } v_1$$
$$= \text{gcd}\left\{f\left(v_n v_1\right), f\left(v_1 v_2\right)\right\}$$
$$=1$$

$$f^*\left(v_i\right) = \text{gcd of labels of all edges incident at } v_i$$
$$= \text{gcd}\left\{f\left(v_{i-1} v_i\right), f\left(v_i v_{i+1}\right), f\left(y_i y'_i\right), f\left(v_i v'_1\right)\right\}$$
$$\text{for} \quad 2 \le i \le n-1$$
$$=1$$
$$f^*\left(v_n\right) = \text{gcd of labels of all edges incident at } v_n$$
$$= \text{gcd}\left\{f\left(v_{n-1} v_n\right), f\left(v_n v_1\right), f\left(v_n v'_n\right), f\left(v_n v'_1\right)\right\}$$
$$=1$$
$$f^*\left(v'_i\right) = \text{gcd of labels of all edges incident at } v'_i$$
$$= \text{gcd}\left\{f\left(y_i v'_i\right), f\left(v'_i v'_1\right)\right\} \quad \text{for} \quad 2 \le i \le n$$
$$=1$$

Thus $f^*\left(v_i\right) = f^*\left(v'_i\right) = 1$ for all i Therefore f admits Vertex Prime labeling. Hence G_k is a Vertex Prime graph. □

3. Conclusion

Here by we have demonstrated the Vertex Prime naming for Sunlet graph S_n. Likewise we talked about Vertex Prime labeling with regards to Duplication, Fusing and Switching of a vertex. In future, this work can be done for different activities to be specific way association and association of Sunlet diagram Sn. To examine comparable outcomes for other graph families and with regards to various naming strategies is an open territory of exploration.

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