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Characterizations of single valued neutrosophic automata using relations

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Abstract
Reducibility and stability relation of single valued neutrosophic automata(SVNA) are introduced and proved that stability relation is equivalence relation in single valued neutrosophic automata.

Keywords
Neutrosophic set, Single Valued neutrosophic set, Single Valued neutrosophic automaton, Stability relation.

AMS Subject Classification
03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

The theory of neutrosophy and neutrosophic set was introduced by Florentin Smarandache in 1999 [6]. The neutrosophic set is the generalization of classical sets, fuzzy set[9], intuitionstic fuzzy set[1], interval valued intuitionistic fuzzy sets [2] and so on. The concept of fuzzy set and intuitionstic fuzzy set unsuccessful when the relation is indeterminate.

The theory of fuzzy sets was introduced by Zadeh in 1965[9] as a generalizations of crisp sets. Since then the fuzzy sets and fuzzy logic are used widely in many applications involving uncertainty. Attanasov introduced the concept of intuitionistic fuzzy sets in 1986 [1] which is an extension of fuzzy set. In intuitionistic fuzzy set, each element of the set representing by a membership grade and non-membership grade. Some other generalizations of fuzzy sets are bipolar valued fuzzy set [4], vague set [3] and so on.

A neutrosophic set N is classified by a Truth membership function T_N, Indeterminacy membership function I_N, and Falsity membership function F_N, where T_N, I_N, andF_N are real standard and non-standard subsets of $]0^-, 1^+[$.

Wang *etal*. [7] introduced the notion of single valued neutrosophic sets.

The notion of the automaton was first fuzzified by Wee [8]. The concept of single valued neutrosophic finite state machine was introduced by Tahir Mahmood [5]. In this paper, the concept of reducibilty and stability relation in single valued neutrosophic automata are introduced. Also proved that stabilty relation is an equivalence relation in single valued neutrosophic automata.

2. Preliminaries

In this section, we recall some definitions and basic results which will be used throughout the paper.

Definition 2.1. *[5] A fuzzy automata is triple $F = (S, A, \alpha)$ where S, A are finite non empty sets called set of states and set of input alphabets and α is fuzzy transition function in $S \times A \times S \to [0,1]$.*

Definition 2.2. *[6] Let U be the universe of discourse. A neutrosophic set (NS) N in U is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N, where T_N, I_N, and F_N are real standard or non-standard subsets of $]0^-, 1^+[$. That is*
$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_N, I_N, F_N \in]0^-, 1^+[\ \}$
and with the condition
$0^- \leq sup\ T_N(x) + sup\ I_N(x) + sup\ F_N(x) \leq 3^+$.
we need to take the interval $[0,1]$ for technical applications instead of $]0^-, 1^+[$.

Definition 2.3. *[6] Let U be the universe of discourse. A single valued neutrosophic set (NS) N in U is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N.*
$N = \{\langle x,(T_N(x),I_N(x),F_N(x))\rangle, x \in U,\ T_N,I_N,F_N \in\ [0,1].\}$

Definition 2.4. *[5] $F = (S,A,N)$ is called single valued neutrosophic automaton (SVNA for short), where S and A are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \eta_N(x),\zeta_N(x),\rho_N(x)\rangle\}$ is an SVNS in $S\times A\times S$. The set of all words of finite length of A is denoted by A^*. The empty word is denoted by ε, and the length of each $x \in A^*$ is denoted by $|x|$.*

Definition 2.5. *[5] $F = (S,\ A,\ N)$ be an SVNA. Define an SVNS $N^* = \{\langle \eta_{N^*}(x),\zeta_{N^*}(x),\rho_{N^*}(x)\rangle\}$ in $S\times A^*\times S$ by*

$$\eta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$$\zeta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

$$\rho_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

$\eta_{N^*}(q_i,xy,q_j) = \vee_{q_r\in Q}[\eta_{N^*}(q_i,x,q_r)\wedge\eta_{N^*}(q_r,y,q_j)]$,
$\zeta_{N^*}(q_i,xy,q_j) = \wedge_{q_r\in Q}[\zeta_{N^*}(q_i,x,q_r)\vee\zeta_{N^*}(q_r,y,q_j)]$,
$\rho_{N^*}(q_i,xy,q_j) = \wedge_{q_r\in Q}[\rho_{N^*}(q_i,x,q_r)\vee\rho_{N^*}(q_r,y,q_j)]$,
$\forall q_i,q_j \in S,\ x\in A^*$ *and* $y \in A$.

3. Characterization of SVNA using relations

Definition 3.1. *Let $F = (S,A,N)$ be an SVNA. If F is said to be deterministic SVNA then for each $q_i \in Q$ and $x \in A$ there exists unique state q_j such that $\eta_{N^*}(q_i,x,q_j) > 0$. $\zeta_{N^*}(q_i,x,q_j) < 1$, $\rho_{N^*}(q_i,x,q_j) < 1$.*

Definition 3.2. *Let $\Theta = p_1,p_2,...,p_z$ be a partition of the states set S such that if $\eta_{N^*}(q_i,x,q_j) > 0$. $\zeta_{N^*}(q_i,x,q_j) < 1$, $\rho_{N^*}(q_i,x,q_j) < 1$. for some $x \in A$ then $q_i \in p_s$and $q_j \in p_{s+1}$. Then Θ will be called periodic partition of order $z \geq 2$. An SVNA F is periodic of period $z \geq 2$ if and only if $z = Maxcard(\Theta)$ where this maximum is taken over all periodic partitions Θ of F. If F has no periodic partition, then F is called aperiodic. Throughout this paper we consider aperiodic SVNA.*

Definition 3.3. *Let $F = (S,A,N)$ be an SVNA. A relation R on a set S is said to be an equivalence relation if it is reflexive, symmetric and transitive.*

Definition 3.4. *Let $F = (S,A,N)$ be an SVNA. An equivalence relation R on a set S is said to be congruence relation if $\forall q_i,q_j \in Q$ and $x \in A$, $q_i\ R\ q_j$ implies that then there exists $q_l,q_k \in S$ such that*
$\eta_{N^*}(q_i,x,q_l) > 0, \eta_{N^*}(q_j,x,q_k) > 0$
$\zeta_{N^*}(q_i,x,q_l) < 1, \zeta_{N^*}(q_j,x,q_k) < 1$ *and*
$\rho_{N^*}(q_i,x,q_l) > 0, \rho_{N^*}(q_j,x,q_k) < 1$.

Definition 3.5. *Let $F = (S,A,N)$ be an SVNA. If q_i and q_j, $q_i,q_j \in S$ are said to be reducible relation and it is denoted by $q_i\Upsilon q_j$ if there exist a word $w \in A^*, q_k \in S$ such that*
$\eta_{N^*}(q_i,w,q_k) > 0 \Leftrightarrow \eta_{N^*}(q_j,w,q_k) > 0$
$(\zeta_{N^*}(q_i,w,q_k) > 0 \Leftrightarrow \zeta_{N^*}(q_j,w,q_k) < 1$
$\rho_{N^*}(q_i,w,q_k) > 0 \Leftrightarrow \rho_{N^*}(q_j,w,q_k) < 1$

Example 3.6. *Let $F = (S,A,N)$ be an single valued neutrosophic automaton, where*
$S = \{q_1,q_2,q_3,q_4\}$, $A = \{x,y\}$, *and N are defined as below.*
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_1,x,q_4) = [0.6,0.4,0.5]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_1,y,q_2) = [0.3,0.5,0.4]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_2,x,q_3) = [0.5,0.1,0.3]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_2,y,q_4) = [0.7,0.4,0.3]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_3,x,q_2) = [0.1,0.7,0.5]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_3,y,q_4) = [0.2,0.4,0.6]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_4,x,q_1) = [0.6,0.2,0.4]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_4,y,q_3) = [0.3,0.4,0.5]$
The states q_2 and q_3 are reducible since $\eta_{N^*}(q_2,xy,q_4) > 0 \Leftrightarrow \eta_{N^*}(q_3,xy,q_4) > 0$ $\zeta_{N^*}(q_2,xy,q_4) < 1 \Leftrightarrow \zeta_{N^*}(q_3,xy,q_4) < 1$, and $\rho_{N^*}(q_2,xy,q_4) < 1 \Leftrightarrow \rho_{N^*}(q_3,xy,q_4) < 1$

Definition 3.7. *Let $F = (S,A,N)$ be an SVNA. If two states q_i and q_j are said to be stability related and it is denoted by $q_i\Omega\ q_j$ if for any word $w_1 \in A^*$ there exists a word $w_2 \in A^*$, $q_k \in S$ such that*
$\eta_{N^*}(q_i,w_1w_2,q_k) > 0 \Leftrightarrow \eta_{N^*}(q_j,w_1w_2,q_k) > 0$
$\zeta_{N^*}(q_i,w_1w_2,q_k) < 1 \Leftrightarrow \zeta_{N^*}(q_j,w_1w_2,q_k) < 1$
$\rho_{N^*}(q_i,w_1w_2,q_k) < 1 \Leftrightarrow \rho_{N^*}(q_j,w_1w_2,q_k) < 1$

Example 3.8. *Let $F = (S,A,N)$ be an single valued neutrosophic automaton, where*
$S = \{q_1,q_2,q_3,q_4\}$, $A = \{x,y\}$, *and N are defined as below.*
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_1,x,q_4) = [0.3,0.4,0.5]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_1,y,q_2) = [0.5,0.2,0.4]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_2,x,q_3) = [0.7,0.1,0.4]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_2,y,q_4) = [0.1,0.6,0.3]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_3,x,q_2) = [0.2,0.5,0.4]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_3,y,q_4) = [0.5,0.2,0.3]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_4,x,q_1) = [0.6,0.3,0.3]$
$(\eta_{N^*},\zeta_{N^*},\rho_{N^*})(q_4,y,q_3) = [0.7,0.3,0.2]$
For anyword $w \in A^*$, there exists a word $xyy \in A^*$ such that
$\eta_{N^*}(q_1,wxyy,q_k) > 0 \Leftrightarrow \eta_{N^*}(q_4,wxyy,q_k) > 0$
$\zeta_{N^*}(q_1,wxyy,q_k) < 1 \Leftrightarrow \zeta_{N^*}(q_4,wxyy,q_k) < 1$
$(\rho_{N^*}(q_1,wxyy,q_k) < 1 \Leftrightarrow \rho_{N^*}(q_4,wxyy,q_k) < 1$*and*
$\eta_{N^*}(q_2,wxyy,q_l) > 0 \Leftrightarrow \eta_{N^*}(q_3,wxyy,q_l) > 0.$
$\zeta_{N^*}(q_2,wxyy,q_l) < 1 \Leftrightarrow \zeta_{N^*},(q_3,wxyy,q_l) < 1.$

$\rho_{N^*}(q_2, wxyy, q_l) > 0 \Leftrightarrow \rho_{N^*}(q_3, wxyy, q_l) < 1$.
The states q_1, q_4 and q_2, q_3 are stability related.

Remark 3.9. *(i) Reducibility relation is not an equivalence relation in SVNA. Since transitive relation does not exists.*

Example 3.10. *Let $F = (S, A, N)$ be a single valued neutrosophic automaton, where*
$S = \{q_1, q_2, q_3, q_4\}$, $A = \{x, y\}$, *and N are defined as below.*
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, x, q_3) = [0.3, 0.5, 0.6]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_1, y, q_1) = [0.5, 0.2, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, x, q_1) = [0.7, 0.2, 0.3]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_2, y, q_1) = [0.1, 0.5, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, x, q_4) = [0.3, 0.4, 0.5]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_3, y, q_4) = [0.5, 0.3, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, x, q_2) = [0.6, 0.2, 0.4]$
$(\eta_{N^*}, \zeta_{N^*}, \rho_{N^*})(q_4, y, q_4) = [0.7, 0.3, 0.2]$

In the above SVNA F $\exists$ a word $yy \in A^*$ such that
$\eta_{N^*}(q_1, yy, q_1) > 0 \Leftrightarrow \eta_{N^*}(q_2, yy, q_1) > 0$
$\zeta_{N^*}(q_1, yy, q_1) < 1 \Leftrightarrow \zeta_{N^*}(q_2, yy, q_1) < 1$
$(\rho_{N^*}(q_1, yy, q_1) < 1 \Leftrightarrow \rho_{N^*}(q_2, yy, q_1) < 1$
Therefore the states q_1 and q_2 are reducible.
Also there exists a word $xy \in A^*$ such that $\eta_{N^*}(q_2, xy, q_1) > 0 \Leftrightarrow \eta_{N^*}(q_4, xy, q_1) > 0$
$\zeta_{N^*}(q_2, xy, q_1) < 1 \Leftrightarrow \zeta_{N^*}(q_4, xy, q_1) < 1$
$(\rho_{N^*}(q_2, xy, q_1) < 1 \Leftrightarrow \rho_{N^*}(q_4, xy, q_1) < 1$
Therefore the states q_2 and q_4 are reducible but q_1 and q_4 are not reducible for anyword $w \in A^*$.
Hence reducibility relation is not transitive.

Theorem 3.11. *Let $F = (S, A, N)$ be an SVNA. Stability relation on SVNA F is an Equivalence relation.*

Proof. Let $F = (S, A, N)$ be an SVNA. Clearly stability relation on SVNA F is reflexive and symmetric. For proving stability relation is an equivalence relation it is enough to prove that it is transitive.
Let $q_i \Omega q_j$ and $q_j \Omega q_k$.
To prove $q_i \Omega q_k$, we need to prove for anyword $u_1 \in A^*$, there exists a word $u \in A^*, q_n \in S$ such that
$\eta_{N^*}(q_i, w_1w_2, q_n) > 0 \Leftrightarrow \eta_{N^*}(q_k, u_1u, q_n) > 0$
$\zeta_{N^*}(q_i, w_1w_2, q_n) < 1 \Leftrightarrow \zeta_{N^*}(q_k, u_1u, q_n) < 1$
$\rho_{N^*}(q_i, w_1w_2, q_n) < 1 \Leftrightarrow \rho_{N^*}(q_k, u_1u, q_n) < 1$.
Since $q_i \,\Omega\, q_j$ for any word $u_1 \in A^*$ there exists a word $u_2 \in A^*$ and a state $q_m \in S$ such that
$\eta_{N^*}(q_i, u_1u_2, q_m) > 0 \Leftrightarrow \eta_{N^*}(q_j, u_1u_2, q_m) > 0$
$\zeta_{N^*}(q_i, u_1u_2, q_m) < 1 \Leftrightarrow \zeta_{N^*}(q_j, u_1u_2, q_m) < 1$
$\rho_{N^*}(q_i, u_1u_2, q_m) < 1 \Leftrightarrow \rho_{N^*}(q_j, u_1u_2, q_m) < 1$.
Since $q_j \,\Omega q_k$ we have for any word $u_1u_2 \in A^*$ there exists a word $u_3 \in A^*, q_n \in S$ such that
$\eta_{N^*}(q_j, u_1u_2u_3, q_n) > 0 \Leftrightarrow \eta_{N^*}(q_k, u_1u_2u_3, q_n) < 1$
$\zeta_{N^*}(q_j, u_1u_2u_3, q_n) < 1 \Leftrightarrow \zeta_{N^*}(q_k, u_1u_2u_3, q_n) > 0$
$\rho_{N^*}(q_j, u_1u_2u_3, q_n) < 1 \Leftrightarrow \rho_{N^*}(q_k, u_1u_2u_3, q_n) < 1$.
$\eta_{N^*}(q_j, u_1u_2u_3, q_n) > 0 \Leftrightarrow \eta_{N^*}(q_i, u_1u_2u_3, q_n) > 0$
$\zeta_{N^*}(q_j, u_1u_2u_3, q_n) < 1 \Leftrightarrow \zeta_{N^*}(q_i, u_1u_2u_3, q_n) < 1$
$\rho_{N^*}(q_j, u_1u_2u_3, q_n) < 1 \Leftrightarrow \rho_{N^*}(q_i, u_1u_2u_3, q_n) < 1$. [Since $q_j \,\Omega\, q_i$].
$\eta_{N^*}(q_i, u_1u_2u_3, q_n) > 0 \Leftrightarrow \eta_{N^*}(q_k, u_1u_2u_3, q_n) > 0$
$\zeta_{N^*}(q_i, u_1u_2u_3, q_n) < 1 \Leftrightarrow \zeta_{N^*}(q_k, u_1u_2u_3, q_n) < 1$
$\rho_{N^*}(q_i, u_1u_2u_3, q_n) < 1 \Leftrightarrow \rho_{N^*}(q_k, u_1u_2u_3, q_n) < 1$.
Now, choose $u_2u_3 = u$.
For any word $u_1 \in A^*$ there exists word $u \in A^*$ and $q_n \in S$such that $\eta_{N^*}(q_i, u_1u, q_n) > 0 \Leftrightarrow \eta_{N^*}(q_k, u_1u, q_n) > 0$
$\zeta_{N^*}(q_i, u_1u, q_n) < 1 \Leftrightarrow \zeta_{N^*}(q_k, u_1u, q_n) < 1$
$\rho_{N^*}(q_i, u_1u, q_n) < 1 \Leftrightarrow \rho_{N^*}(q_k, u_1u, q_n) < 1$.
Hence, $q_i \,\Omega\, q_k$. □

4. Conclusion

In this paper we introduce reducibility and stability relation in single valued neutrosophic automata. We have shown by example that reducibility relation is not an equivalence relation and prove that stability relation is an equivalence relation.

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