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Pythagorean fuzzification and defuzzification functions

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Abstract

This paper addresses the concept of Pythagorean fuzzification and defuzzification. It is the process of converting crisp quantity into a Pythagorean fuzzy set and Pythagorean fuzzy set into crisp respectively. While various method for the fuzzification and defuzzification of fuzzy sets and Intuitionistic fuzzy sets have been devised no such idea has been found in the case of Pythagorean fuzzy sets. An attempt has been made to introduce various type of Pythagorean fuzzification and defuzzification functions which will be more useful in modeling real world situations in Pythagorean fuzzy environment.

Keywords

Pythagorean fuzzy set, fuzzification, defuzzification, triangular, trapezoidal and Gaussian functions.

AMS Subject Classification

03E72.

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Contents

1. Introduction

The model of fuzzy set was familiarized by L.A Zadeh[18]. In fuzzy set Zadeh discussed about the membership element, which is known as the degree of membership. It allows elements to be a partial membership in the set. He also developed many applications of the fuzzy set theory in many fields, such as computer science, engineering and management science, etc. Due to insufficiency in the availability of information, the evaluation of membership is not enough up to our satisfaction. Atanassov [3] introduced intuitionistic fuzzy sets with the property that the sum of the membership and non membership degrees is less than or equal to one. The notion of IFSs provides a versatile framework to elaborate uncertainty and vagueness. The idea of IFS seems to be resourceful in modeling many real-life situations like medical diagnosis[7]. There are situations where $\mu+\nu \geq 1$ unlike the cases capture in IFSs. This limitation in IFS naturally lead to construct Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in[14–17] is a new tool to handle vagueness considering the membership grade, μ and non membership grade, ν satisfying the conditions $\mu+\nu \leq 1$ or $\mu+\nu \geq 1$, and also, it follows that $\mu^2+\nu^2+\pi^2=1$, where π is the Pythagorean fuzzy set index. The concept of Pythagorean fuzzy set could be a novel mathematical framework within the fuzzy family with higher ability to tackle uncertainty embedded in decision-making. It is useful to represent ambiguous and unsure decision information.

Fuzzification is that the process of converting classical set to fuzzy. Defuzzification is the process of converting fuzzy value to crisp value. Real world problems sometimes emerge to complex owing to uncertainty parameters or in situation in which problem occurs. A fuzzification function provides a flexible model to elaborate vagueness and uncertainty in real life applications. Mean while defuzzification has attracted far less attention than other process involved in fuzzy systems and technologies. Sometimes it is necessary to convert fuzzy in to crisp for further processing. Radhika C. and Parvathi R.[12, 13] provides Intuitionistic fuzzification and defuzzification methods. Pythagorean fuzzification and defuzification is the process of converting Pythagorean fuzzy to crisp and crisp to Pythagorean fuzzy respectively. In this paper some standard fuzzification and defuzzification methods were proposed. The remaining part of the work is organized as follows: Some preliminary concepts of Pythagorean fuzzy sets are overviewed in section 2. In section 3, various Pythagorean fuzzification methods like triangular, trapezoidal and Gaussian Pythagorean fuzzification functions were presented. In section 4, defuzzification methods like triangular, trapezoidal and Gaussian Pythagorean fuzzification functions were presented. In section 5 all the three types of Pythagorean fuzzification and defuzzification functions are illustrated with real life applications. Finally, the paper is concluded with section 6.

2. Preliminaries

Definition 2.1. *[2] Let the universal set X be fixed. An intuitionistic fuzzy set A in X is defined as an object of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ *where the functions* $\mu_A(x)$ *and* $\nu_A(x)$ *are mappings from X to* $[0,1]$ *defines the degree of membership and non-membership of the element* $x \in X$ *respectively, and for every* $x \in X$ *in* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2. *[16] Let M be a fixed set, then a Pythagorean fuzzy set in M can be defined as follows:* $P = \{(m, \lambda_p(m), \eta_p(m))/m \in$ *M where and are mappings from M to* $[0,1]$ *with conditions* $0 \leq \lambda_p(m) \leq 1$, $0 \leq \eta_p(m) \leq 1$ *and also* $0 \leq \lambda_p^2(m) + \eta_p^2(m) \leq 1$ *for all* $m \in M$*, and they denote the degree of membership and degree of non membership of element* $m \in M$ *to set P, respectively. Let* $\pi_p(m) = \sqrt{1 - \lambda_p^2(m) - \eta_p^2(m)}$*, then it is called the Pythagorean fuzzy index of element* $m \in M$ *to set P, representing the degree of indeterminacy of m to P. Also for* $0 \leq \pi_p(m) \leq 1$*, for every* $m \in M$.

3. Pythagorean fuzzy fuzzification functions

Definition 3.1. *Pythagorean fuzzy triangular function*

Let A_{PF} be a Pythagorean fuzzy set. Then the Pythagorean fuzzy triangular membership function is of the form

$$\lambda_{A_{PF}}(x) = \begin{cases} 0 & : x \leq a_1 \\ \sqrt{\dfrac{x-a_1}{a_2-a_1} - \varepsilon^2} & : a_1 \leq x \leq a_2 \\ \sqrt{\dfrac{a_3-x}{a_3-a_2} - \varepsilon^2} & : a_2 \leq x \leq a_3 \\ 0 & : x \geq a_3. \end{cases}$$

where $a_1 \leq a_2 \leq a_3$.The precise appearence of the function is determined by the choice of paramaters a_1, a_2, a_3 which in turns form a triangle. Here a_1 and a_3 locate the feet of the triangles and the parameter a_2 locates the peak.

The corresponding Pythagorean fuzzy triangular non-membership function is of the form,

$$\eta_{A_{PF}}(x) = \begin{cases} \sqrt{1-\varepsilon^2} & : x \leq a_1 \\ \sqrt{\dfrac{x-a_1}{a_2-a_1} - \varepsilon^2} & : a_1 \leq x \leq a_2 \\ \sqrt{\dfrac{a_3-x}{a_3-a_2} - \varepsilon^2} & : a_2 \leq x \leq a_3 \\ \sqrt{1-\varepsilon^2} & : x \geq a_3. \end{cases}$$

Note 3.2. ε *is an arbitrary parameter chosen in such a way that* $\lambda^2_{APF}(x) + \eta^2_{APF}(x) + \varepsilon^2 = 1$ *and* $0 \leq \varepsilon^2 < 1$

Definition 3.3. *Pythagorean fuzzy trapezoidal function*

Let A_{PF} be a Pythagorean fuzzy set. Then the Pythagorean fuzzy trapezoidal function is of the form

$$\lambda_{A_{PF}}(x) = \begin{cases} 0 & : x \leq a_1 \\ \sqrt{\dfrac{x-a_1}{a_2-a_1} - \varepsilon^2} & : a_1 < x \leq a_2 \\ \sqrt{1-\varepsilon^2} & : a_2 \leq x \leq a_3 \\ \sqrt{\dfrac{a_4-x}{a_4-a_3} - \varepsilon^2} & : a_3 \leq x < a_4 \\ 0 & : x \geq a_4. \end{cases}$$

The corresponding Pythagorean fuzzy trapezoidal non-membership function is of the form,

$$\eta_{A_{PF}}(x) = \begin{cases} \sqrt{1-\varepsilon^2} & : x \leq a_1 \\ \sqrt{1 - \dfrac{x-a_1}{a_2-a_1} - \varepsilon^2} & : a_1 < x \leq a_2 \\ 0 & : a_2 \leq x \leq a_3 \\ \sqrt{1 - \dfrac{a_4-x}{a_4-a_3} - \varepsilon^2} & : a_3 \leq x < a_4 \\ \sqrt{1-\varepsilon^2} & : x \geq a_4. \end{cases}$$

Note 3.4. *The graph of Pythagorean fuzzy trapezoidal function may be symmetric or asymmetric in shape. Pythagorean fuzzy triangular function is a special of Pythagorean fuzzy trapezoidal function.*

Definition 3.5. *Pythagorean fuzzy Gaussian function*

Pythagorean fuzzy Gaussian function is defined with the help of two parameters, central value c, width $w > 0$. The smaller the width, the narrower the curve is.

The membership function of Pythagorean fuzzy Gaussian function is defined by

$$\lambda_{A_{PF}}(x) = \sqrt{e^{-\left(\frac{(x-c)^2}{2w^2}\right)} - \varepsilon^2}.$$

The non-membership function of Pythagorean fuzzy Gaussian function is defined by

$$\eta_{A_{PF}}(x) = \sqrt{1 - e^{-\left(\frac{(x-c)^2}{2w^2}\right)}}.$$

4. Pythagorean fuzzy defuzzification functions

Definition 4.1. *Pythagorean fuzzy triangular function*

Let A_{PF} be a Pythagorean fuzzy set. Then the Pythagorean fuzzy triangular defuzzification function is defined by

$$D(y) = \begin{cases} \leq a_1 & : y = 0 \\ a_1 + (a_2 - a_1)(y + \varepsilon^2) & : 0 < y \leq \sqrt{\frac{x-a_1}{a_2-a_1} - \varepsilon^2} \\ a_3 + (a_2 - a_1)(y + \varepsilon^2) & : \sqrt{\frac{x-a_1}{a_2-a_1} - \varepsilon^2} \leq y \leq \sqrt{\frac{a_3-x}{a_3-a_2} - \varepsilon^2} \\ \geq a_3 & : y = 0. \end{cases}$$

where $y = \lambda^2_{PF}(x)$ is the fuzzified value which lies between 0 to 1, and ε is an arbitrary parameter chosen in such a way that $\lambda^2_{A_{PF}(x)} + \eta^2_{A_{PF}(x)} + \varepsilon^2 = 1$ and $0 \leq \varepsilon^2 < 1$.

Definition 4.2. *Pythagorean fuzzy trapezoidal function*

Let A_{PF} be a Pythagorean fuzzy set. Then the Pythagorean fuzzy trapezoidal defuzzification function is defined by

$$D(y) = \begin{cases} \leq a_1 & : y = 0 \\ a_1 + (a_2 - a_1)(y + \varepsilon^2) & : 0 < y \leq \sqrt{\frac{x-a_1}{a_2-a_1} - \varepsilon^2} \\ a_2 \leq x \leq a_3 & : y = \sqrt{1-\varepsilon^2} \\ a_4 + (a_3 - a_4)(y + \varepsilon^2) & : \sqrt{1-\varepsilon^2} \leq y < \sqrt{\frac{a_4-x}{a_4-a_3} - \varepsilon^2} \\ \geq a_4 & : y = 0. \end{cases}$$

Definition 4.3. *Pythagorean fuzzy Gaussian function*

Pythagorean fuzzy Gaussian defuzzification function is defined by

$$D(y) = \begin{cases} c - \omega\sqrt{-2log(y+\varepsilon^2} & : x \leq c \\ c + \omega\sqrt{-2log(y+\varepsilon^2} & : x > c \end{cases}$$

5. Case Study

5.1 Bacterial Group

Among the bacterial groups extremophile is a bacteria which can be able to survive in extreme environment. Here we have chosen one extremophile named as Psychropiles.

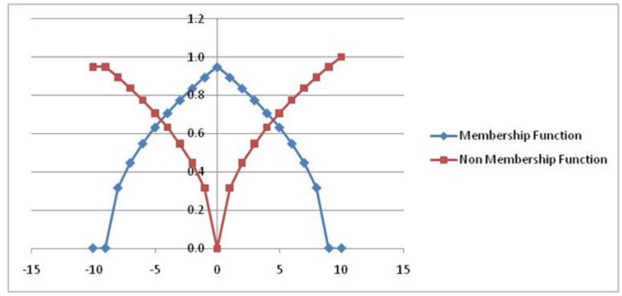

Figure 1. Pythagorean Fuzzy triangular function

It means the organism are capable of growth in reproduction in low temperature ranging from -10 degree Celsius to $+20$ degree Celsius. Suppose the bacteria Psychropiles growth temperature varies from -10 degree Celsius to $+10$ degree Celsius then the corresponding membership and non membership function is as follows:

$$\lambda_{A_{PF}}(x) = \begin{cases} 0 & : x \leq -10 \\ \sqrt{\left(\frac{x+10}{10}\right) - 0.1} & : -10 \leq x \leq 0 \\ \sqrt{\left(\frac{10-x}{10}\right) - 0.1} & : 0 \leq x \leq 10 \\ 0 & : x \geq 10. \end{cases}$$

$$\eta_{A_{PF}}(x) = \begin{cases} \sqrt{1-0.1} & : x \leq -10 \\ \sqrt{1 - \left(\frac{x+10}{10}\right)} & : -10 \leq x \leq 0 \\ \sqrt{1 - \left(\frac{10-x}{10}\right)} & : 0 \leq x \leq 10 \\ \sqrt{1-0.1} & : x \geq 10. \end{cases}$$

where $a_1 = 10, a_2 = 0, a_3 = 10$ and $\varepsilon^2 = 0.1$. The graph of Pythagorean Fuzzy triangular function is represented in Figure 1.

5.2 Lemon Grass

Let us consider the growth of Lemon grass according to their height. The trapezoidal Pythagorean Fuzzy function is specified by the parameters $a_1 = 40cm, a_2 = 50cm, a_3 = 60cm, a_4 = 70cm$ and $\varepsilon^2 = 0.1$. The membership function and non-membership function is defined as

$$\lambda_{A_{PF}}(x) = \begin{cases} 0.1 & : x \leq 40 \\ \sqrt{\left(\frac{x-40}{10}\right) - 0.1} & : 40 < x \leq 50 \\ \sqrt{1-0.1} & : 50 \leq x \leq 60 \\ \sqrt{\left(\frac{70-x}{10}\right) - 0.1} & : 60 \leq x < 70 \\ 0.1 & : x \geq 70. \end{cases}$$

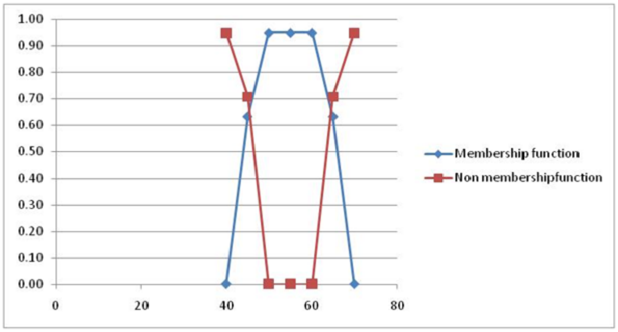

Figure 2. Pythagorean Fuzzy trapezoidal function

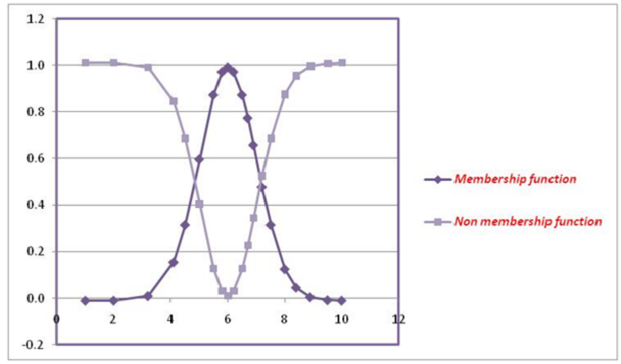

Figure 3. Pythagorean Fuzzy Gaussian function

$$\eta_{A_{PF}}(x) = \begin{cases} \sqrt{1-0.1} & : x \leq 40 \\ \sqrt{1-\left(\frac{x-40}{10}\right)} & : 40 < x \leq 50 \\ 0 & : 50 \leq x \leq 60 \\ \sqrt{1-\left(\frac{70-x}{10}\right)} & : 60 \leq x < 70 \\ \sqrt{1-0.1} & : x \geq 70. \end{cases}$$

The graph of Pythagorean Fuzzy triangular function is represented in Figure 2.

5.3 Vigna Mungo Linn

Let us consider the growth of Black Gram(**Vigna Mungo Linn**). The growth of first 40 days has to be noted and it can be expressed as Pythagorean fuzzy Gaussian function, where the parameters $c = 6.5$ and $w = 1$.
The membership function of Pythagorean fuzzy Gaussian function is defined by

$$\lambda_{A_{PF}}(x) = \sqrt{e^{-\left(\frac{(x-6.5)^2}{2}\right)} - 0.1}.$$

The non-membership function of Pythagorean fuzzy Gaussian function is defined by

$$\eta_{A_{PF}}(x) = \sqrt{1 - e^{-\left(\frac{(x-6.5)^2}{2}\right)}}.$$

The graph of Pythagorean Fuzzy triangular function is represented in Figure 3.

5.4 Pythagorean Fuzzy defuzzification.

Let us consider the 3×3 matrix

$$A= \begin{bmatrix} 10 & 50 & 100 \\ 90 & 45 & 71 \\ 60 & 84 & 30 \end{bmatrix}$$

From definition 3.1 Pythagorean Fuzzy triangular fuzzified matrix is given by

$$\lambda_{A_{PF}}(x) = \begin{bmatrix} 0 & 0.9 & 0 \\ 0.3 & 0.9 & 0.7 \\ 0.8 & 0.5 & 0.7 \end{bmatrix}$$

$$\eta_{A_{PF}}(x) = \begin{bmatrix} 0.95 & 0.6 & 0.3 \\ 0 & 0.4 & 0.6 \\ 0.8 & 0.9 & 0.95 \end{bmatrix}$$

The corresponding Pythagorean fuzzy triangular defuzzified matrix is given by

$$A= \begin{bmatrix} 10 & 50 & 100 \\ 90 & 45 & 71 \\ 60 & 84 & 30 \end{bmatrix}$$

After Pythagorean Fuzzy triangular fuzzification and Pythagorean fuzzy triangular defuzzification the output remains the same.

6. Conclusion

A simple method has been made to formulate fuzzification and defuzzification functions of Pythagorean Fuzzy set. The proposed Pythagorean fuzzification and defuzzification function method is quite easy to use. In the end we used real life applications to demonstrate the performance of our methods.

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