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Anti-duplication self vertex switching in some graphs

C. Jayasekaran [1]* and M. Ashwin Shijo[2]

Abstract

For a finite undirected simple graph $G(V,E)$, duplication of a vertex $v \in V(G)$ forms a new graph G' by introducing a new vertex v' such that $N_{G'}(v') = N_G(v)$. We define anti-duplication of a vertex v in G by introducing a new vertex v' which produces a new graph G' such that $N_{G'}(v') = [N_G(v)]^c$. In this paper, we find the number $adss_1(G)$ when G is $P_n, C_n, K_n, K_{n,m}, S_n, B_n$ and $D(n,m)$.

Keywords

Anti-duplication, anti-duplication self switching vertex.

AMS Subject Classification

05C07, 05CXX.

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Contents

1. Introduction

By a graph G we mean a finite undirected simple graph. The subgraph of G obtained by removing the vertex v and all the edges incident with v is called the subgraph obtained by the removal of the vertex v and is denoted by $G--v$. The degree of a vertex v in a graph G is the number of edges incident with v. The degree of v is represented by $d_G(v)$. A cycle of length n is represented by C_n and a path with n vertices is denoted by P_n. A graph is said to be connected if there exists a path between any two vertices in G. If a graph which is not connected is said to be disconnected graph. The set of vertices adjacent to v in G is denoted by $N_G(v)$, the neighbours of v in G. The complete graph K_p has every pair of its vertices adjacent. A bipartite graph or bigraph G is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every line of G joins a point of V_1 to a point of V_2. If G contains every line joining V_1 and V_2, then G is called complete bipartite graph. If V_1 has m vertices and V_2 has n vertices, we write $G = K_{m,n}$. Clearly, $K_{m,n}$ have $m+n$ vertices and mn edges. $K_{1,n}$ is called a star for $n \geq 1$. It is denoted by S_n. The Book B_n is the graph $S_n \times P_2$, where S_n is the star with $n+1$ vertices. A $(n,m)-dragon$ is formed by joining an end point of a path P_m to a point of cycle C_n. It is denoted by $D(n,m)$.

In [7] Lint and Seidel introduces the vertex switching. For a finite undirected graph $G(V,E)$ and a subset $S \subset V$, the switching of G by S is defined as the graph $G^S(V,E')$ which is obtained from G by removing all edges between S and its complement $V–S$ and adding as edges all non edges between S and $V-S$. For $S = \{v\}$, we write G^v instead of $G^{\{v\}}$ and the corresponding switching is called as vertex switching [3]. Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N_{G'}(v') = N_G(v)$ [5]. In [1], C. Jayasekaran and M. Ashwin Shijo introduced the concept anti-duplication of a vertex in a graph and studied its properties. In [2]C. Jayasekaran and M. Ashwin Shijo introduced the concept anti-duplication graph for a given graph and studied its properties. For standard definitions and notations we follow Harary[6].

In this paper, we introduce anti-duplication self vertex switching in a graph and calculate the number of anti-duplication self switching vertices $adss_1(G)$, for the graph

G when G is a path P_n, a cycle C_n, a complete graph K_n, a star graph S_n, a book graph B_n and a dragon graph $D(n,m)$ has been found.

2. Anti-duplication of a vertex in a Graph

Definition 2.1. *[1] Anti-duplication of a vertex v in G produces a new graph G' by adding a new vertex v' such that $N_{G'}(v') = [N_G[v]]^c$.*

The graph obtained from G by anti-duplication of the vertex v is denoted by $AD(vG)$.

Example 2.2. *Consider the graph G given in figure 2.1. The anti-duplication of the vertices v_1,v_2, v_3 and v_4 are given in figures 2.2, 2.3, 2.4 and 2.5, respectively.*

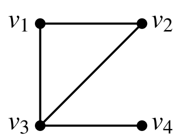

Fig 2.1. G

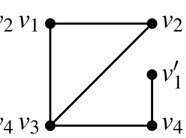

Fig 2.2. $AD(v_1G)$

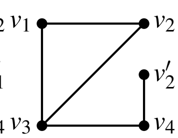

Fig 2.3. $AD(v_2G)$

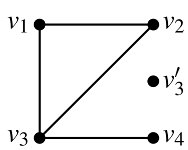

Fig 2.4. $AD(v_3G)$

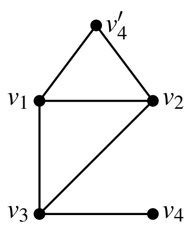

Fig 2.5. $AD(v_4G)$

We recall the theorems which are used in the next section.

Theorem 2.3. *[8] If v is a self vertex switching of a graph G of order p, then $d_G(v) = (p-1)/2$.*

Theorem 2.4. *[1] Let G be a graph of order p. Let v be any vertex in G and v' be the anti-duplication vertex of v in $AD(vG) = G'$. Then $d_{G'}(v) = d_G(v)$*

Theorem 2.5. *[1] Let G be a graph of order p. Let v be any vertex in G and v' be the anti-duplication vertex of v in $AD(vG) = G'$. Then $d_{G'}(v') = p-1-d_G(v)$.*

Notation 2.6. *[4] Let us consider a cycle $C_r = (v_1, v_2, v_3, \ldots, v_r)$ (clockwise). For our convenience we denote it by $C_{r(v_1)}$. Identifying an end vertex of paths P_m at v_i and P_s at v_j in $C_{r(v_1)}$ is denoted by $C_{r(v_1)}(0, \ldots, P_m, 0, \ldots, P_s, 0, \ldots, 0)$. Thus the graph G given in figure 2.4 is $C_{4(v)}(P_2, 0, P_3, 2P_2)$*

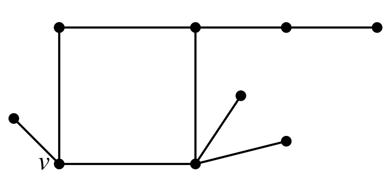

Fig 2.6. $C_{4(v)}(P_2, 0, P_3, 2P_2)$

3. Anti-duplication self vertex switching in a graph

Definition 3.1. *Let G be a graph and let v' be the anti-duplication vertex of v in $AD(vG)$. A vertex v is called anti-duplication self switching vertex of a graph G if v is a self vertex switching of$AD(vG)$.*

The set of all anti-duplication self-switching vertices is denoted by $ADSS_1(G)$.

The number of all anti-duplication self-switching vertices is denoted by $adss_1(G)$.

Example 3.2. *The graph G given in figure 2.7 has v_1 as an anti-duplication self-switching vertex since v_1 is a self vertex switching of $AD(v_1G)$. Clearly v_2 and v_4 are the other anti-duplication self-switching vertices of G as $AD(v_2G) \cong [AD(V_2G)]^{v_2}$ and $AD(V_4G) \cong [AD(V_4G)]^{v_4}$ and hence $adss_1(G)$ $= 3$.*

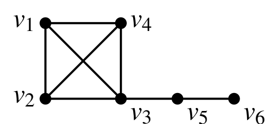

Fig 3.1. G

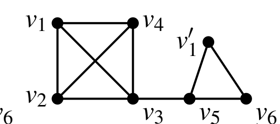

Fig 3.2. $AD(v_1G)$

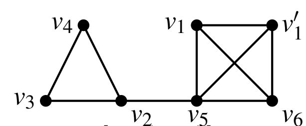

Fig 3.3. $[AD(v_1G)]^{v_1}$

Theorem 3.3. *If v is an anti-duplication self-switching vertex of a graph G of order p, then p is even and $d_G(v) = p/2$.*

Proof. Let v be an anti-duplication self-switching vertex of a graph G of order p and let v' be the anti-duplication vertex of v in $AD(vG)$. Then v is a self-switching vertex of $AD(vG)$. By definition $AD(vG)$ is of order $p+1$. By Theorem 2.3, $p+1$ is odd and $d_{AD(vG)}(v) = (p+1-1)/2 = p/2$. This implies that p is even and by Theorem 2.4 $d_G(v) = d_{AD(vG)}(v) = p/2$. Hence the theorem. □

Theorem 3.4. *If v is an anti-duplication self-switching vertex of a graph G, then v is non-adjacent to minimum degree vertices in G.*

Proof. Let v be an anti-duplication self-switching vertex of graph G and v' be the anti-duplication vertex of v in $AD(vG)$. Let u be a minimum degree vertex in G. Suppose u is adjacent to v in G. Then by definition, u is adjacent to v but non-adjacent to v' in $AD(vG)$. This implies that $d_{AD(vG)}(u) = d_G(u)$. In $[AD(vG)]^v, v$ is non-adjacent to u and hence $d_{[AD(vG)]^v}(u) = d_{AD(vG)}(u) - 1 = d_G(u) - 1$. Clearly u is a minimum degree vertex with degree equal to $d_G(u) - 1$ in $[AD(vG)]^v$. This implies that $AD(vG)$ is not isomorphic to $[AD(vG)]^v$ which is a contradiction to v is anti-duplication self-switching vertex. Hence v is non-adjacent to the minimum degree vertices. □

4. $adss_1(G)$ when G is $P_n, C_n, K_n, K_{n,m}, S_n, B_n$ and $D(n,m)$.

In this section we find the number of anti-duplication self switching vertices for the graphs $P_n, C_n, K_n, K_{n,m}, S_n, B_n$ and $D(n,m)$.

Theorem 4.1. *For path P_n,*

$$adss_1(P_n) = \begin{cases} 2 \; if \; n = 2 \\ 0 \; otherwise \end{cases}$$

Proof. By definition n must be even. We consider the following cases.

Case 1. $n = 2$

It is clear that both the vertices of P_2 are anti-duplicating self-switching vertices and hence $adss_1(P_2) = 2$.

Case 2. $n = 4$

Let $v_1v_2v_3v_4$ be the path P_4. Clearly v_1 and v_4 have degree 1 and hence they are not anti-duplicating self-switching vertices of P_4. As v_2 is adjacent with v_1 and v_3 is adjacent with v_4 and v_1 and v_4 are the vertices with minimum degree, by Theorem 3.4, v_2 and v_3 are not anti-duplicating self-switching vertices in P_4 and hence $adss_1(P_4) = 0$.

Case 3. $n \neq 2, 4$

Clearly P_n does not contain a vertex with degree $n/2$ and hence by Theorem 3.3, $adss_1(P_n) = 0$.

The theorem follows from cases 1, 2 and 3. □

Theorem 4.2. *For cycle $C_n, adss_1(C_n) = 0$.*

Proof. We consider the following two cases.

Case 1. $n = 4$

Let $v_1v_2v_3v_4v_1$ be the cycle C_4. Clearly $AD(v_iC_4) = C_{4(v_i)}(0,0,P_2,0)$ and $[AD(v_iC_4)]^{v_i} = C_{3(v_i)}(0,2P_2,0), 1 \leq i \leq 4$. This implies that v_i is not a self vertex switching of $AD(v_iC_4)$ and hence v_i is not an anti-duplicating self-switching vertex of C_4. This implies that $adss_1(C_4) = 0$.

Case 2. $n \neq 4$

Clearly C_n does not contain a vertex with degree $n/2$ and hence by Theorem 3.3, $adss_1(C_n) = 0$.

The theorem follows from cases 1 and 2. □

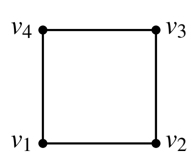

Fig 4.1. $G = C_4$

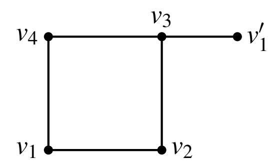

Fig 4.2. $AD(v_1C_4) = C_{4(v_1)}(0,0,P_2,0)$

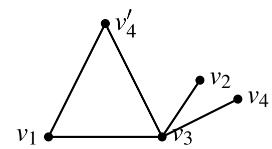

Fig 4.3. $[AD(v_1C_4)]^{v_1} = C_{3(v_1)}(0,2P_2,0)$

Theorem 4.3. *For a complete graph K_n,*

$$adss_1(K_n) = \begin{cases} 2 \; if \; n = 1 \\ 0 \; otherwise \end{cases}$$

Proof. If $n = 2$, then $K_2 = P_2$ and hence by Theorem 4.1, $adss_1(K_2) = 2$.

If $n \neq 2$, then K_n does not contain any vertex with degree $p/2$ and hence $adss_1(K_n) = 0$. Hence the proof. □

Theorem 4.4. *For a complete bipartate graph $K_{m,n}$,*

$$adss_1(K_{m,n}) = \begin{cases} 2 \; if \; n = m = 1 \\ 0 \; otherwise \end{cases}$$

Proof. Let $K_{n,m}$ be a (p,q) graph where $n + m = p$. We consider the following three cases.

Case 1. $n = m = 1$

Clearly $K_{1,1} = P_2$ and by Theorem 4.1, we have $adss_1(K_{1,1}) = 2$.

Case 2. $n = m \neq 1$

Let $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ be the vertices of $K_{n,n} = G$ and $E(G) = u_iv_j, 1 \leq i, j \leq n$. Clearly $d_G(v_i) = d_G(u_j) = n = p/2$. Let v_i' be the anti-duplication vertex of the vertex v_i in $AD(v_iG), 1 \leq i \leq n$. Now v_i is non-adjacent to $v_k, 1 \leq k \leq n, i \neq k$ and adjacent to $u_j, 1 \leq j \leq n$ and hence v_i' is adjacent to v_k and non-adjacent to u_j in $AD(v_iG)$. By Theorem 2.5, $d_{AD(v_iG)}(v_i') = 2n - n - 1 = n - 1$. Clearly $d_{AD(v_iG)}(v_i) = d_G(v_i) = n$ and $d_{AD(v_iG)}(v_k) = n + 1; 1 \leq i, j, k \leq n, i \neq k$. Now in $[AD(v_iG)]^{v_i}$ we have $d^*(v_i) = 2n - n - 1 + 1 = n$ and $d^*(v_i') = n - 1 + 1 = n; d^*(v_k) = n + 2$ and $d^*(u_j) = n - 1, 1 \leq j, k \leq n, i \neq k$, where $d^*(v_i)$ denotes the degree of the vertex v_i in $AD(v_iG)^{v_i}$. Clearly $[AD(v_iG)]^{v_i}$ contains a vertex v_k with degree $n + 2$ but $AD(v_iG)$ doesnot contain any vertices with degree $n + 2$. This implies that v_i is not a self vertex switching of $AD(v_iG)$ and hence v_i is not an anti-duplication self vertex switching of $G = K_{n,n}$. As v_i is interchange similar to all other vertices of $K_{n,n}$, there doesnot exist any vertex with anti-duplication self vertex switching in $K_{n,n}$ and hence $adss_1(K_{n,n}) = 0$.

Case 3. $n \neq m$

In this case, $K_{n,m}$ doesnot contain any vertices of degree $p/2$ and hence by Theorem 3.3, $adss_1(K_{n,m}) = 0$.

From cases 1,2 and 3, the theorem follows. □

Corollary 4.5. *For the star graph $S_n = K_{1,n}$,*

$$adss_1(S_n) = \begin{cases} 2\ if\ n=1 \\ 0\ otherwise \end{cases}$$

Proof. The star graph S_n is a complete bipartate graph $K_{1,n}$. Hence by Theorem 4.4, $adss_1(S_n) = \begin{cases} 2\ if\ n=1 \\ 0\ otherwise \end{cases}$. □

Theorem 4.6. *For the book graph $B_n (n \geq 2), adss_1(B_n) = 0$.*

Proof. When $n = 1, B_1 = C_4$ and hence by Theorem 4.2, $adss_1(B_1) = 0$. Let $n \geq 2$. Let $v_0, v_1, v_2, ..., v_n$ and $u_0, u_1, u_2, ..., u_n$ be the vertices of two copies of the star graph $S_n = K_{1,n}$ where u_0 and v_0 are the central vertices respetively. Join u_i and $v_i, 0 \leq i \leq n$. The resultant graph is the book graph B_n with vertex set $V(S_n) = \{u_i, v_i; 0 \leq i \leq n\}$ and edge set $E(G) = \{u_0u_i, v_0v_i, u_i, v_i; 0 \leq i \leq n\}$. This implies that B_n has $p = 2n+2$ vertices and $q = 3n+1$ edges. Clearly, $d_{B_n}(u_i) = d_{B_n}(v_i) = 2, 1 \leq i \leq n$ and $d_{B_n}(u_0) = d_{B_n}(v_0) = n+1 = p/2$. Since $n \geq 2, p/2 = n+1 \geq 3$ Since $d_{B_n}(v_i) = d_{B_n}(u_j) = 2 \neq 3$, by Theorem 3.3, u_i and $v_i, 1 \leq i \leq n$ are not anti-duplication self-switching vertices of B_n. Also u_0 and v_0 are adjacent to the minimum degree vertices u_i and v_i, respectively, by Theorem 3.4, u_0 and v_0 are not anti-duplication self-switching vertices of B_n. Thus $adss_1(B_n) = 0$. Hence the proof. □

Theorem 4.7. *For the dragon graph $G = D(n,m), n \geq 3$ and $m \geq 1$*

$$adss_1(D_{n,m}) = \begin{cases} 2\ if\ n=3, m=1 \\ 0\ otherwise \end{cases}$$

Proof. Let $v_1v_2...v_m$ be the path P_m and let $u_1u_2...u_nu_1$ be the cycle C_n. Join u_1 and v_1. The resultant graph G is $(n,m)-dragon$. The vertex set $V(G) = \{v_i, u_j; 1 \leq i \leq m; 1 \leq j \leq n\}$ and edge set $E(G) = \{v_iv_{i+1}, u_ju_{j+1}, v_1u_1, u_1u_n; 1 \leq i \leq m-1; 1 \leq j \leq n-1\}$. Clearly G has $p = n+m \geq 4$ vertices and $q = n+m$ edges. Also $d_G(v_i) = d_G(u_j) = 2$ for $1 \leq i \leq m-1, 2 \leq j \leq n, d_G(v_m) = 1$ and $d_G(u_1) = 3$. Now we consider the following cases.

Case 1. $p = 4$

In this case, $D(3,1)$ is the only possible dragon graph and hence $m = 3, n = 1$. Clearly, u_2 and u_3 are the two anti-duplicating self-switching vertices of $D(3,1)$. Hence $adss_1(D(3,1)) = 2$.

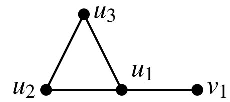

Fig 4.4. $G = D(3,1)$

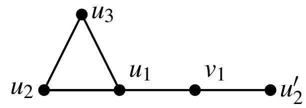

Fig 4.5. $AD(u_2G)$

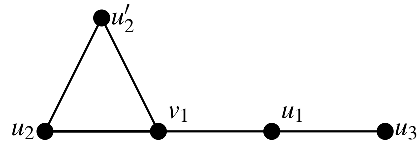

Fig 4.6. $[AD(u_2G)]^{u_2}$

Case 2. $p = 6$

The possible dragon graphs with 6 vertices are $D(3,3)$, $D(4,2)$ and $D(5,1)$. We consider the following subcases.

Subcase 2.a. $G = D(3,3)$

Clearly, the vertex set is $V(G) = \{v_i, u_j; 1 \leq i \leq 3; 1 \leq j \leq 3\}$ and edge set is $E(G) = \{v_1v_2, v_2v_3, u_1u_2, u_2u_3, u_1u_3, v_1u_1\}$. Clearly,$d_G(u_1) = 3 = p/2$ and $d_G(v) \neq p/2$ for $v \neq u_1$ in G. Hence by Theorem 3.3, v is not an anti-duplication self-switching vertex. Let u_1' be the anti-duplication vertex of u_1. By definition of anti-duplication of a vertex, u_1' is adjacent with only v_2 and v_3 in $AD(uG)$. Now, in $[AD(u_1G)]^{u_1}, u_1$ is not adjacent with u_2 and u_3. This implies that the vertices u_2 and u_3 forms a component K_2 in $[AD(u_1G)]^{u_1}$. Hence u_1 is not a self vertex switching of $AD(u_1G)$. This implies that u_1 is not an anti-duplicatiion self-switching vertex of the dragon graph $D(3,3)$.

Subcase 2.b. $G = D(4,2)$

Clearly, the vertex set is $V(G) = \{v_i, u_j; 1 \leq i \leq 2; 1 \leq j \leq 4\}$ and edge set is $E(G) = \{v_1v_2, u_1u_2, u_2u_3, u_3u_4, u_4u_1, v_1u_1\}$. Clearly, by Theorem 3.3, the vertices u_2, u_3, u_4, v_1 and v_2 are not anti-duplication self-switching vertices. Let u' be the anti-duplication vertex of u_1. By definition of anti-duplication of a vertex, u_1' is adjacent with u_3 and v_2 in $AD(u_1G)$ and hence the vertices u_1 and u_3 are of degree 3 and all other vertices are of degree 2. Now, in $[AD(u_1G)]^{u_1}, u_1$ is not adjacent with u_2 and u_4. This implies that both the vertices are of degree 1. But $AD(u_1G)$ does not contain any vertex with degree 1. Hence there doesnot exist an isomorphism between $AD(u_1G)$ and $[AD(u_1G)]^{u_1}$. This implies that u_1 is not an anti-duplication self-switching vertex of the dragon graph $(4,2)$.

Subcase 2.c. $G = D(5,1)$

In this case, $V(G) = \{u_1, u_2, u_3, u_4, u_5, v_1\}$ and $E(G) = \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1, v_1u_1\}$. Clearly, by Theorem 3.3, the vertices u_2, u_3, u_4, u_5 and v_1 are not anti-duplication self-switching vertices. Also by Theorem 3.4, u_1 is not an anti-duplication self-switching vertex as u_1 is adjacent to v_1, the minimum degree vertex of G. Thus from the above three cases we have $adss_1(G) = 0$.

Case 3. $p \neq 4, 6$.

Clearly $D(n,m)$ doesnot contain any vertices with degree $p/2$ and hence by Theorem 3.3, $adss_1(D(n,m)) = 0$.

The theorem follows from cases 1, 2 and 3. □

5. Conclusion

In this paper, we have found the number of anti-duplication self-switching vertices in some standard graphs like Path, Cycle, Complete graph, Book graph, Dragon graph and Star graph. One can find the number $adss_1(G)$ for some more graphs.

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