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Epiregularity in generalized topological space

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Abstract

The notion of generalized epiregularity in generalized topological space is introduced and investigate some of its properties in this paper.

Keywords

Generalized Epiregular, Generalized Hausdorff, Generalized Completely Hausdorff, Generalized Paracompact.

AMS Subject Classification

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1. Introduction

One of the important development of general topology in recent years is the theory of generalized topological spaces defined by A. Csaszar [2]. In particular, he introduced the basic operators in generalized topology. Noiri and B. Roy [1] in 2011 introduced a new kind of sets called generalized μ-closed set in a topological space by using the concept of generalized μ-open set introduced by Csaszar. In 2011, M. S. Sarsak [13] studied separation axioms and B. Roy [1] introduced regularity and normality on generalized topological spaces using generalized μ-closed and μ-open sets. Samirah A. Alzahrani[12] introduced the concept of epiregularity in topological spaces and studied its topological properties. We introduced the concept of epiregular space in generalized topology and studied some of its basic properties using the concept of generalized μ-open and generalized μ-closed sets.

2. Preliminaries

In this section, we recall some definitions and basic results used throughout the paper.

Definition 2.1. *[14] Let topology τ on a set Y contains another topology τ' on Y (that is, every member of τ' is a member of τ), we say that τ is a stronger or finer topology than τ', or that τ' is a weaker or coarser topology than τ.*

Definition 2.2. *[3] A generalized topological space (Y, μ) is said to be generalized paracompact if every μ-open covering of Y has μ-locally finite μ-open refinement that covers Y.*

Definition 2.3. *[4] A generalized topological space (Y, μ) is said to be generalized Hausdorff if for any two distinct points m and n, there exists disjoint μ-open sets A and B such that m $\in$ A and n $\in$ B.*

Definition 2.4. *[1] A generalized topological space (Y, μ) is said to be generalized regular (generalized T_3) if for each μ-closed set F of Y not containing m, there exist disjoint μ-open sets A and B such that m $\in$ A and F $\subseteq$ B.*

Definition 2.5. *[1] A generalized topological space (Y, μ) is generalized normal (generalized T_4) if for any pair of disjoint μ-closed subsets G and H of Y, there exist disjoint μ-open sets A and B such that G $\subseteq$ A and H $\subseteq$ B.*

3. Generalized Epiregularity

Definition 3.1. *A generalized topological space (Y, μ) is called generalized epiregular if there is a generalized coarser topology (Y,μ') on Y such that (Y, μ') is generalized T_3.*

Example 3.2. *Let $Y = \{p,q,r\}$, $\mu = \{\phi,\{p\},\{q\},\{r\},\{p,q\},\{q,r\},\{p,r\},\{p,q,r\}\}$ and $\mu' = \{\phi,\{p\},\{q\},\{p,q\},\{q,r\},\{p,q,r\}\}$. Hence (Y,μ') is generalized T_3 (generalized regular Hausdorff) space. Here (Y,μ) is generalized epiregular.*

Theorem 3.3. *Every generalized epiregular space is generalized Hausdorff.*

Proof. Proof directly follows from the definition 3.1. □

Remark 3.4. *Converse of the above theorem need not be true which is given in the following example.*

Example 3.5. *Let* $Y = \{p,q\}$ *and* $\mu = \{\phi, Y, \{p\}, \{q\}, \{r\}, \{p,q\}\}$. *Here* (Y,μ) *is generalized Hausdorff but it is not generalised epiregular. Here there does not exist coarser topology* (Y,μ') *such that* (Y,μ') *is generalized* T_3.

Definition 3.6. *A generalized topological space (Y, μ) is said to be generalized completely Hausdorff if for any $x \neq y$, there exists disjoint μ-open sets A and B such that $x \in$ cl (A), $y \in$ cl (B).*

Theorem 3.7. *Every generalized epiregular space is generalized completely Hausdorff.*

Proof. Let (Y, μ) be any generalized epiregular space, and let m, n be any two distinct points in Y, then there exists generalized coarser topology μ' on Y such that (Y, μ') is generalized T_3. By generalized T_2 of μ', there exists P, Q $\in$ (Y, μ') such that m $\in$ P, n $\in$ Q and P $\cap$ Q = ϕ. Also by generalized regularity of μ', there exists A, B $\in$ (Y, μ') such that $m \in A \subseteq (cl(A))^{\mu'} \subseteq P$ and $n \in B \subseteq (cl(B))^{\mu'} \subseteq Q$, where $(cl(A))^{\mu'} = \{m \in Y : D \cap A \neq \phi, \forall\ open\ D\ in\ (Y,\mu'), m \in D\}$ similarly $(cl(B))^{\mu'}$. Since $(cl(G))^{\mu} \subseteq (cl(G))^{\mu'}$, for any G $\subseteq$ Y, this implies that $(cl(A))^{\mu} \subseteq (cl(A))^{\mu'}$. As $(cl(A))^{\mu'} \cap (cl(B))^{\mu'} = \phi$ then $(cl(A))^{\mu} \cap (cl(B))^{\mu} = \phi$. Thus (Y, μ) is generalized completely Hausdorff. □

Remark 3.8. *Converse of the above theorem need not be true which is given in the following example.*

Example 3.9. *Let* $Y = \{q,r\}$ *and* $\mu = \{\phi, \{q\}, \{r\}, \{q,r\}\}$. *Here (Y, μ) is generalized completely Hausdorff but it is not generalized epiregular. Here there does not exist coarser topology (Y, μ') such that (Y, μ') is generalized T_3.*

Theorem 3.10. *If the generalized coarser topology (Y, μ') of the generalized epiregular space (Y, μ) is generalized paracompact, then (Y, μ) is generalized T_4.*

Proof. Proof of the theorem follows definition from 2.3 and 2.4. □

Theorem 3.11. *Any generalized epiregular compact space is generalized T_4.*

Proof. Let the generalized epiregular compact space be (Y, μ) then generalized coarser topology (Y, μ') is generalized T_3 which implies that it is generalized T_2. Also every generalized epiregular compact space is generalized T_4. □

Theorem 3.12. *In an generalized epiregular space, for every generalized compact set G and every m $\notin$ G, there exist disjoint μ-open sets A, B such that G $\subseteq$ A and m $\in$ B.*

Proof. Let (Y, μ) be generalized epiregular space, then there exists a generalized coarser topology μ' on Y such that (Y, μ') is generalized T_3. Let G be any generalized compact set in (Y, μ) and let m $\notin$ G, hence G is μ-closed in (Y, μ') and m $\notin$ G, by generalized regularity of (Y, μ'), there exists A, B $\in$ μ' such that G $\subseteq$ A, m $\in$ B and A $\cap$ B = ϕ. □

Theorem 3.13. *If G and H are disjoint generalized compact sets in an generalized epiregular space (Y, μ), then there exists disjoint μ-open sets A and B such that G $\subseteq$ A, H $\subseteq$ B.*

Proof. Let (Y, μ) be generalized epiregular space, then there exists a generalized coarser topology μ' on Y such that (Y, μ') is generalized T_3. Let G, H be any disjoint generalized compact subsets of (Y, μ) , hence they are disjoint generalized compact subsets of (Y, μ') and by theorem 3.10 for each $p \in$ G and generalized compact set H, there exist μ-open sets A_a, B_a such that $p \in A_a$, H $\subseteq B_a$ and $A_a \cap B_a = \phi$. Now consider G as an arbitrary generalized compact set disjoint from H. (By theorem 3.10), for each p in G, disjoint μ-open set A_{a_i} containing p and B_{a_i} containing H, such that A = $\cup_{i=1}^{n}$ A_{a_i} is μ-open set containing G and disjoint from B = $\cap_{i=1}^{n}$ B_{a_i} which is a μ-open set containing H. □

Theorem 3.14. *Generalized epiregularity is a generalized topological property.*

Proof. Let (Y, μ) be generalized epiregular space. Assume that (X, μ_1) $\cong$ (X, μ_2). Let μ_1' be a generalized coarser topology on Y such that (Y, μ_1') is generalized T_3. Let f : (Y, μ_1) $\rightarrow$ (X, μ_2) be a generalized homeomorphism. Define a generalized topology μ_2' on by (X, μ_2') = {f (A) : A $\in$ μ_1'}. Then μ_2' is a coarser than μ_2 and (X, μ_2') is generalized T_3. Hence (X, μ_2) is generalized epiregular. □

Theorem 3.15. *Generalized epiregularity is a generalized hereditary property.*

Proof. Let (Y, μ) be generalized epiregular space, and let (X, μ_X) be a subspace of (Y, μ). Let μ' be a generalized coarser topology on Y such that (Y, μ') is generalized T_3. Since generalized T_3 is generalized hereditary, (X, μ_X') is generalized T_3 and $\mu_X' \subseteq \mu_X$. Therefore (X, μ_X) is generalized epiregular. □

Theorem 3.16. *Generalized epiregular is an generalized additive property.*

Proof. Let (Y_α, μ_α) be an generalized epiregular space for each $\alpha \in \Lambda$, let μ_α', be a generalized coarser topology on Y_α, coarser than μ_α such that (Y_α, μ_α') is generalized T_3. Since generalized T_1 and generalized epiregularity are both generalized additive, $\oplus_{\alpha\in\Lambda}$ (Y_α, μ_α') is generalized T_3, and its generalized topology is coarser than the generalized topology on $\oplus_{\alpha\in\Lambda}$ (Y_α, μ_α). □

Theorem 3.17. *Let $\{(Y_\alpha, \mu_\alpha) : \alpha \in \Lambda\}$ be a family of generalized topological spaces, and let $Y = \Pi_{\alpha\in\Lambda} Y_\alpha$. Then (Y, μ) is generalized epiregular, where μ is the Tychonoff product* *generalized topology, if and only if (Y_α, μ_α) is generalized epiregular for each $\alpha \in \Lambda$.*

Proof. Assume (Y, μ) is generalized epiregular, and let $\beta \in \Lambda$, by theorem 3.13, every subspace of (Y, μ) is generalized epiregular. Then there is a subspace of (Y, μ) that is generalized homeomorphic to (Y_β, μ_β). Since generalized epiregularity is a generalized topological property (Y_β, μ_β) is generalized epiregular. Assume (Y_α, μ_α) is generalized epiregular space for each $\alpha \in \Lambda$, let μ'_α be a generalized topology on Y_α, coarser than μ_α such that (Y_α, μ'_α) is generalized T_3. Since generalized T_3 is multiplicative, $\Pi_{\alpha\in\Lambda} Y_\alpha$ is generalized T_3 with respect of the generalized product topology of μ'_α's, and its generalized topology is coarser than the generalized topology on $\Pi_{\alpha\in\Lambda}$ (Y_α, μ_α). Hence (Y, μ), Y = $\Pi_{\alpha\in\Lambda} Y_\alpha$ is generalized epiregular. □

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