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On some topological indices of line graphs

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Abstract

Topological indices are real values assigned to graphs.Different toplogical indices have been defined so far.This paper discusses Arithmetico-geometrico index and Harmonic mean index of line graph of certain graphs including sudivision graphs.

Keywords

Arithmetico-geometrico index, Harmonic mean index, line graph, sudivision graph.

AMS Subject Classification

05C07, 05C76, 92E10.

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Contents

1. Introduction

Tracing its roots to Gutman and Trinajstic in 1972 [1],various topological indices are defined till date.The different topological indices are Wiener index [4],Arithmetico-geometrico index [3] and Harmonic mean index [2] , to name a few.This paper intends to discuss the Arithmetico-geometrico index and Harmonic mean index of line graph of certain graphs including sudivision graphs.

Here we consider only simple connected graphs which are connected graphs without loops or multiple edges .For a graph G,V(G) and $E(G)$ denote the set of all vertices and edges respectively.For a graph G the degree of a vertex v is the number of edges incident to v and is denoted by $\delta(v)$.Here AG Index and HM index of line graphs of some standard graphs and line graphs of some subdivision graphs are discussed.

Some basic definitions used in the paper.

Definition 1.1. *Arithmetico-Geometrico topological index for a non-empty graph G is denoted by AG(G) and is defined as* $AG(G)=\sum_{rs\in E(G)} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}}$ *where* $\delta(r)$ *and* $\delta(s)$ *denote the degrees of vertices of the edge rs in G*

Definition 1.2. *Harmonic mean index of a graph G is denoted and defined as*

$$HM(G)=\sum_{rs\in E(G)} \frac{2\delta(r)\delta(s)}{\delta(r)+\delta(s)}$$

where $\delta(r)$ *and* $\delta(s)$ *denote the degrees of vertices of the edge rs in G.*

2. Main Results

Theorem 2.1. *Let* τ *be the line graph of the subdivision graph of the wheel graph* W_{p+1}*.Then its AG index is*

$$AG(\tau)=\frac{3+p}{2}\sqrt{\frac{p}{3}}+\frac{p^2+7p}{2}$$

$$HM(\tau)=\frac{1}{2(3+p)}[p^4+2p^3+33p^2+72p]$$

Proof. Consider wheel graph W_{p+1} obtained by placing K_1 in the centre of C_p and joining every edge of the cycle with K_1.Then W_{p+1} will have $p+1$ vertices and $2p$ edges.Its subdivision graph can be obtained as inserting a vertex into each of the $2p$ edges,so that its has $3p+1$ vertices and $4p$ edges.The line graph of this subdivision graph contain as many vertices as the edges in the subdivion graph.Hence there are $4p$ vertices and we can easily see that there are $\frac{p^2+9p}{2}$ edges.

Let us denote the line graph by τ.The edge set of τ can be partitioned into three compartments taking the degrees of their vertices into consideration.They are as follows:

1. $E_1 = \{rs \in E(\tau) | \delta(r) = 3 = \delta(s)\}$

2. $E_2 = \{rs \in E(\tau) | \delta(r) = 3, \delta(s) = p\}$

3. $E_3 = \{rs \in E(\tau) | \delta(r) = p = \delta(s)\}$

Note that $\delta(r), \delta(s)$ represent the degrees of r and s respectively.There are $4p$ edges in E_1, p edges in E_2 and $\frac{p^2-p}{2}$ edges in E_3.
Now,

$$
\begin{aligned}
AG(\tau) &= \sum_{rs\in E(\tau)} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} \\
&= \sum_{rs\in E_1} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} + \sum_{rs\in E_2} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} \\
&+ \sum_{rs\in E_3} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} \\
&= \sum_{rs\in E_1} \frac{3+3}{2\sqrt{3.3}} + \sum_{rs\in E_2} \frac{3+p}{2\sqrt{3.p}} + \sum_{rs\in E_3} \frac{p+p}{2\sqrt{p.p}} \\
&= \sum_{rs\in E_1} 1 + \sum_{rs\in E_2} \frac{3+p}{2\sqrt{3.p}} + \sum_{rs\in E_3} 1 \\
&= 4p + p.\frac{3+p}{2\sqrt{3p}} + \frac{p^2-p}{2} \\
&= \frac{p^2+7p}{2} + \frac{3+p}{2}\sqrt{\frac{p}{3}}
\end{aligned}
$$

$$
\begin{aligned}
HM(\tau) &= \sum \frac{2\delta(r)\delta(s)}{\delta(r)+\delta(s)} \\
&= \sum_{rs\in E_1} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} + \sum_{rs\in E_2} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} \\
&+ \sum_{rs\in E_3} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}} \\
&= \sum_{rs\in E_1} \frac{2.3.3}{3+3} + \sum_{rs\in E_2} \frac{2.3.p}{3+p} + \sum_{rs\in E_3} \frac{2.p.p}{p+p} \\
&= 4p.3 + p.\frac{6p}{3+p} + \frac{p^2-p}{2}.\frac{2p^2}{2p} \\
&= 12p + \frac{6p^2}{3+p} + \frac{(p^2-p)p}{2} \\
&= \frac{1}{2(3+p)}[24p(3+p) + 12p^2 + (3+p)(p^3-p^2)] \\
&= \frac{1}{2(3+p)}[p^4 + 2p^3 + 33p^2 + 72p]
\end{aligned}
$$

Hence the theorem. □

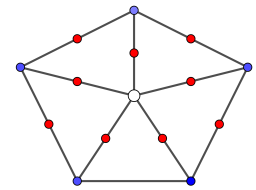

Figure 1. subdivision graph of W_6

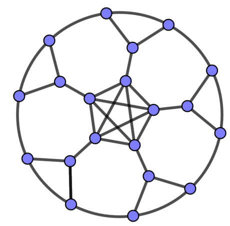

Figure 2. Line graph of subdivision graph of W_6

A class of graphs called Tadpole graph can be generated by augmenting a path to a cycle.ie, $\tau_{p,k}$ is C_p augmented with path of length k.It has $p+k$ vertices and edges.The subdivision graph of $\tau_{p,k}$ will have $2(p+k)$ vertices and edges.The line graph of this subdivision graph is denoted by τ_1 and will have the number of vertices as equal to the edges of the parent graph and hence has $2(p+k)$ vertices and claim to have an edge between two vertices if the corresponding edges share a common vertex in the parent graph.Hence one can easily see that there are $2(p+k)+1$ edges.

Theorem 2.2. *The AG index of line graph τ_1 can be obtained as* $AG(\tau_1) = 2(p+k-3) + \frac{3+5\sqrt{3}}{2\sqrt{2}}$
$HM(\tau_1) = 4(p+k) - \frac{7}{15}$ *where the parent tadpole graph has C_p attached with a path of length k.*

Proof. The $2(p+k)+1$ edges of τ_1 can be compartmentalised to four as follows: E_1 with one edge defined as $\{rs \in E(\tau_1) | \delta(r) = 1, \delta(s) = 2\}$
E_2 with $2(p+k)-6$ edges defined as $\{rs \in E(\tau_1) | \delta(r) = \delta(s) = 2\}$
E_3 with 3 edges defined as $\{rs \in E(\tau_1) | \delta(r) = 2, \delta(s) = 3\}$
and E_4 with 3 edges defined as $\{rs \in E(\tau_1) | \delta(r) = \delta(s) = 3\}$

Then

$$AG(\tau_1) = \sum_{rs\in E(\tau_1)} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}}$$
$$= \{\sum_{rs\in E_1} + \sum_{rs\in E_2} + \sum_{rs\in E_3} + \sum_{rs\in E_4}\} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}}$$
$$= \sum_{rs\in E_1} \frac{1+2}{2\sqrt{1.2}} + \sum_{rs\in E_2} \frac{2+2}{2\sqrt{2.2}} + \sum_{rs\in E_3} \frac{2+3}{2\sqrt{2.3}}$$
$$+ \sum_{rs\in E_4} \frac{3+3}{2\sqrt{3.3}}$$
$$= (\frac{3}{2\sqrt{2}} \times 1) + (1\times(2(p+k)-6)) + (\frac{5}{2\sqrt{6}} \times 3)$$
$$+ (3\times 3)$$
$$= \frac{3}{2\sqrt{2}} + (2(p+k)-6) + \frac{5}{2}\sqrt{\frac{3}{2}}$$
$$= (2(p+k)-6) + \frac{3+5\sqrt{3}}{2\sqrt{2}}$$

$$HM(\tau_1) = \sum \frac{2\delta(r)\delta(s)}{\delta(r)+\delta(s)}$$
$$= (\sum_{rs\in E_1} + \sum_{rs\in E_2} + \sum_{rs\in E_3} + \sum_{rs\in E_4}) \frac{2\delta(r)\delta(s)}{\delta(r)+\delta(s)}$$
$$= \frac{4}{3} + (2p+2k-6)2 + \frac{36}{5} + 9$$
$$= 4(p+k) - \frac{7}{15}$$

□

3. Topological indices of line graphs of some standard graphs

Linegraph of cycles are cycles them self that is $L(C_n) = C_n$ and $AG(C_n) = n$.
Line graphs of paths are paths $L(P_{n+1}) = P_n$ and line graphs of stars $K_{1,n}$ is K_n

Theorem 3.1. *Let line graphs of paths be denoted by $L(P_{n+1}) = \mathscr{P}_n$.Then for $\mathscr{P}_n$,the following results can be obtained*

$$AG(\mathscr{P}_n) = n-2+\frac{3}{\sqrt{2}}$$

Proof. $\mathscr{P}_n$ has two sets of edges,one set(E_1) having the two end edges whose vertices are of degrees 1 and 2 and the second set(E_2) with $n-2$ edges both of its vertices with degree 2.Hence

$$AG(\mathscr{P}_n) = \sum_{rs\in E(\mathscr{P}_n)} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}}$$
$$= \sum_{rs\in E_1} \frac{1+2}{2\sqrt{1.2}} + \sum_{rs\in E_2} \frac{2+2}{2\sqrt{2.2}}$$
$$= n-2+\frac{3}{\sqrt{2}}$$

□

Theorem 3.2.

$$AG(K_n) = n$$

where K_n is a complete graph on n vertices.

Proof. For K_n there are $\frac{n(n-1)}{2}$ each incident with vertices of degree $n-1$.Hence

$$AG(K_n) = \sum_{rs\in E(K_n)} \frac{\delta(r)+\delta(s)}{2\sqrt{\delta(r).\delta(s)}}$$
$$= n.\frac{(n-1)+(n-1)}{2\sqrt{(n-1)(n-1)}} \qquad = n$$

□

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