Malaya Journal of Matematik, Vol. 9, No. 1, 385-390, 2021
https://doi.org/10.26637/MJM0901/0064

Solving equality and inequality constraints FLPP using intuitionistic fuzzy numbers with different techniques

Kirtiwant Ghadle[1], Mayuri Deshmukh [2]* and Omprakash Jadhav[3]

Abstract
This article explains discovery fuzzy optimal solution without transforming into crisp LP Problems and ranking method. On the other hand, an attempt has been made by newly introducing the triangular and pentagonal IFN to deal with FLP problems with all types of constraints. The maximization and minimization FLP problems are solved by different Big-M techniques which are further compared, and an optimal solution has been found. The purpose of this paper is to arrive at the finest technique for unraveling FLP problems with triangular IFN and pentagonal IFN. The various Big-M methods are compared to solve FLP problems and it is observed that the answers are identical in each case but Ghadle et.al (Alternative Big-M Method) requires less time and minimum iterations with desired IFOS.

Keywords
Fuzzy set, Linear programming, Pentagonal fuzzy number, Triangular intuitionistic fuzzy number, Big-M technique.

AMS Subject Classification
90C05, 03E72, 94D05.

[1,2] *Department of Mathematics Dr. Babasaheb Ambedkar Marathwada University, Aurangabad -431004 (M.S.), India.*
[3] *Department of Statistics Dr. Babasaheb Ambedkar Marathwada University, Aurangabad -431004 (M.S.), India.*
***Corresponding author**: [1] jagtapmayuri01@gmail.com
Article History: Received **24** October **2020**; Accepted **09** January **2021**

Contents

Abbreviation

LP- Linear Programming
FS- Fuzzy Set
FN- Fuzzy Number
TP- Transportation Problem
FLP- Fuzzy Linear Programming
IFS- Intuitionistic Fuzzy Set
IFN- Intuitionistic Fuzzy Number
FFLP- Fully Fuzzy Linear Programming
TIFN- Triangular Intuitionistic Fuzzy Number
PIFN- Pentagonal Intuitionistic Fuzzy Number
IFTP- Intuitionistic Fuzzy Transportation Problem
FOP- Fuzzy Optimal Solution
IFOP- Intuitionistic Fuzzy Optimal Solution .

1. Introduction

Its quiet possible to transform the real-world issues or problems into LP model as this model is applicable to transportation and manufacturing. The finest result of LP depends upon a limited amount of constraints, and so, ample of the collected facts has a very minute influence on the result, therefore for having certainty and meticulousness of given information and to handle ample information, one have to think about fuzzy data.

Fuzzy sets, offered by Zadeh (1965), gives a flexible structure for handling non-statistical vagueness ideas. It has been aimed to symbolize uncertainty and ambiguity mathematically to deliver formalized tools for dealing with the imprecision inherent many real-life problems. FS theory is having various applications in clustering, image processing, decision making etc. Yet, fuzzy sets couldn't tackle the circumstances, where the unclear information comprises some degree of hesitancy which essentially arises from the vague data. Atanassov (1986) the idea of the IFS as a simplification of FS, and is more useful in netting the unclear, partial or undefined information that involves some amount of hesitancy and is relevant in several fields of research.

In [5] suggested a innovative technique for discovery the FOS of Fully FLP problems with equality constraints. A novel alternative algorithm is offered for simplex and two-phase simplex method in [6] which reduces the number of iterations while solving LP problems. An innovative efficient method is proposed in [15] based on crisp nonlinear programming for Fully FLP problems with equality constraints using unrestricted variables and parameters. In [10], introduced octagonal IFN with its membership and non-membership functions and offered an innovative algorithm for MODI method to arrive an optimal solution for IFTP using a ranking function. A new algorithm in [11] is developed to solve balanced and unbalanced fuzzy TP which uses mixed constraint with trapezoidal and trivial fuzzy numbers.

2. Preliminaries

In this section, we recall some definitions.

Definition 2.1. *[12] A FN is a generalized of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function.*
A FN $\tilde{A}$ is a convex normalized FS on the real line R such that
i)There exist at least one $x \in R$ with $\mu_{\tilde{A}}(x) = 1$.
ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.2. *[?] A FN of $\tilde{A} = (j_1, j_2, j_3)$ is said to be a triangular FN if its membership function is given below*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le j_1, \\ \frac{x-j_1}{j_2-j_1}, & j_1 \le x \le j_2 \\ 1, & x = j_2 \\ \frac{j_3-x}{j_3-j_2}, & j_2 \le x \le j_3 \\ 0, & x > j_3 \end{cases} \tag{2.1}$$

Definition 2.3. *[13] A Pentagonal FN $\tilde{A}_p = (j_1, j_2, j_3, j_4, j_5)$ where j_1, j_2, j_3, j_4 and j_5 are real numbers and $j_1 \le j_2 \le j_3 \le j_4 \le j_5$ with membership function is given below*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le j_1, \\ \frac{x-j_1}{j_2-j_1}, & j_1 \le x \le j_2 \\ \frac{x-j_2}{j_3-j_2}, & j_2 \le x \le j_3 \\ 1, & x = j_3 \\ \frac{j_4-x}{j_4-j_3}, & j_3 \le x \le j_4 \\ \frac{j_5-x}{j_5-j_4}, & j_4 \le x \le j_5 \\ 0, & x > j_5 \end{cases} \tag{2.2}$$

Definition 2.4. *[14] Let X denote the universe of discourse, then an IFS $\bar{A}^I$ in X is given by $\bar{A}^I = \{x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x); x \in X\}$ where $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) : X \to [0,1]$ are function such that $0 \le \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \le 1$ for all $x \in X$, For each x the membership function $\mu_{\bar{A}^I}(x)$ and $\nu_{\bar{A}^I}(x)$ represent the degree of membership and non-membership of the element $x \in X$ to $A \subset X$ respectively.*

Definition 2.5. *[14] An IF subset $\bar{A}^I = \{x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x); x \in X\}$ of the real line R is called an IFN if the following holds:*
i) There exists $m \in R, \mu_{\bar{A}^I}(m) = 1$ and $\nu_{\bar{A}^I}(m) = 0$, m is called the mean value of $\bar{A}^I$.
ii) $\mu_{\bar{A}^I}$ is a continuous mapping from R to the closed interval [0,1] and for all $x \in R$ the relation $0 \le \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \le 1$ holds.
The membership and non-membership function of $\bar{A}^I$ is of the following form

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0, & -\infty < x \le m - \alpha, \\ f_1(x), & x \in [m-\alpha, m] \\ 1, & x = m \\ g_1(x), & x \in [m, m+\beta] \\ 0, & m + \beta \le x \le \infty \end{cases} \tag{2.3}$$

$$\nu_{\bar{A}^I}(x) = \begin{cases} 1, & -\infty < x \le m - \alpha', \\ f_2(x), & x \in [m-\alpha', m], 0 \le f_1(x) + f_2(x) \le 1 \\ 0, & x = m \\ g_2(x), & x \in [m, m+\beta'], 0 \le g_1(x) + g_2(x) \le 1 \\ 1, & m + \beta' \le x \le \infty \end{cases} \tag{2.4}$$

Here m is the mean value of $\bar{A}^I(x)$, α and β are called left and right spreads of membership function $\mu_{\bar{A}^I}(x)$ respectively. α', β' represent left and right spreads of non-membership function $\nu_{\bar{A}^I}(x)$ respectively.

Definition 2.6. *[2] A triangular IFN $\bar{A}^I(x)$ is an IF subset in R with the following membership function $\mu_{\bar{A}^I}(x)$ and non-membership function $\nu_{\bar{A}^I}(x)$*

$$\mu_{\bar{A}^I}(x) = \begin{cases} \frac{x-j_1}{j_2-j_1}, & j_1 \le x \le j_2 \\ \frac{j_3-x}{j_3-j_2}, & j_2 \le x \le j_3 \\ 0, & otherwise \end{cases} \tag{2.5}$$

$$\nu_{\bar{A}^I}(x) = \begin{cases} \frac{j_2-x}{j_2-j_1'}, & j_1' \le x \le j_2 \\ \frac{x-j_2}{j_3'-j_2}, & j_2 \le x \le j_3' \\ 1, & otherwise \end{cases} \tag{2.6}$$

where, $j_1' \le j_1 \le j_2 \le j_3 \le j_3'$ and $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) \le 0.5$ for $\mu_{\bar{A}^I}(x) = \nu_{\bar{A}^I}(x)$ for all $x \in X$. TIFN $\bar{A}^I$ is denoted by $(j_1, j_2, j_3; j_1', j_2, j_3')$.

Definition 2.7. *[3] A pentagonal IFN $\bar{A}^I$ is defined as $\bar{A}^I = j_1, j_2, j_3, j_4, j_5; j_1', j_2', j_3, j_4', j_5'$. Where all $j_1, j_2, j_3, j_4, j_5; j_1'$,*

j'_2, j_3, j'_4, j'_5 are real numbers such that $j'_1 \le j_1 \le j'_2 \le j_2 \le j_3 \le j_4 \le j'_4 \le j_5 \le j'_5$ and its membership function $\mu_{\bar{A}^I}(x)$, non-membership function $\nu_{\bar{A}^I}(x)$ are given below

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0, & x < j_1, \\ \frac{x-j_1}{j_2-j_1}, & j_1 \le x \le j_2 \\ \frac{x-j_2}{j_3-j_2}, & j_2 \le x \le j_3 \\ 1, & x = j_3 \\ \frac{j_4-x}{j_4-j_3}, & j_3 \le x \le j_4 \\ \frac{j_5-x}{j_5-j_4}, & j_4 \le x \le j_5 \\ 0, & x > j_5 \end{cases} \tag{2.7}$$

$$\nu_{\bar{A}^I}(x) = \begin{cases} 1, & x < j'_1, \\ \frac{j'_2-x}{j'_2-j'_1}, & j'_1 \le x \le j'_2 \\ \frac{j_3-x}{j_3-j'_2}, & j'_2 \le x \le j_3 \\ 0, & x = j_3 \\ \frac{x-j_3}{j'_4-j_3}, & j_3 \le x \le j'_4 \\ \frac{x-j'_4}{j'_5-j'_4}, & j'_4 \le x \le j'_5 \\ 1, & x > j'_5 \end{cases} \tag{2.8}$$

Definition 2.8. *Operations on IFN*
i) Triangular IFN: [14] *Let$\bar{A}^I = (p_1, p_2, p_3; p'_1, p_2, p'_3)$ and $\bar{B}^I = (r_1, r_2, r_3; r'_1, r_2, r'_3)$ be two triangular IFN the arithmetic operations on $\bar{A}^I$ and $\bar{B}^I$ is given below:*

Addition:$(p_1, p_2, p_3; p'_1, p_2, p'_3) + (r_1, r_2, r_3; r'_1, r_2, r'_3) = (p_1 + r_1, p_2 + r_2, p_3 + r_3; p'_1 + r'_1, p_2 + r_2, p'_3 + r'_3)$

Subtraction:$(p_1, p_2, p_3; p'_1, p_2, p'_3) - (r_1, r_2, r_3; r'_1, r_2, r'_3) = (p_1 - r_3, p_2 - r_2, p_3 - r_1; p'_1 - r'_3, p_2 - r_2, p'_3 - r'_1)$

Multiplication:$(p_1, p_2, p_3; p'_1, p_2, p'_3) * (r_1, r_2, r_3; r'_1, r_2, r'_3) = (p_1 r_1, p_2 r_2, p_3 r_3; p'_1 r'_1, p_2 r_2, p'_3 r'_3)$

ii)Pentagonal IFN: [1] *Let$\bar{A}^I = (p_1, p_2, p_3, p_4, p_5; p'_1, p'_2, p_3, p'_4, p'_5)$ and $\bar{B}^I = (r_1, r_2, r_3, r_4, r_5; r'_1, r'_2, r_3, r'_4, r'_5)$ be two Pentagonal IFN the arithmetic operations on $\bar{A}^I$ and $\bar{B}^I$ is given below:*

Addition : $(p_1, p_2, p_3, p_4, p_5; p'_1, p'_2, p_3, p'_4, p'_5) + (r_1, r_2, r_3, r_4, r_5; r'_1, r'_2, r_3, r'_4, r'_5) = (p_1 + r_1, p_2 + r_2, p_3 + r_3, p_4 + r_4, p_5 + r_5, ; p'_1 + r'_1, p'_2 + r'_2, p_3 + r_3, p'_4 + r'_4, p'_5 + r'_5)$

Subtraction : $(p_1, p_2, p_3, p_4, p_5; p'_1, p'_2, p_3, p'_4, p'_5) - (r_1, r_2, r_3, r_4, r_5; r'_1, r'_2, r_3, r'_4, r'_5) = (p_1 - r_5, p_2 - r_4, p_3 - r_3, p_4 - r_2, p_5 - r_1; p'_1 - r'_5, p'_2 - r'_4, p_3 - r_3, p'_4 - r'_2, p'_5 - r'_1)$

Multiplication : $(p_1, p_2, p_3, p_4, p_5; p'_1, p'_2, p_3, p'_4, p'_5) * (r_1, r_2, r_3, r_4, r_5; r'_1, r'_2, r_3, r'_4, r'_5) = (p_1 r_1, p_2 r_2, p_3 r_3, p_4 r_4, p_5 r_5; p'_1 r'_1, p'_2 r'_2, p_3 r_3, p'_4 r'_4, p'_5 r'_5)$

3. Numerical Examples

3.1 Triangular IFN

Example 3.1.1 Explain FLP problem

$$\begin{aligned} \text{Maximize}\tilde{Z} =& (2,4,6;0,4,8)\tilde{x}_1 + (4,6,8;2,6,10)\tilde{x}_2 \\ \text{Subject to}: & 4\tilde{x}_1 + 4\tilde{x}_2 \le (48,54,60;42,54,66) \\ & 3\tilde{x}_1 + 6\tilde{x}_2 = (42,48,54;36,48,60) \\ & \tilde{x}_1, \tilde{x}_2 \ge 0 \end{aligned}$$

Solution: Standard form of FLP problem:

$$\begin{aligned} \text{Maximize}\tilde{Z} =& (2,4,6;0,4,8)\tilde{x}_1 + (4,6,8;2,6,10)\tilde{x}_2 + \\ & 0\tilde{S}_1 - M\tilde{A}_1 \\ \text{Subject to}: & 4\tilde{x}_1 + 4\tilde{x}_2 + \tilde{S}_1 = (48,54,60;42,54,66) \\ & 3\tilde{x}_1 + 6\tilde{x}_2 + \tilde{A}_1 = (42,48,54;36,48,60) \\ & \tilde{x}_1, \tilde{x}_2, \tilde{S}_1, \tilde{A}_1 \ge 0 \end{aligned}$$

By applying Ghadle et al.,[4] we get

Table 1. Iteration table

		$\tilde{c}_j$	$(2,4,6;0,4,8)$	$(4,6,8;2,6,10)$	0	-M
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{S}_1$	$\tilde{A}_1$
0	$\tilde{S}_1$	$(48,54,60;42,54,66)$	4	4	1	0
$-M$	$\tilde{A}_1$	$(42,48,54;36,48,60)$	3	6^*	0	1
0	$\tilde{S}_1$	$(12,22,32;2,22,42)$	2^*	0	1	$-\frac{2}{3}$
$(4,6,8;2,6,10)$	$\tilde{x}_2$	$(7,8,9;6,8,10)$	$\frac{1}{2}$	1	0	$\frac{1}{6}$
$(2,4,6;0,4,8)$	$\tilde{x}_1$	$(6,11,16;1,11,21)$	1	0	$\frac{1}{2}$	$-\frac{1}{3}$
$(4,6,8;2,6,10)$	$\tilde{x}_2$	$(-1,2.5,6;-4.5,2.5,9.5)$	0	1	$-\frac{1}{4}$	$\frac{1}{3}$

IFOS is $\tilde{x}_1 = (6,11,16;1,11,21), \tilde{x}_2 = (-1,2.5,6;-4.5,2.5,9.5)$ and Maximize $\tilde{Z} = (8,59,144;0,59,263)$

Table 2. Comparison Table

Different Big-M Method	Ghadle etc al.[4]	Muralidaran etc al.[9]	Khobragade etc al. [7]	Lokhande etc al.[8]
Example 3.1.1 IFOS	(8,59,144; 0,59,263)	(8,59,144; 0,59,263)	(8,59,144;, 0,59,263)	(8,59,144; 0,59,263)

Table 2 describes the evaluation as well as confirmation results with different Big-M methods. It is evident from the outcomes that, for **Maximization type of case with TIFN,** though intuitionistic fuzzy optimal values are similar, Ghadle et al (Alternative Big-M Method) proves to be the better method as it gives the required IFOS within minimum time.

Example 3.1.2 Explain FLP problem

$$\begin{aligned}\text{Minimize}\tilde{Z} =&(3,4,5;2,4,6)\tilde{x}_1+(1,2,3;0,2,4)\tilde{x}_2+\\&(1,2,3;0,2,4)\tilde{x}_3\\ \text{Subject to}:&3\tilde{x}_1+\tilde{x}_2-2\tilde{x}_3=(24,27,30;21,27,33)\\&\tilde{x}_1+\tilde{x}_2=(27,30,33;24,30,36)\\&1\tilde{x}_1+2\tilde{x}_1-\tilde{x}_3=(27,30,33;24,30,36)\\&\tilde{x}_1,\tilde{x}_2,\tilde{x}_3\geq 0\end{aligned}$$

Solution: Standard form of FLP problem:

$$\begin{aligned}\text{Maximize}\tilde{Z} =&-(3,4,5;2,4,6)\tilde{x}_1-(1,2,3;0,2,4)\tilde{x}_2\\&-(1,2,3;0,2,4)\tilde{x}_3-M\tilde{A}_1-M\tilde{A}_2\\&-M\tilde{A}_3\\ \text{Subject to}:&3\tilde{x}_1+\tilde{x}_2-2\tilde{x}_3+\tilde{A}_1=(24,27,30;21,27,33)\\&\tilde{x}_1+\tilde{x}_2+\tilde{A}_2=(27,30,33;24,30,36)\\&1\tilde{x}_1+2\tilde{x}_1-\tilde{x}_3+\tilde{A}_3=(27,30,33;24,30,36)\\&\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{A}_1,\tilde{A}_2,\tilde{A}_3\geq 0\end{aligned}$$

By applying Ghadle et al.,[4] we get

Table 3. Iteration table

		$\tilde{c}_j$	$-\binom{3,4,5;}{2,4,6}$	$-\binom{1,2,3;}{0,2,4}$	$-\binom{1,2,3;}{0,2,4}$	-M	-M	-M
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$
$-M$	$\tilde{A}_1$	$\binom{24,27,30}{21,27,33}$	3^*	1	-2	1	0	0
$-M$	$\tilde{A}_2$	$\binom{27,30,33;}{24,30,36}$	1	1	0	0	1	0
$-M$	$\tilde{A}_3$	$\binom{27,30,33;}{24,30,36}$	1	2	-1	0	0	1
$-\binom{3,4,5;}{2,4,6}$	$\tilde{x}_1$	$\binom{8,9,10;}{7,9,11}$	1	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	0	0
$-M$	$\tilde{A}_2$	$\binom{17,21,25;}{13,21,29}$	0	$\frac{2}{3}$	0	$-\frac{1}{3}$	1	0
$-M$	$\tilde{A}_3$	$\binom{17,21,25;}{13,21,29}$	0	$\frac{5}{3}^*$	-1	$-\frac{1}{3}$	0	1
$-\binom{3,4,5;}{2,4,6}$	$\tilde{x}_1$	$\binom{3,4.8,6.6;}{1.2,4.8,8.4}$	1	0	$-\frac{7}{15}$	$\frac{2}{5}$	0	$-\frac{1}{5}$
$-M$	$\tilde{A}_2$	$\binom{7,12.6,18.2;}{1.4,12.6,23.8}$	0	0	$\frac{2}{5}^*$	$-\frac{1}{5}$	1	$-\frac{2}{5}$
$-\binom{1,2,3;}{0,2,4}$	$\tilde{x}_2$	$\binom{10.2,12.6,15;}{7.8,12.6,17.4}$	0	1	$-\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{3}{5}$
$-\binom{3,4,5;}{2,4,6}$	$\tilde{x}_1$	$\binom{11.1,19.5,27.8;}{2.8,19.5,36.1}$	1	0	0	$\frac{1}{6}$	$\frac{7}{6}$	$-\frac{2}{3}$
$-\binom{1,2,3;}{0,2,4}$	$\tilde{x}_3$	$\binom{17.5,31.5,45.5;}{3.5,31.5,59.5}$	0	0	1	$-\frac{1}{2}$	$\frac{5}{2}$	-1
$-\binom{1,2,3;}{0,2,4}$	$\tilde{x}_2$	$\binom{20.7,31.5,42.2;}{9.9,31.5,53.1}$	0	1	0	$-\frac{1}{2}$	$\frac{3}{2}$	0

IFOS is $\tilde{x}_1=(11.1,19.5,27.8;2.8,19.5,36.1)$, $\tilde{x}_2=(20.7,31.5,42.2;9.9,31.5,53.1)$ and $\tilde{x}_3=(17.5,31.5,45.5;3.5,31.5,59.5)$ and Minimize $\tilde{Z}=(71.6,204,402.4;5.6,204,667)$

Table 4. Comparison Table

Different Big-M Method	Ghadle etc al.[4]	Muralidaran etc al.[9]	Khobragade etc al. [7]	Lokhande etc al.[8]
Example 3.1.2 IFOS	(71.6,204, 402.4;5.6, 204,667)	(71.6,204, 402.4;5.6, 204,667)	(71.6,204, 402.4;5.6, 204,667)	(71.6,204, 402.4;5.6, 204,667)

Table 4 describes the evaluation as well as confirmation results with different Big-M methods. It is evident from the outcomes that, for **Minimization type of case with TIFN,** though intuitionistic fuzzy optimal values are similar, Ghadle et al (Alternative Big-M Method) proves to be the better method as it gives the required IFOS within minimum time.

3.2 Pentagonal IFN

Example 3.2.1 Explain FLP problem

$$\begin{aligned}\text{Maximize}\tilde{Z} =&(16,18,20,22,24;12,14,20,26,28)\tilde{x}_1+\\&(26,28,30,32,34;22,24,30,36,38)\tilde{x}_2\\ \text{Subject to}:&1\tilde{x}_1+4\tilde{x}_2\leq(36,38,40,42,44;32,34,40,\\&46,48)\\&2\tilde{x}_1+1\tilde{x}_2=(26,28,30,32,34;22,24,30,\\&36,38)\\&\tilde{x}_1,\tilde{x}_2,\geq 0\end{aligned}$$

Solution: Standard form of FLP problem:

$$\begin{aligned}\text{Maximize}\tilde{Z} =&(16,18,20,22,24;12,14,20,26,28)\tilde{x}_1+\\&(26,28,30,32,34;22,24,30,36,38)\tilde{x}_2+0\tilde{S}_1\\&-M\tilde{A}_1\\ \text{Subject to}:&1\tilde{x}_1+4\tilde{x}_2+\tilde{S}_1=(36,38,40,42,44;32,34,\\&40,46,48)\\&2\tilde{x}_1+1\tilde{x}_2+\tilde{A}_1=(26,28,30,32,34;22,24,\\&30,36,38)\\&\tilde{x}_1,\tilde{x}_2,\tilde{S}_1,\tilde{A}_1\geq 0\end{aligned}$$

By applying Ghadle et al.,[4] we get

Table 5. Iteration table

		$\tilde{c}_j$	$\binom{16,18,20,22,24;}{12,14,20,26,28}$	$\binom{26,28,30,32,34}{22,24,30,36,38}$	0	-M
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{S}_1$	$\tilde{A}_1$
0	$\tilde{S}_1$	$\binom{36,38,40,42,44;}{32,34,40,46,48}$	1	4^*	1	0
-M	$\tilde{A}_1$	$\binom{26,28,30,32,34;}{22,24,30,36,38}$	2	1	0	1
$\binom{26,28,30,32,34;}{22,24,30,36,38}$	$\tilde{x}_2$	$\binom{9,9.5,10,10.5,11;}{8,8.5,10,11.5,12}$	$\frac{1}{4}$	1	$\frac{1}{4}$	0
-M	$\tilde{A}_1$	$\binom{15,17.5,20,22.5,25;}{10,12.5,20,27.5,30}$	$\frac{7}{4}^*$	0	$-\frac{1}{4}$	1
$\binom{26,28,30,32,34;}{22,24,30,36,38}$	$\tilde{x}_2$	$\binom{5.4,6.3,7.1,8,8.9;}{3.7,4.5,7.1,9.7,10.5}$	0	1	$\frac{2}{7}$	$-\frac{1}{7}$
$\binom{16,18,20,22,24;}{12,14,20,26,28}$	$\tilde{x}_1$	$\binom{8.5,10,11.4,12.8,14.2;}{5.7,7.1,11.4,15.7,17.1}$	1	0	$-\frac{1}{7}$	$\frac{4}{7}$

IFOS is $\tilde{x}_1=(8.5,10,11.4,12.8,14.2;5.7,7.1,11.4,15.7,17.1)$, $\tilde{x}_2=(5.4,6.3,7.1,8,8.9;3.7,4.5,7.1,9.7,10.5)$ and Maximize $\tilde{Z}$=$(276.4,356.4,441,537.6,643.4;149.8,207.4,441,757.4,877.8)$

Table 6. Comparison Table

Different Big-M Method	Ghadle etc al.[4]	Muralidaran etc al.[9]	Khobragade etc al. [7]	Lokhande etc al.[8]
Example 3.2.1 IFOS	(276.4,356.4 441, 537.6, 643.4;149.8, 207.4,441, 757.4,877.8)	(276.4,356.4 441, 537.6, 643.4;149.8, 207.4,441, 757.4,877.8)	(276.4,356.4 441, 537.6 643.4;149.8, 207.4,441, 757.4,877.8)	(276.4,356.4 441, 537.6 643.4;149.8, 207.4,441, 757.4,877.8)

Table 6 describes the evaluation as well as confirmation results with different Big-M methods. It is evident from the outcomes that, for **Maximization type of case with PIFN,** though intuitionistic fuzzy optimal values are similar, Ghadle et al (Alternative Big-M Method) proves to be the better method as it gives the required IFOS within minimum time.

Example 3.2.2 Explain FLP problem

$$\text{Minimize}\tilde{Z} = (8,10,12,14,16;4,6,12,18,20)\tilde{x}_1 + (26,28,30,32,34;22,24,30,36,38)\tilde{x}_2$$
$$\text{Subject to}: 6\tilde{x}_1 + 8\tilde{x}_2 = (80,90,100,110,120;60,70,100,130,140)$$
$$0\tilde{x}_1 + 12\tilde{x}_2 = (100,110,120,130,140;80,90,120,150,160)$$
$$\tilde{x}_1, \tilde{x}_2, \geq 0$$

Solution: Standard form of FLP problem:

$$\text{Maximize}\tilde{Z} = -(8,10,12,14,16;4,6,12,18,20)\tilde{x}_1 - (26,28,30,32,34;22,24,30,36,38)\tilde{x}_2 - M\tilde{A}_1 - M\tilde{A}_2$$
$$\text{Subject to}: 6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{A}_1 = (80,90,100,110,120;60,70,100,130,140)$$
$$0\tilde{x}_1 + 12\tilde{x}_2 + \tilde{A}_2 = (100,110,120,130,140;80,90,120,150,160)$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{A}_1, \tilde{A}_2 \geq 0$$

By applying Ghadle et al.,[4] we get

Table 7. Iteration table

		$\tilde{c}_j$	$-\begin{pmatrix}8,10,12,14,16;\\4,6,12,18,20\end{pmatrix}$	$-\begin{pmatrix}26,28,30,32,34\\22,24,30,36,38\end{pmatrix}$	-M	-M
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{A}_1$	$\tilde{A}_2$
-M	$\tilde{A}_1$	$\begin{pmatrix}80,90,100,110,120;\\60,70,@100,130,140\end{pmatrix}$	6	8	1	0
-M	$\tilde{A}_2$	$\begin{pmatrix}100,110,120,130,140;\\80,90,120,150,160\end{pmatrix}$	0	12^*	0	1
-M	$\tilde{A}_1$	$\begin{pmatrix}-12.8,3.6,20,@37.2,53.6;\\-46.6,-30,20,70,87.2\end{pmatrix}$	$\frac{4}{3}^*$	0	1	$\frac{2}{3}$
$-\begin{pmatrix}26,28,30,32,34;\\22,24,30,36,38\end{pmatrix}$	$\tilde{x}_2$	$\begin{pmatrix}8.3,9.1,10,10.8,11.6;\\6.6,7.5,10,12.5,13.3\end{pmatrix}$	0	1	0	$\frac{1}{12}$
$-\begin{pmatrix}8,10,12,14,16;\\4,6,12,18,2\end{pmatrix}$	$\tilde{x}_1$	$\begin{pmatrix}-9.6,2.7,15,27.9,40.2;\\-34.8,-22.5,15,52.5,65.5\end{pmatrix}$	1	0	$\frac{3}{4}$	$\frac{1}{2}$
$-\begin{pmatrix}26,28,30,32,34;\\22,24,30,36,38\end{pmatrix}$	$\tilde{x}_2$	$\begin{pmatrix}8.3,9.1,10,10.8,11.6;\\6.6,.7.5,10,12.5,13.3\end{pmatrix}$	0	1	0	$\frac{1}{12}$

IFOS is $\tilde{x}_1 = (-9.6,2.7,15,27.9,40.2;-34.8,-22.5,15,52.5,65.5), \tilde{x}_2 = (8.3,9.1,10,10.8,11.6;6.6,7.5,10,12.5,13.3)$ and Minimize $\tilde{Z}_= (139,281.8,480,736.2,1037.6;6,45,480,1395,1813.4)$

Table 8. Comparison Table

Different Big-M Method	Ghadle etc al.[4]	Muralidaran etc al.[9]	Khobragade etc al. [7]	Lokhande etc al.[8]
Example 3.2.2 IFOS	(139,281.8, 480,736.2, 1037.6;6, 45,480, 1395,1813.4)	(139,281.8, 480,736.2, 1037.6;6, 45,480, 1395,1813.4)	(139,281.8, 480,736.2, 1037.6;6, 45,480, 1395,1813.4)	(139,281.8, 480,736.2, 1037.6;6, 45,480, 1395,1813.4)

Table 8 describes the evaluation as well as confirmation results with different Big-M methods. It is evident from the outcomes that, for **Minimization type of case with PIFN,** though intuitionistic fuzzy optimal values are similar, Ghadle et al (Alternative Big-M Method) proves to be the better method as it gives the required IFOS within minimum time.

4. Conclusion

In this article we have solved FLP problems without converting into CLP problems. The triangular and pentagonal IFLP problems for maximization and minimization cases by different Big-M methods have been solved. Again, the methods of Muralidaran et al., Ghadle et al. (Alternative Big-M Method), Khobragade et al., Lokhande et al. all are compared. We found that Ghadle et al (Alternative Big-M Method) is easy to solve, requires less time and it minimizes the iterations. Therefore it is superior from other method.

References

[1] M. Christi and B. Kasturi, Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers Solved Using Ranking Technique and Russell's method, *Journal of Engineering Research and Applications,* 6(2016), 82-86.

[2] M. Deshmukh, K. Ghadle and O. Jadhav, Optimal Solution of Fully Fuzzy LPP with Symmetric HFNs. Computing in Engineering and Technology, *Advances in Intelligent Systems and Computing,* Springer Nature, Singapore 1025(2020), 387-395.

[3] M. Deshmukh, K. Ghadle and O. Jadhav, An Innovative Approach for Ranking Hexagonal Fuzzy Numbers to Solve Linear Programming Problems, *International Journal on Emerging Technologies,* 11(2020), 385–388.

[4] K. Ghadle, T. Pawar and N. Khobragade, Solution of Linear Programming Problem by New Approach, *International Journal of Engineering and Innovative Technology,* 3(2013), 301-307.

[5] A. Kumar,J. Kaur and P. Sing, A New Method for Solving Fully Fuzzy Linear Programming Problems, *Applied Mathematical Modelling,* 35(2011), 817-823.

[6] N. Khobragade, N. Vaidya and N. Lamba, Approximation Algorithm for Optimal Solution to the Linear Programming Problem, *International Journal of Mathematics in Operational Research,* 6(2014), 139-154.

[7] N. Khobragade, Alternative Approach to the Simplex Method-1, *Bulletin of Pure and Applied Sciences,* 23E (2004), 35-40.

[8] K. Lokhande, N. Khobragade and P. Khot, Simplex Method: An Alternative Approach, *International Journal of Engineering and Innovative Technology,* 3(2013), 426-428.

[9] C. Muralidaran, B. Venkateswarlu, Direct Solving Method of Fully Fuzzy Linear Programming Problems with Equality Constraints Having Positive Fuzzy Numbers, *Advances in Algebra and Analysis*, Trends in Mathematics, Springer Nature Switzerland (2018) 301-307.

[10] G. Menaka, Ranking of Octagonal Intuitionistic Fuzzy Numbers, *IOSR Journal of Mathematics,* 13(2017), 63-71.

[11] P. Pathade, A. Hamoud and K. Ghadle, A Systematic Approach for Solving Mixed Constraints Fuzzy Balanced and Unbalanced Transportation Problem, *Indone-*

sian Journal of Electrical Engineering and Computer Science, 19(2020), 85-90.

[12] P. Pathade, K. Ghadle, Transportation Problem with Triangular Mixed Intuitionistic Fuzzy Numbers Solved by BCM, *International Journal Fuzzy Mathematical Archive,* 15(2018), 55-61.

[13] T. Pathinathan, K. Ponnivalavan, Pentagonal Fuzzy Numbers, *International Journal of Computing Algorithm,* 3(2014), 1003-1005.

[14] K. Pramila, G. Uthra, Optimal Solution of an Intuitionistic Fuzzy Transportation Problem, *Annals of Pure and Applied Mathematics,* 8(2014), 67-73.

[15] Saberi, H. Najafi, S. Edalatpanah and H. Dutta, A Nonlinear Model for Fully Fuzzy Linear Programming with Fully Unrestricted Variables and Parameters, *Alexandria Engineering Journal* 55(2016), 2589-2595.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆