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A mathematical model for apportionment and reapportionment using F26A graph and the magic labeling

Md. Shahira Sulthana[1]*, G. Margaret Joan Jebarani[2] and L. Sujatha[3]

Abstract

The Allotment of a set of integers to vertices and edges of a graph satisfying certain conditions termed as labelling of a graph. It was introduced in the late 1960s by (5). The concept of magic labelling which is one of the many labellings itself has a variety. By fixing some natural number (here some atomic numbers) to the edges of F26A graph, it is proved to be a magic graph.

Keywords

F26A graph, Magic Labeling, Apportionment, Reapportionment, Atomic Number of Elements with their symbols.

[1] *Department of Mathematics, Auxilium College, Vellore, Tamil Nadu, India.*
[2] *Department of Mathematics, Auxilium College, Vellore, Tamil Nadu, India.*
[3] *PG and Research Department of Mathematics, Auxilium College, Vellore, Tamil Nadu, India.*
***Corresponding author**: [1] mdshahira.mphil@gmail.com; [2] gmjoanj@gmail.com [3] sujathajayasankar@yahoo.co.in

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1. Introduction

The applications of concepts of graph theory keep increasing day by day. In particular ,the concept of Graph labelinghas a noteworthy contribution in such applications.The Authors of this paperwere inspired to work in graph labelings,the source being " A Dynamic survey of graph labeling ",the Electronic Journal of combinatorics,19(2009),1-219 by J.A.Gallian [1]. It is natural to be attracted towards Magic and the authors decide to use the magic labelingand present an application and hence this paper. By choosing the Graph F26A and the Magic Labeling they have proposed a Mathematical model for Apportionment and Reapportionment. Inorder to maintain secrecy with respect to the detailsof the apportionment,Atomic Numbers and symbols of certain elements are used.

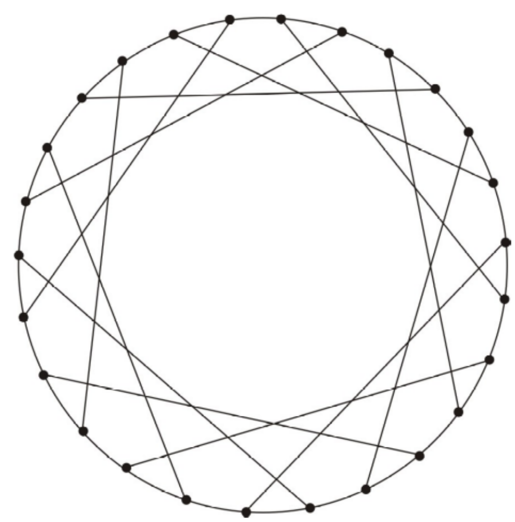

Figure 1. F26A

2. Pre-requisites

Definition 2.1 (F26A [3])**.** *The F26A graph is a cubic graph with 26 vertices and 39 edges.*

Definition 2.2 (Magic Graph [5])**.** *By assigning natural numbers to the edges such that the sum total of the edge values* *incident at a vertex is a constant (same for all vertices) is defined to be a magic labelling (one of many).*

Definition 2.3 (Atomic Number of Elements [4]). *The number of protons present in the nuclease of each atom denotes the atomic number which is unique for each element and it is denoted by the letter Z.*

Definition 2.4 (Apportionment [7]). *The division of the properties or wealth owned by a person among two or more people in same proportion or ratio is known as Apportionment.*

Definition 2.5 (Reapportionment [7]). *To redistribute or reallocate the sum that is already apportioned.*

3. Findings

Theorem 3.1. *The F26A graph admits the magic labeling, where the edge labels are assigned positive integers.*

Proof. There are 26 vertices and 39 edges in the F26A. Three edges pass through each v_i.

The magic total, that is the sum of distinct edge labels incident with the vertex v_i is denoted by $S(v_i)$ should be the same for $i = 1, 2, \ldots, 26$ for the graph to be a magic graph.

Let take the magic sum as x. Define a function which is injective from $E(G)$ to N. $f(e_1) = x_1$, $f(e_2) = x_2$, and $f(e_3) = x - (x_1 + x_2)$ where e_1, e_2 and e_3 are incident with the vertex v_i.

Adding,

$$\begin{aligned} S(v_1) &= f(e_1) + f(e_2) + f(e_3) \\ &= x_1 + x_2 + x - (x_1 + x_2) \\ &= x \end{aligned}$$

By repeating the same procedure, avoiding repetition for edge rules, the graph is found to admit the magic labeling and so becomes a magic graph. □

3.1 Verification

The edge labels assume any positive integer. To be specific take the magic sum as 200. Take

$$f(e_i) = (2q+3) - i \text{ for } i = 1, 2 \ldots, 8 \tag{3.1}$$

Therefore,

$$\begin{aligned} f(e_1) &= 80, \\ f(e_2) &= 79, \\ &\ldots \\ f(e_8) &= 73 \end{aligned}$$

As the vertex v_9 is connected to v_2 by the edge e_28 (forming a cycle), The formula 3.1 fails for $i = 9$ onwards. $f(e_9) = 72$ according to 3.1. According to the condition,

$$\begin{aligned} f(e_1) + f(e_2) + f(e_{28}) &= 200 \\ \Rightarrow 80 + 79 + f(e_{28}) &= 200 \\ \Rightarrow f(e_{28}) &= 41 \end{aligned}$$

The edges e_{28}, e_8, e_9 are incident at v_9 so,

$$\begin{aligned} f(e_{28}) + f(e_8) + f(e_9) &= 200 \\ \Rightarrow 41 + 73 + f(e_9) &= 200 \end{aligned}$$

So, $f(e_9) \neq 72$ but $f(e_9) = 86$. From this stage, the edge labels are to be alloted vertex by vertex.

$$\begin{aligned} f(e_8 + i) &= 2q + 7 + i \ (i = 1, 2) \ e_9, e_{10} \text{ labeled} \\ f(e_{10} + i) &= 2q - 11 + i \ (i = 1, 2) \ e_{11}, e_{12} \text{ labeled} \\ f(e_{12} + i) &= 2q + 3 + i \ (i = 1, 2) \ e_{13}, e_{14} \text{ labeled} \\ f(e_{15} + i) &= 2q - 14 + 2i \ (i = 0, 1) \ e_{15}, e_{16} \text{ labeled} \\ f(e_{17} + 2i) &= 3q - 10 - i \ (i = 0, 1) \ e_{17}, e_{18} \text{ labeled} \\ f(e_{18} + 2i) &= q - 7 + 2i \ (i = 0, 1) \ e_{19}, e_{20} \text{ labeled} \end{aligned}$$

The rest of the edges e_{21} to e_{26} cannot be combined even two by two

$$\begin{aligned} f(e_{21}) &= 132 = 3q + 15 \\ f(e_{22}) &= 25 = q - 14 \\ f(e_{23}) &= 105 = 3q - 12 \\ f(e_{24}) &= 48 = q + 9 \\ f(e_{25}) &= 90 = 2q + 12 \\ f(e_{26}) &= 59 = 2q - 19 \end{aligned}$$

The edges are labelled 27 to 34

$$\begin{aligned} f(e_{27}) &= 61 = 2q - 17 \\ f(e_{35}) &= 27 = q - 11 \\ f(e_{36}) &= 63 = 2q - 15 \\ f(e_{37}) &= 35 = q - 3 \\ f(e_{38}) &= 70 = 2q - 8 \\ f(e_{39}) &= 62 = 2q - 16 \\ f(e_{28} + i) &= q + 2 + 2i \ (i = 0, 1, \ldots, 6) \\ f(e_{35}) &= 27 = q - 11 \\ f(e_{36}) &= 63 = 2q - 15 \\ f(e_{37}) &= 35 = q - 3 \\ f(e_{38}) &= 70 = 2q - 8 \\ f(e_{39}) &= 62 = 2q - 16 \end{aligned}$$

Here the vertex sum is checked just for two vertices

1. $s(v_4) = f(e_3) + f(e_4) + f(e_{30}) = 78 + 77 + 45 = 200$ and
2. $s(v_{23}) = 25 + 105 + 70 = 200$

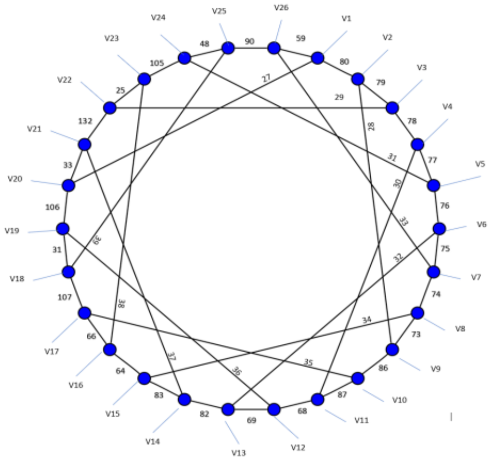

Figure 2. F26A Graph with Magic labeling

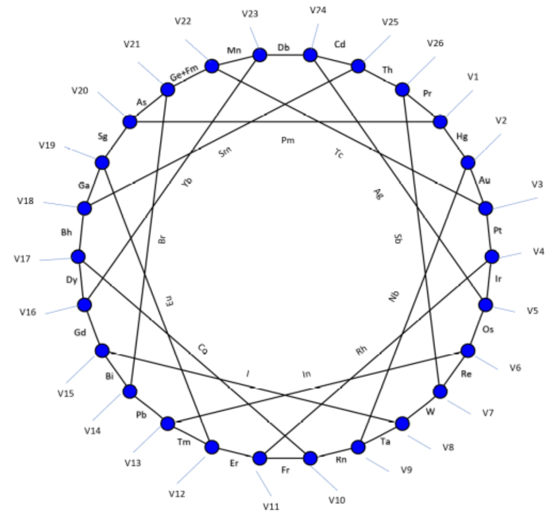

Figure 3. F26A Graph with symbols

So the F26A graph with respect to the suggested Magic labeling is found to satisfy the required conditions.

Hence it is a Magic graph.

4. Description of Mathematical Model

Description of Mathematical Model is given below:

A mathematic model using the magic labeling on F26A graph. The great business magnet of South India BMB wants to distribute a very huge sum to 25 people. He contacts the Veeru-Kanchan (VKA) and describes the mode of distribution with certain condition,allowing VKA to have a certain degree of freedom.

1) To divide the huge sum into 200 parts and to apportion it to three persons (named say P_1, P_2, P_3) in the ratio 80:61:59

The first two have to reapportion to two different persons while the third need not reapportion to anyone. He enjoys the apportionment from BMB and reapportionment through two persons (of course it is again the fortune from BMB)

The person who receives a portion as reapportionment needs to divide it into 200 parts, keeping with him the same number of parts (which will be specified to him, not the same amount) and reapportion it to different persons. A total of 14 persons have to reapportion to two persons, 10 of them to one person (not the same person) and one to none .

The VKA works at the problem and recognizes it in the form of F26A graph and the magic labeling and submits the solution to BMB.

The above diagram is the solution proposed by VKA (It is not unique; it is one among many, according to freedom given to VKA, they find the solution). The great BMB,scrutinizes and accepts and decides the persons to receive fixing people for the vertices starting V1with VKA .

According to the request from BMB, the VKA makes the solution secretive by alloting the names of the elements instead of numbers which are atomic numbers of such elements. The diagram is given below.

5. Conclusion

The Authors have made an effort and have come out with a Mathematical model for Apportionment ,using a graph and a graph labeling.They believe that this opens an avenue for such and many more problems with different graphs and different labeling.

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