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Graceful labeling in a graph consisting chord with quadrilateral snake

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Abstract

We studied graphs which are quadrilateral with one chord, barycentric subdivision of quadrilateral with one chord, double quadrilateral snake and alternate double quadrilateral snake. We proved that Graph obtained by joining quadrilateral with one chord and double quadrilateral snake, Graph obtained by joining quadrilateral with one chord and alternate double quadrilateral snake and Graph obtained by joining barycentric subdivision of quadrilateral with one chord and quadrilateral snake are graceful.

Keywords

Graceful labeling, Quadrilateral snake, Barycentric subdivision , Double quadrilateral snake , Alternate double quadrilateral snake.

AMS Subject Classification

05C78.

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Contents

1. Introduction

The concept of graceful labeling was introduced by Rosa [5] in 1967 and for numbering in graph was defined by S.W.Golomb [2]. Many researchers have studied gracefulness of graphs, refer Gallian survey [1]. A good number of papers are found with variety of applications in coding theory, radar communication, cryptography etc. A depth details about applications of graph labeling is found in Bloom and Golomb [2]. We accept all notations and terminology from Harary [3]. We recall some definitions which are use in this paper.

A function f is called graceful labeling of a graph $G=(V,E)$ if $f: V \rightarrow \{0,1,...,q\}$ is injective and the induce function $f^*: E \rightarrow \{1,...,q\}$ defined as $f^*(e)=|f(u)-f(v)|$ is bijective for every edge $e=(u,v) \in E(G)$. A graph G is called graceful graph if it admits a graceful labeling.

Cycle is a closed trail in which the "first vertex = last vertex"

The quadrilateral snake Q_n is obtained from the path P_n by replacing every edge of a path by cycle C_4.

A chord of a quadrilateral is an edge joining two non-adjacent vertices of quadrilateral.

The double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path

An alternate double quadrilateral snake ADQ_n is consist of two alternate double quadrilateral snakes that have common path.

Let G = (V, E) be a graph. Let e = uv be an edge of G, and w is not a vertex of G. The edge e is subdivided when it is replaced by edges $e^{'}=uw$ and $e^{"}=wv$.

Let G = (V, E) be a graph if every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of graph G. In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of original graph. The barycentric subdivision of any graph G is denoted by S(G). It is easy to observe that $|VS(G)| = |V(G)| + |E(G)|$ and $|ES(G)| = 2|E(G)|$.

In this paper we introduced gracefulness of (i) Graph obtained by joining quadrilateral with one chord and double quadrilateral snake (ii) Graph obtained by joining quadrilateral with one chord and alternate double quadrilateral snake (iii) Graph obtained by joining barycentric subdivision of quadrilateral with one chord and quadrilateral snake. For detail survey of graph labeling, we refer Gallian [1].

2. Main Results:

2.1 Theorem

The graph obtained by joining quadrilateral with one chord and double quadrilateral snake is graceful.

***Proof*:**

Let G = (V, E) be the graph, obtained by joining two graphs. Quadrilateral with one chord G_1 and double quadrilateral snake G_2 by a path P_k of length k. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of G_1 and $\{w_1,w_2,...,w_k,w'_1,...,w'_k,x_1,x_2,...,x_k\}$ be vertices of G_2 and $\{v_1,v_2,...,v_k\}$ be the vertices of P_k with $v_1 = u_4$ and $v_k = x_1$ and for G_2 join x_i to x_{i+1} (alternatively) to four new vertices w_i to w_{i+1}, w'_i and w'_{i+1} by the edges x_iw_i, w_iw_{i+1}, $w_{i+1}x_{i+1}$, $x_{i+1}w'_{i+1}$, $w'_iw'_{i+1}$ and $x_iw'_i$. (i = 1,2,...j-1)
Here $|V(G)|$= 5j+5, $|E(G)|$= 7j+6

***Case-1*:** k is odd.
$f : v \to \{0,1,....q\}$ where q = 7j + (2l+4).
For vertices

$f(u_1)$ = 7j + (2l+4) $\quad$ $f(u_2) = 0$
$f(u_3)$ = 7j + (2l+2) $\quad$ $f(u_4) = 1$

$f(v_1) = 1$ $\quad$ $f(v_2)$ = 7j + (2l+1)
$f(v_3) = 2$ $\quad$ $f(v_4)$ = 7j + (2l)
.
$f(v_{2m-1})$ = m $\quad$ $f(v_{2m})$ = 7j + (2l+(2-m))
.
$f(v_{2i})$ = 7j + (i+2)

$f(w_{4l-3})$ = 7l - (6-m) $\quad$ $f(w'_{4l-3})$ = 7l - (4-m)
$f(w_{4l-2})$ = 7j + ((8+m)-7l) $\quad$ $f(w'_{4l-2})$ = 7j + ((6+m)-7l)
$f(w_{4l-1})$ = 7j + ((5+m)-7l) $\quad$ $f(w'_{4l-1})$ = 7j + ((3+m)-7l)
$f(w_{4l})$ = 7l - (2-m) $\quad$ $f(w'_{4l})$ = 7l + m

$f(x_{2l-1})$ = 7j + ((9+m)-7l) $\quad$ $f(x_{2l})$ = 7l - (3-m)

2.2 Illustration

Graceful labeling of the graph obtained by joining quadrilateral with one chord and double quadrilateral snake.

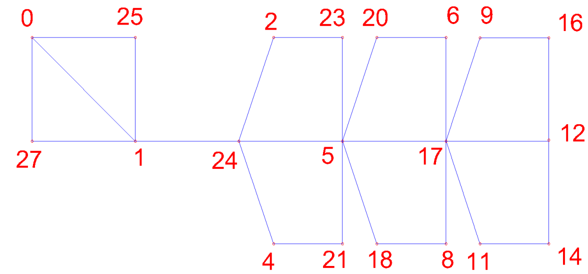

figure 1: The graph obtained joining by 3 copies of double quadrilateral snake and quadrilateral with one chord with p = 20 and q = 27 is graceful labeling.

***Case-2*:** k is even.
$f : v \to \{0,1,....q\}$ where q = 7j + (2l+5).
For vertices

$f(u_1)$ = 7j + (2l+5) $\quad$ $f(u_2) = 0$
$f(u_3)$ = 7j + (2l+3) $\quad$ $f(u_4) = 1$

$f(v_1) = 1$ $\quad$ $f(v_2)$ = 7j + (2l+2)
$f(v_3) = 2$ $\quad$ $f(v_4)$ = 7j + (2l+1)
.
$f(v_{2m-1})$ = m $\quad$ $f(v_{2m})$ = 7j + (2l+(3-m))
.
$f(v_{2i+1})$ = i + 1

$f(w_{4l-3})$ = 7j + ((9+m)-7l) $\quad$ $f(w'_{4l-3})$ = 7j + ((7+m)-7l)
$f(w_{4l-2})$ = 7l - (5-m) $\quad$ $f(w'_{4l-2})$ = 7l - (3-m)
$f(w_{4l-1})$ = 7l - (2-m) $\quad$ $f(w'_{4l-1})$ = 7l + m
$f(w_{4l})$ = 7j + ((5+m)-7l) $\quad$ $f(w'_{4l})$ = 7j + ((3+m)-7l)

$f(x_{2l-1})$ = 7l - (6-m) $\quad$ $f(x_{2l})$ = 7j + ((6+m)-7l)

$(\forall\ j = 1,2,....,\forall\ l = 1,2,...,\forall\ m = 1,2,...,\forall\ i = 1,2,....,)$.
(j = no. of snakes in double quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph, i = no. of vertices in a graph)

Here $f : V \to \{0,1,...,q\}$ is injective and the induce function $f^* : E \to \{1,...,q\}$ is bijective. So graph G is graceful.

2.3 Theorem

The graph obtained by joining quadrilateral with one chord and alternate double quadrilateral snake is graceful.

***Proof*:**

Let G = (V, E) be the graph, obtained by joining two graphs. Quadrilateral with one chord G_1 and alternate double quadrilateral snake G_2 by a path P_k of length k. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of G_1 and $\{w_1,w_2,...,w_k,w'_1,...,w'_k,x_1,x_2,...,x_k\}$ be vertices of G_2 and $\{v_1,v_2,...,v_k\}$ be the vertices of P_k with $v_1 = u_4$ and $v_k = x_1$ and for G_2 join x_i to x_{i+1} (alternatively) to four new vertices w_i to w_{i+1}, w'_i and w'_{i+1} by the edges x_iw_i, w_iw_{i+1}, $w_{i+1}x_{i+1}$, $x_{i+1}w'_{i+1}$, $w'_iw'_{i+1}$ and $x_iw'_i$. (i = 1,2,...j-1)
Here $|V(G)|$= 6j+4, $|E(G)|$= 8j+5

***Case-1*:** k is odd.
$f : v \to \{0, 1, ...q\}$ where q = 8j + (2l+3).
For vertices

$f(u_1)$ = 8j + (2l+3) $f(u_2) = 0$
$f(u_3)$ = 8j + (2l+1) $f(u_4) = 1$

$f(v_1) = 1$ $f(v_2)$ = 8j + (2l)
$f(v_3) = 2$ $f(v_4)$ = 8j + (2l-1)

$f(v_{2m-1})$ = m $f(v_{2m})$ = 8j + (2l+(1-m))

$f(v_{2i})$ = 8j + (i+1)

$f(w_{2l-1})$ = 4l - (3-m) $f(w'_{2l-1})$ = 4l - (1-m)
$f(w_{2l})$ = 8j + ((4+m)-4l) $f(w'_{2l})$ = 8j + ((2+m)-4l)

$f(x_{2l-1})$ = 8j + ((5+m)-4l) $f(x_{2l})$ = 4l + m

2.4 Illustration

Graceful labeling of the graph obtained by joining quadrilateral with one chord and alternate double quadrilateral snake.

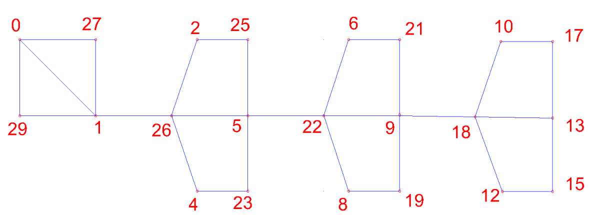

figure 2: The graph obtained joining by 3 copies of alternate double quadrilateral snake and quadrilateral with one chord with p = 22 and q = 29 is graceful labeling.

***Case-2*:** k is even.
$f : v \to \{0, 1, ...q\}$ where q = 8j + (2l+4).
For vertices

$f(u_1)$ = 8j + (2l+4) $f(u_2) = 0$
$f(u_3)$ = 8j + (2l+2) $f(u_4) = 1$

$f(v_1) = 1$ $f(v_2)$ = 8j + (2l+1)
$f(v_3) = 2$ $f(v_4)$ = 8j + (2l)

$f(v_{2m-1})$ = m $f(v_{2m})$ = 8j + (2l+(2-m))

$f(v_{2i+1})$ = i + 1

$f(w_{2l-1})$ = 8j + ((5+m)-4l) $f(w'_{2l-1})$ = 8j - (4l-(3+m))
$f(w_{2l})$ = 4l - (m-2) $f(w'_{2l})$ = 4l + m

$f(x_{2l-1})$ = 4l - (3-m) $f(x_{2l})$ = 8j + ((2+m)-4l)

$(\forall\ j = 1, 2, ..., \forall\ l = 1, 2, ..., \forall\ m = 1, 2, ...)$
(j = no. of snakes in alternate double quadrilateral, l = labeling in j^{th} graph, m = labeling in l^{th} graph)

Hence $f : V \to \{0, 1, ..., q\}$ is injective and the induce function $f^* : E \to \{1, ..., q\}$ is bijective. So graph G is graceful.

2.5 Theorem

The graph obtained by joining barycentric subdivision of quadrilateral with one chord and quadrilateral snake is graceful.

***Proof*:**
Let G = (V, E) be the graph, obtained by joining two graphs. barycentric subdivision of quadrilateral with one chord G_1 and quadrilateral snake G_2 by a path P_k of length k. Let $\{u_1, u_2, u_3, u_4\}$ be vertices of quadrilateral with one chord and $\{x_1, x_2, x_3, x_4, x_5\}$ are inserted vertices due to barycentric subdivision $\{u_1x_1u_2x_2u_3x_3u_4x_4, u_2x_5u_4\}$ are of G_2 and $\{w_1, w_2, ..., w_k\}$ be vertices of quadrilateral snake G_2 and $\{v_1, v_2, ..., v_k\}$ be the vertices of P_k with $v_1 = u_4$ and $v_k = w_1$ we consider the following two cases.

***Case-1*:** k is odd
$f : v \to \{0, 1, ...q\}$ where q = 4j + (9+2l).

For vertices

$f(u_1)$ = 4j + (9+2l) $f(u_2)$ = 4j + (8+2l)
$f(u_3)$ = 4j + (7+2l) $f(u_4)$ = 4j + (5+2l)

$f(v_1)$ = 4j + (5+2l) $f(v_2) = 6$
$f(v_3)$ = 4j + (4+2l) $f(v_4) = 7$

$f(v_{2m-1})$ = 4j + ((6-m)+2l) $f(v_{2m})$ = 5+m

$f(v_{2i})$ = 5+i

$f(w_{6l-5})$ = 4l+(m+1) $f(w_{6l-2})$ = 4j - (4l - (8+m))
$f(w_{6l-4})$ = 4j-(4l-(9+m)) $f(w_{6l-1})$ = 4l+(4+m)
$f(w_{6l-3})$ = 4l+(3+m) $f(w_{6l})$ = 4j - (4l - (6+m))

$f(x_1) = 0$ $f(x_2) = 1$ $f(x_5) = 5$
$f(x_3) = 3$ $f(x_4) = 4$

2.6 Illustration

Graceful labeling of the graph obtained by joining barycentric subdivision of quadrilateral with one chord and quadrilateral snake

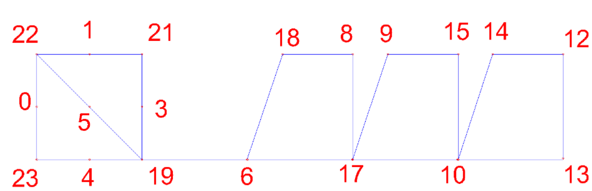

figure 3: The graph obtained joining by 3 copies of quadrilateral snake and barycentric subdivision of quadrilateral with one chord with p = 19 and q = 23 is graceful labeling.

***Case-2*:** k is even
$f : v \to \{0, 1, ..., q\}$ where q = 4j + (10+2l).

For vertices
$f(u_1)$ = 4j + (10+2l) $f(u_2)$ = 4j + (9+2l)
$f(u_3)$ = 4j + (8+2l) $f(u_4)$ = 4j + (6+2l)

$f(v_1)$ = 4j + (6+2l) $f(v_2)$ = 6
$f(v_3)$ = 4j + (5+2l) $f(v_4)$ = 7
.
$f(v_{2m-1})$ = 4j + ((7-m)+2l) $f(v_{2m})$ = 5+m
.
$f(v_{2i+1})$ = 4j + (6+i)

$f(w_{6l-5})$ = 4j + ((10+m) - 4l) $f(w_{6l-2})$ = 4l + (3+m)
$f(w_{6l-4})$ = 4l + (2+m) $f(w_{6l-1})$ = 4j + ((7+m) - 4l)
$f(w_{6l-3})$ = 4j + ((8+m) - 4l) $f(w_{6l})$ = 4l + (5+m)

$f(x_1) = 0$ $f(x_2) = 1$ $f(x_5) = 5$
$f(x_3) = 3$ $f(x_4) = 4$

Hence $f : V \to \{0, 1, ..., q\}$ is injective and the induce function $f^* : E \to \{1, ..., q\}$ is bijective. So graph G is graceful.

2.7 Concluding Remark

Present work contributes some new results. We discussed gracefulness of graph obtained by joining (barycentric subdivision of quadrilateral with one chord and quadrilateral snake, quadrilateral with one chord and double quadrilateral snake, quadrilateral with one chord and alternate double quadrilateral snake). The labeling pattern is demonstrated by means of illustrations which provide better understanding to derived results.

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