Malaya Journal of Matematik, Vol. 9, No. 1, 419-424, 2021

https://doi.org/10.26637/MJM0901/0071

P_4-Decomposition in Boolean function graph of $B_3(G)$

S. Muthammai[1] and S. Dhanalakshmi[2*]

Abstract

For any graph G, let $V(G)$ and $E(G)$ denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{Kp}, \overline{L(G)}, NINC)$ of G is a graph with vertex set $V(G) \cup E(G)$ and two vertices in $B(\overline{Kp}, \overline{L(G)}, NINC)$ are adjacent if and only if they correspond to two non adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_3(G)$. In this paper, P_4- decomposition in Boolean Function Graph $B(\overline{Kp}, \overline{L(G)}$, NINC) of some standard graphs and corona graphs are obtained.

Keywords

Boolean Function graph, Edge Domination Number, Decomposition.

[1] *Department of Mathematics, Principal [Retd], Alagappa Government Arts College, Karaikudi-630003, Tamil Nadu, India.*

[2] *Department of Mathematics, Government Arts College for Women (Autonomous) [Affiliated to Bharathidasan University, Tiruchirappalli-620024, Tamil Nadu], Pudukkottai-622001, Tamil Nadu, India.*

*****Corresponding author**: [2] dhanalakshmi8108@gmail.com

Article History: Received **01** December **2020**; Accepted **04** February **2021**

Contents

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. A graph with p vertices and q edges is denoted by $G(p, q)$. The corona G_1 o G_2 of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2, and then joining the ith vertex of G_1 to every vertex of in the ith copy of G_2. For any graph G, GoK_1 is denoted by G^+.

A decomposition of a graph G is a family of edge-disjoint subgraphs $\{G_1, G_2, \ldots, G_k\}$ such that $E(G) = E(G_1) \cup E(G_2) \cup \cup \ldots \cup E(G_k)$. If each G_i is isomorphic to H, for some subgraph H of G, then the decomposition is called a H-decomposition of G. In particular, a P_4-decomposition of a graph G is a partition of the edge set of G into paths of length 3. In this case, G is said to be P_4-decomposable. Several authors studied various types of decomposition by imposing conditions on Gi in the decomposition. Heinrich, Liu and Yu[3] proved that a connected 4-regular graph admits a P_4-decomposition if and only if $|E(G)| \equiv 0(\bmod 3)$. Sunil Kumar [10] proved that a complete r- partite graph is P_4-decomposable if and only if its size is a multiple of 3. P. Chithra Devi and J. Paulraj Joseph [1] gave a necessary and sufficient condition for the decomposition of the total graph of standard graphs and corona of graphs into paths on three edges. Janakiraman et al., introduced the concept of Boolean function graphs $[4-6]$. For any graph G, let $V(G)$ and $E(G)$ denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{Kp}, \overline{L(G)}, NINC)$ of G is a graph with vertex set $V(G) \cup E(G)$ and two vertices in $B(\overline{Kp}, \overline{L(G)}, NINC)$ are adjacent if and only if they correspond to two non adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_3(G)$.

In this paper, P_4-decomposition in Boolean Function Graph $B(\overline{Kp}, \overline{L(G)}, NINC)$ of some standard graphs are obtained.

2. Prior Results

Observation 2.1 ([6]). *Let G be a graph with p vertices and q edges.*

1. *$\overline{L(G)}$ is an induced subgraph of $B_3(G)$ and the subgraph of $B_3(G)$ induced by vertices of G in $B_3(G)$ is totally disconnected.*

2. *If $d_i = \deg_G(v_i), v_i \in V(G)$, then the number of edges in $B_3(G)$ is $(q/2)(2p+q-3) - 1/2 \sum_{1 \leq i \leq p} d_i^2$*

3. *The degree of a vertex of v in $B_3(G)$ is $q - \deg_G(v)$ and the degree of a vertex e' of $L(G)$ in $B_3(G)$ is $\deg_{L(G)}(e') + p - 2$*

4. *Both G and $B_3(G)$ are regular if and only if either G is totally connected or G is complete.*

3. Main Results

In the following, P_4- decomposition of $B_3(P_n), B_3(C_n), B_3(K_{1,n})$ and corona graphs are found.

Theorem 3.1. *Let $n \geq 3$*

1. *If n is odd, then $B_3(P_n) - ((n+1)/2)K_2$ is P_4- decomposable.*

2. *If n is even, then $B_3(P_n) - ((n-2)/2)K_2$ is P_4- decomposable.*

Proof. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = (v_i, v_{i+1}), i = 1, 2, \ldots, n-1$ be the edges of P_n. Then

$$v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1} \in V(B_3(P_n)).B_3(P_n)$$

has $(2n-1)$ vertices and $\left(\left(3n^2 - 11n + 10\right)/2\right)$ edges. It is to be noted that, in all the sets the suffixiin y_i is integer modulo n and j in ej is integer modulo $n-1, v_0 = v_n$ and $e_0 = e_n - 1$.
Case 1. n is odd, $n \geq 7$.
Then the edge set of $B_3(P_n)$ can be decomposed into $((n^2 - 4n + 3)/2)P_4$ and $((n+1)/2)K_24$ The edge set of $((n+1)/2)K_2$ is given by the set

$$\left\{U_{i=1}^{(n-3)/2}\{(v_n, e_i)\} \cup \{(e_1, e_{n-1}), (v_n - 1, e_1)\}.\right\}$$

The edge set of $\left(\left(n^2 - 4n + 3\right)/2\right) P_4$ is given by the edge set $A^{(1)}, i = 1, 2, \ldots, 4$, where,

$$A^{(1)} = U_{j=1}^{(n-5)/2}\left\{U_{i=1}^{n-1} A_{ji}^{(1)}\right\},$$
$$A_{ji}^{(1)} = \left\{\left(v_i, e_{i+j+1}\right), \left(e_{i+j+1}, e_i\right), \left(e_j, v_{i+j+1}\right)\right\}$$
$$A^{(2)} = \bigcup_{i=1}^{(n-1)/2} A_i^{(2)},$$
$$A_i^{(2)} = \left\{\left(v_i, e_{i+(n-1)/2}\right), \left(e_{i+(n-1)/2}, e_i\right), \left(e_i, v_{i+(n-1)/2}\right)\right\}$$
$$A^{(3)} = U_{i=1}^{(n-3)/2} A_i^{(3)},$$
$$A_i^{(3)} = \left\{\left(e_{i+1}, v_i\right), \left(v_i, e_i + (n+1)/2\right), \left(e_{i+(n+1)/2}, v_{i+(n-1)/2}\right)\right\}$$
$$A^{(4)} = \left\{\left(v_{(n-1)2}, e_{(n+1)2}\right), \left(e_{(n+1)/2}, v_n\right), \left(v_n, e_{(n-1)/2}\right)\right\}$$

Here,

$$\langle A^{(1)}\rangle \cong ((n-1)(n-5))/2P_4, \langle A^{(2)}\rangle \cong ((n-1)/2)P_4,$$
$$\langle A^{(3)}\rangle \cong ((n-3)/2)P_4, \ \langle A^{(4)}\rangle \cong P_4$$

Hence, $B_3(P_n) - ((n+1)/2)K_2$ is P_4- decomposable.
Case 2. n is even, $n \geq 6$
Then the edge set of $B_3(P_n)$ can be decomposed into $((n^2 - 4n + 4)/2)P_4$ and $((n-2)/2)K_2$. The edge set of $((n-2)/2)K_2$ is given by the set $\left\{U_{i=1}^{(n-4)/2}\{(v_n, e_{i+2})\} \cup \{(e_1, e_n - 1)\}\right.$. The edge set of $\left(\left(n^2 - 4n + 4\right)/2\right) P_4$ is given by the edge sets $A^{(5)}, A^{(6)}$ and $A^{(7)}$ where

$$A^{(5)} = U_{j=1}^{(n-4)/2}\left\{U_{i=1}^{n-1} A_{ji}^{(5)}\right\},$$
$$A_j(5) = \left\{\left(v_i, e_{i+j+1}\right), \left(e_{i+j+1}, e_i\right), \left(e_i, v_{i+j+1}\right)\right\}$$
$$A^{(6)} = U_{i=1}^{(n-2)/2} A_i^{(6)},$$
$$A_i^{(6)} = \left\{\left(e_{i+1}, v_i\right), \left(v_i, e_i + (n/2)\right), \left(e_{i+(n/2)}, v_{i+(n-2)2}\right)\right\}$$
$$A^{(7)} = \left\{\left(e_2, v_n\right), \left(v_n, e_1\right), \left(e_1, v_{n-1}\right)\right\}$$

Here, $\langle A^{(5)}\rangle \cong (((n-1)(n-4))/2)P_4, \langle A^{(6)}\rangle \cong ((n-2)/2)P_4$, $\langle A^{(3)}\rangle \cong P_4$. Hence, $B_3(P_n) - ((n-2)/2)K_2$ is P_4- decomposable. ☐

Theorem 3.2. *For $n \geq 6$, the graph $B_3(C_n) - nK_2$ is P_4- decomposable.*

Proof. Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = (v_i, v_{i+1}), i = 1, 2, \ldots, n-1, e_n = (v_n, v_1)$ be the edges of C_n. Then

$$v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n \in V(B_3(C_n)) \cdot B_3(C_n)$$

has $2n$ vertices and $\frac{1}{2}\left(3n^2 - 7n\right)$ edges. It is to be noted that, in all the sets the suffices in y_i and j in ej are integers modulo $n, v_0 = v_n$ and $e_0 = e_n$.
Case 1. n is odd, $n \geq 7$.
Then the edge set of $B_3(C_n)$ can be decomposed into

$$\frac{1}{2}\left(n^2 - 3n\right) P_4$$

and nK_2. The edge set of nK_2 is given by the set

$$U_{i=1}^{n}\{(v_i, e_{i+n-2})\}.$$

The edge set of $\left(\left(n^2 - 3n\right)/2\right) P_4$ is given by the edge set $B^{(1)}$, where

$$B^{(1)} = \bigcup_{j=1}^{(n-3)/2}\left\{U_{i=1}^{n} B_{ji}^{(1)}\right\},$$
$$B_{ij}^{(1)} = \left\{\left(e_{i+2j-1}, v_i\right)\left(v_i, e_{i+2j}\right)\left(e_{i+2j}, e_{i+j-1}\right)\right\}$$

Here, $\langle B^{(1)}\rangle \cong (n(n-3)/2)P_4$. Hence, $B_3(C_n) - nK_2$ is P_4- decomposable.
Case 2. n is even, $n \geq 6$
Then the edge set of $B_3(C_n)$ can be decomposed into

$$\left(\left(n^2 - 3n\right)/2\right) P_4$$

and nK_2. The edge set of nK_2 is given by the set

$$\bigcup_{i=1}^{n}\{(v_i, e_{i+n-2})\}.$$

The edge set of $\left(\left(n^2-3n\right)/2\right)P_4$ is given by the edge sets $B^{(2)}$ and $B^{(3)}$, where

$$B^{(2)}=\bigcup_{j=1}^{(n-4)/2}\left\{U_{i=1}^{n}B_{ji}^{(2)}\right\},$$
$$B_{ji}^{(2)}=\left\{\left(e_{i+2j-1},v_i\right),\left(v_i,e_{i+2j}\right),\left(e_{i+2j},e_{i+j-1}\right)\right\}$$
$$B^{(3)}=\bigcup_{i=1}^{n/2}B_i^{(3)},$$
$$B_i^{(3)}=\left\{\left(v_i,e_{i+n-3}\right),\left(e_{i+n-3},e_{i+(n-6)2}\right),\left(e_{i+(n-6)2},v_{(n+2)/2+i-1}\right)\right\}$$

Here, $\langle B^{(2)}\rangle\cong(n(n-4)/2)P_4$ and $\langle B^{(3)}\rangle\cong(n/2)P_4$. Therefore, $B_3(C_n)-nK_2$ is P_4- decomposable. □

Theorem 3.3. *Let $n\geq 4$.*

1. *If $n\equiv 0(mod 3)$, then $B_3(K_{1,n})-nP_3$ is P_4-decomposable.*
2. *If $n\equiv 1(\bmod 3)$, then $B_3(K_{1,n})$ is P_4- decomposable.*
3. *If $n\equiv 2(\bmod 3)$, then $B_3(K_{1,n})-nK_2$ is P_4- decomposable.*

Proof. Let $V(K_{1,n})=\{v,v_1,v_2,\ldots,v_n\}$, where v is the central vertex and $e_i=(v,v_i), i=1,2,\ldots,n$ be the edges of $K_{1,n}$. Then $v,v_1,v_2,\ldots,v_n,e_1,e_2,\ldots,e_n\in V(B_3(K_{1n}))\cdot B_3(K_{1,n})$ has $(2n+1)$ vertices and $\left(n^2-n\right)$ edges.

In all the sets defined below, the suffices are integers modulo n and $v_0=v_n, e_0=e_n$.

Case 1. $n\equiv 0(\bmod 3), n\geq 3$.

Then the edge set of $B_3(K_{1,n})$ can be decomposed into

$$\left(\left(n^2-3n\right)/3\right)P_4$$

and nP_3. The edge set of nP_3 is given by the set

$$\bigcup_{i=1}^{n}\left\{\left(e_{i+1},v_i\right),\left(v_i,e_{i+2}\right)\right\}.$$

The edge set of $\left(\left(n^2-3n\right)/3\right)P_4$ is given by the edge set

$$C^{(1)}=U_{j=1}^{(n-3)/3}\left\{U_{i=1}^{n}C_{ji}^{(1)}\right\},$$
$$C_{ji}^{(1)}=\left\{\left(e_{i+2j+1},v_i\right),\left(v_i,e_{i+2j+2}\right),\left(e_{i+2j+2,}v_{i+3j+2}\right)\right\}$$

Here, $\langle C^{(1)}\rangle\cong((n(n-3)/3)P_4$. Therefore, $B_3(K_{1,n})-nP_3$ is P_4- decomposable.

Case 2. $n\equiv 1(\bmod 3), n\geq 4$

Then the edge set of $B_3(K_{1,n})$ can be decomposed into

$$\left(\left(n^2-n\right)/3\right)P_4.$$

The edge set of $\left(\left(n^2-n\right)/3\right)P_4$ is given by the edge set $C^{(2)}$, where

$$C^{(2)}=\bigcup_{j=1}^{(n-1)/3}\left\{U_{i=1}^{n}C_{ji}^{(2)}\right\},$$
$$C_{ji}^{(2)}=\left\{\left(e_{i+2j-1},v_i\right),\left(v_i,e_{i+2j}\right),\left(e_{i+2j},v_{i+3j}\right)\right\}$$

Here, $\langle C^{(2)}\rangle\cong((n(n-1)/3)P_4)$ Therefore, $B_3(K_{1,n})$ is P_4- decomposable.

Case 3. $n\equiv 2(\bmod 3), n\geq 5$.

Then the edge set of $B_3(K_{1,n})$ can be decomposed into

$$\left(\left(n^2-2n\right)/3\right)P_4$$

and nK_2. The edge set of nK_2 is given by the set

$$U_{i=1}^{n}\left\{\left(v_i,e_{i+1}\right)\right\}.$$

The edge set $\left(\left(n^2-2n\right)/3\right)P_4$ is given by the edge set $C^{(3)}$, where

$$C^{(3)}=U_{j=1}^{(n-2)/3}\left\{U_{i=1}^{n}C_{ji}^{(3)}\right\},$$
$$C_{ji}^{(3)}=\left\{\left(e_{i+2j},v_i\right),\left(v_i,e_{i+2j+1}\right),\left(e_{i+2j+1},v_{i+3j+1}\right)\right\}$$

Here, $\langle C^{(3)}\rangle\cong(n(n-2)/3)P_4$ Therefore, $B_3(K_{1,n})-nK_2$ is P_4- decomposable. □

Theorem 3.4. *Let $n\geq 6$.*

1. *If n is even, then $B_3(P_n^+)-((3n-4)/2)K_2$ is P_4- decomposable.*
2. *If n is odd, then $B_3(P_n^+)-((3n-1)/2)K_2$ is P_4- decomposable.*

Proof. Let $V(P_n^+)=\{v_1,v_2,\ldots v_n,u_1,u_2,\ldots,u_n\}$ where $v_1,v_2,\ldots v_n$ are the vertices of P_n and $u_1,u_2,\ldots,u_n$ are the pendant vertices of P_n+ and $e_i=(v_i,v_{i+1})\, i=1,2,\ldots,n-1$ and $f_i=(v_i,u_i), i=1,2,\ldots,n$ be the edges of P_n+. Then $v_1,v_2,\ldots v_n,u_1,u_2,\ldots u_n,e_1,e_2,\ldots,e_{n-1},f_1,f_2\ \ldots,f_n\in V(B_3(P_n^+).B_3(P_n^+)$ has $4n-1$ vertices and $\left(6n^2-12n+7\right)$ edges.

In all the sets, suffix j in ej is integer modulo $n-1$ and the suffix k in f_k,u_k,v_k is integer modulo $n, u_0=u_n, f_0=f_n, v_0=v_n$ and $e_0=e_{n-1}$.

Case 1. n is even, $n\geq 6$.

Then the edge set of $B_3({P_n}^+)$ can be decomposed into

$$\left(\left(4n^2-9n+6\right)/2\right)P_4$$

and $((3n-4)/2)K_2$. The edge set of $((3n-4)/2)K_2$ is given by the set

$$\left(U_{i=1}^{n-1}\left\{\left(u_n,f_i\right)\right\}\right)\cup\left(U_{i=1}^{\frac{n-4}{2}}\left\{\left(v_{i+3}e_{i+1}\right)\right\}\right)\cup\left\{\left(v_2,e_{n-1}\right)\right\}$$

The edge set of $\left(\left(4n^2-9n+6\right)/2\right)P_4$ is given by the edge sets $M^{(1)}, i=1,2,\ldots,7$, where

$$M^{(1)} = U_{j=1}^{(n-4)/2}\left\{U_{i=1}^{n-1} M_{ji}^{(1)}\right\},$$
$$M_{ji}^{(1)} = \left\{\left(e_{i+2j-1}, v_i\right), \left(v_i, e_{i+2j}\right), \left(e_{i+2j}, e_{i+j-1}\right)\right\}$$
$$M^{(2)} = U_{j=1}^{(n-2)/2}\left\{U_{i=1}^{n} M_{ji}^{(2)}\right\},$$
$$M_{ji}^{(2)} = \left\{\left(v_i, f_{i+j}\right), \left(f_{itj}, f_i\right), \left(f_i, v_{i+j}\right)\right\}$$
$$M^{(3)} = \bigcup_{i=1}^{n/2} M_i^{(3)},$$
$$M_i^{(3)} = \left\{\left(v_i, f_{i+n/2}\right), \left(f_{i+n/2}, f_i\right), \left(f_i, v_{i+n/2}\right)\right\}$$
$$M^{(4)} = \bigcup_{j=1}^{(n-2)}\left\{\bigcup_{i=1}^{n-1} M_{ji}^{(4)}\right\},$$
$$M_{ji}^{(4)} = \left\{\left(e_i, f_{i+j+1}\right), \left(f_{i+j+1}, u_i\right), \left(u_i, e_{i+j}\right)\right\}$$
$$M^{(5)} = U_{i=1}^{n-1} M_i^{(5)}, M_i^{(5)} = \left\{\left(u_n, e_i\right), \left(e_i, u_i\right), \left(u_i, f_{i+1}\right)\right\}$$
$$M^{(6)} = U_{i=1}^{n-1} M_i^{(6)},$$
$$M_i^{(6)} = \left\{\left(e_i, v_n\right), \left(v_n, e_{i+(n-2)/2}\right), \left(e_{i+(n-2)/2}, v_{i+(n+2)/2}\right)\right\}$$
$$M^{(7)} = \left\{\left(v_1, e_{n-1}\right), \left(e_{n-1}, e_1\right), \left(e_1, v_3\right)\right\}$$

Here, $\langle M^{(1)}\rangle \cong (((n-1)(n-4))/3)P_4, \langle M^{(2)}\rangle \cong (n(n-2)/2)$ P_4, $\langle M^{(3)}\rangle \cong (n/2)P_4$ and $\langle M^{(4)}\rangle \cong ((n-2)(n-1))P_4$, $\langle M^{(5)}\rangle \cong (n-1)P_4, \langle M^{(6)}\rangle \cong (n-1)P_4, \langle M^{(7)}\rangle \cong P_4$. Hence, $B_3\left(Pn^{+}\right) - ((3n-4)/2)K_2$ is P_4- decomposable.

Case 2. n is odd, $n \geq 9$.
Then the edge set of $B_3\left(Pn^{+}\right)$ can be decomposed into

$$\left(\left(4n^2-9n+5\right)/2\right) P_4$$

and $((3n-1)/2)K_2$ The edge set of $((3n-1)/2)K_2$ is given by the set

$$\left(U_{i=1}^{n-1}\left\{\left(u_n, f_i\right)\right\}\right) \cup \left(U_{i=1}^{\frac{n-5}{2}}\left\{\left(v_{i+3}, e_{i+1}\right)\right\}\right) \cup \left\{\left(v_n, e_{n-2}\right),\right.$$
$$\left.\left(v_1, e_{n-1}\right), \left(v_2, e_{n-1}\right)\right\}$$

The edge set of $\left(\left(4n^2-9n+5\right)/2\right) P_4$ is given by the edge sets $M^{(4)}, M^{(5)}, M^{(7)}$ as in Case land the sets $M^{(8)}, M^{(9)}, M^{(10)}$ and $M^{(11)}$, where,

$$M^{(8)} = U_{j=1}^{(n-5)/2}\left\{U_{i=1}^{n-1} M_{ji}^{(8)}\right\},$$
$$M_{ji}^{(8)} = \left\{\left(e_{i+2j-1}, v_i\right), \left(v_i, e_{i+2j}\right), \left(e_{i+2j}, e_{i+j-1}\right)\right\}$$
$$M^{(9)} = U_{j=1}^{(n-1)/2}\left\{U_{i=1}^{n} M_{ji}^{(9)}\right\},$$
$$M_{ji}^{(9)} = \left\{\left(v_i, f_{i+j}\right), \left(f_{i+j}, f_i\right), \left(f_i, v_{i+j}\right)\right\}$$
$$M^{(10)} = U_{i=1}^{(n-3)/2} M_i^{(10)},$$
$$M_i^{(10)} = \left\{\left(e_i, v_n\right), \left(v_n, e_{i+(n-1)2}\right), \left(e_{i+(n-1)2}, v_{i+(n+3)2}\right)\right\}$$
$$M^{(11)} = U_{i=1}^{(n-1)/2} M_i^{(11)},$$
$$M_i^{(11)} = \left\{\left(v_i, e_{i+(n-4)}\right), \left(e_{i+(n-4)}, e_{i+(n-7)2}\right),\right.$$
$$\left.\left(e_{i+(n-7)/2}, v_{i+(n-1)/2}\right)\right\}$$

Here, $\langle M^{(8)}\rangle \cong ((n-1)(n-5))/2P_4, \langle M^{(9)}\rangle \cong (n(n-1)/2)P_4$, $\langle M^{(10)}\rangle \cong ((n-3)/2)P_4$ and $\langle M^{(11)}\rangle \cong ((n-1)/2)P_4$ Hence, $B_3\left(P_n^{+}\right) - ((3n-1)/2)K_2$ is P_4-decomposable. □

Theorem 3.5. *For $n \geq 6$, the graph $B_3\left(C_n^{+}\right) - nK_2$ is P_4- decomposable.*

Proof. Let $V\left(C_n^{+}\right) = \{v_1, v_2, \ldots v_n, u_1, u_2, \ldots, u_n\}$, where $v_1, v_2, \ldots, v_n$ are the vertices of C_n and $u_1, u_2, \ldots, u_n$ are the pendant vertices of Cn^{+} and $e_i = \left(v_i, v_{i+1}\right) i = 1,2,\ldots,n-1, \quad e_n = \left(v_n, v_1\right)$ and $f_i = \left(v_i, u_i\right), i = 1,2,\ldots,n$ be the edges of C_n+. Then $v_1, v_2, \ldots v_n, u_1, u_2, \ldots u_n, e_1, e_2, \ldots, e_n, f_1, f_2, \ldots f_n \in V$ $\left(B_3\left(C_n^{+}\right) \cdot B_3\left(C_n^{+}\right)\right)$ has $4n$ vertices and $\left(6n^2-8n\right)$ edges.

It is to be noted that in all the sets, all the suffices are integers modulo $n, f_0 = f_n, v_0 = v_n$ and $e_0 = e_n, u_0 = u_n$.

Case 1. n is even, $n \geq 6$
Then the edge set of $B_3\left(Cn^{+}\right)$ can be decomposed into

$$\left(2n^2-3n\right) P_4$$

and nK_2. The edge set of nK_2 is given by the set $U_{i=1}^{n}\left\{\left(u_i, f_{i+1}\right)\right\}$. The edge set of $\left(2n^2-3n\right) P_4$ is given by the edge sets $N^{(i)}$ $i = 1,2,\ldots,6$, where

$$N^{(1)} = \bigcup_{j=1}^{(n-4)/2}\left\{U_{i=1}^{n} N_{ji}^{(1)}\right\},$$
$$N_{ji}^{(1)} = \left\{\left(e_{i+2j-1,} v_i\right), \left(v_i, e_{i+2j}\right), \left(e_{i+2j}, e_{i+j-1}\right)\right\}$$
$$N^{(2)} = \bigcup_{i=1}^{n/2} N_i^{(2)},$$
$$N_i^{(2)} = \left\{\left(v_i, e_{i+(n-3)}\right), \left(e_{i+(n-3)}, e_i + (n-6)/2\right),\right.$$
$$\left.\left(e_{i+(n-6)/2}, V_{i+n/2}\right)\right\}$$
$$N^{(3)} = \bigcup_{j=1}^{(n-2)/2}\left\{U_{i=1}^{n} N_{ji}^{(3)}\right\},$$
$$N_{ji}^{(3)} = \left\{\left(v_i, f_{i+j}\right), \left(f_{i+j}, f_i\right), \left(f_i, v_{i+j}\right)\right\}$$
$$N^{(4)} = \bigcup_{i=1}^{n/2} N_i^{(4)},$$
$$N_i^{(4)} = \left\{\left(v_i, f_{i+n/2}\right), \left(f_{i+n/2}, f_i\right), \left(f_i, v_{i+n/2}\right)\right\}$$
$$N^{(5)} = \bigcup_{j=1}^{(n-2)}\left\{U_{i=1}^{n} N_{ji}^{(5)}\right\},$$
$$N_{ji}^{(5)} = \left\{\left(e_i, f_{i+j+1}\right), \left(f_{i+j+1}, u_i\right), \left(u_i, e_{i+j-1}\right)\right\}$$
$$N^{(6)} = \bigcup_{i=1}^{n} N_i^{(6)},$$
$$N_i^{(6)} = \left\{\left(e_{i+(n-2)}, u_i\right), \left(u_i, e_{i+(n-1)}\right), \left(e_{i+(n-1)}, v_{i+1}\right)\right\}$$

Here, $\langle N^{(1)}\rangle \cong (n(n-4)/2)P_4, \langle N^{(2)}\rangle \cong (n/2)P_4$, $\langle N^{(3)}\rangle \cong (n(n-2)/2)P_4$ and $\langle N^{(4)}\rangle \cong (n/2)P_4, \langle N^{(5)}\rangle \cong (n(n-2))P_4$, $\langle N^{(6)}\rangle \cong nP_4$. Hence, $B_3\left(C_n^{+}\right) - nK_2$ is P_4-decomposable.

Case 2. n is odd, $n \geq 5$.
Then the edge set of $B_3\left(C_n{}^+\right)$ can be decomposed into

$$\left(2n^2-3n\right)P_4$$

and nK_2. The edge set of nK_2 is given by the set

$$U_{i=1}^{n}\left\{(u_i, f_{i+1})\right\}.$$

The edge set of $\left(2n^2-3n\right)P_4$ is given by the edge sets $N^{(5)}$, $N^{(6)}$ as in Case 1 and the sets $N^{(7)}$ and $N^{(8)}$, where

$$N^{(7)} = \bigcup_{j=1}^{(n-3)/2}\left\{\bigcup_{i=1}^{n} N_{ji}^{(7)}\right\},$$
$$N_{ji}^{(7)} = \left\{\left(e_{i+2j-1,}v_i\right), \left(v_i, e_i+2j\right), \left(e_{i+2j}, e_{i+j-1}\right)\right\}$$
$$N^{(8)} = \bigcup_{j=1}^{(n-1)/2}\left\{U_{i=1}^{n} N_{ji}^{(8)}\right\},$$
$$N_{ji}^{(8)} = \left\{\left(v_i, f_{itj}\right), \left(f_{i+j}, f_i\right), \left(f_i, v_{i+j}\right)\right\}$$

Here, $\langle N^{(7)}\rangle \cong ((n(n-3)/2)P_4$ and $\langle N^{(8)}\rangle \cong ((n(n-1)/2)P_4$. Hence, $B_3\left(C_n^+\right) - nK_2$ is P_4-decomposable. □

Theorem 3.6. *For $n \geq 3$ and $n \equiv 0(\bmod 3)$, the graph*

$$B_3\left(K_{1,n}^+\right) - nK_2$$

is P_4- decomposable.

Proof. Let

$$V\left(K_{1,n}^+\right) = \left\{v, v_1, v_2, \ldots v_n, u, u_1, u_2, \ldots, u_n\right\},$$

where v is the central vertex and $\langle\{v_1, v_2, \ldots v_n\}\rangle \cong K_{1,n}$ and $u, u_1, u_2, \ldots, u_n$ are the pendant vertices of $K_{1,n}+$ and $e_i = (v, v_i)$, $\quad i = 1, 2, \ldots, n$ and $f = (v, u), f_i = (v_i, u_i), i = 1, 2, \ldots, n$ be the edges of $K_{1,n}{}^+$ Then $v, v_1, v_2, \ldots v_n, u, u_1, u_2, \ldots u_n, e_1, e_2, \ldots, e_n, f, f_1, f_2, \ldots, f_n \in V\left(B_3\left(K_{1,n}^+\right)\right.$ $B_3\left(K_1, n^+\right)$ has $(4n+1)$ vertices and $\left(\left(11n^2-n\right)/2\right)$ edges. In all the sets, suffix in vi, u e_i and f_i is integers modulo n, $f_0 = f_n, v_0 = v_n$ and $e_0 = e_n$, $u_0 = u_n$.
Case 1. n is even, $n \geq 6$.
Then the edge set of $B_3\left(K_1, n^+\right)$ can be decomposed into $\left(\left(11n^2-3n\right)/6\right)P_4$ and nK_2. The edge set of nK_2 is given by the set $U_{i=1}^{n}\left\{(v, f_i)\right\}$. The edge set of $\left(\left(11n^2-3n\right)/6\right)P_4$ is given by the edge sets $Q^{(1)}, \ldots, Q^{(6)}$, where

$$Q^{(1)} = U_{j=1}^{(n-3)/3}\left\{U_{i=1}^{n} Q_{ji}^{(1)}\right\},$$
$$Q_j i^{(1)} = \left\{\left(e_{i+j}, v_i\right), \left(v_i, e_{i+j+1}\right), \left(e_{i+j+1}, v_{i+3j}\right)\right\}$$
$$Q^{(2)} = U_{j=1}^{(n-2)/2}\left\{U_{i=1}^{n} Q_{ji}^{(2)}\right\},$$
$$Q_{ji}^{(2)} = \left\{\left(v_i, f_{i+j}\right), \left(f_{i+j}, f_i\right), \left(f_i, v_{i+j}\right)\right\}$$
$$Q^{(3)} = U_{i=1}^{n/2} Q_i^{(3)},$$
$$Q_i^{(3)} = \left\{\left(v_i, f_{i+n/2}\right), \left(f_{i+n/2}, f_i\right), \left(f_i, v_{i+n/2}\right)\right\}$$
$$Q^{(4)} = \bigcup_{j=1}^{(n-1)}\left\{U_{i=1}^{n} Q_{ji}^{(4)}\right\},$$
$$Q_{ji}^{(4)} = \left\{\left(e_i, f_{i+i}\right), \left(f_{i+j}, u_i\right), \left(u_i, e_{i+j}\right)\right\}$$
$$Q^{(5)} = U_{i=1}^{n} Q_i^{(5)},$$
$$Q_i^{(5)} = \left\{\left(v_i, f\right), \left(f, f_i\right), \left(f_i, u\right)\right\}$$
$$Q^{(6)} = U_{i=1}^{n} Q_i^{(6)},$$
$$Q_i^{(6)} = \left\{\left(e_{i+(2n-3)/3}, v_i\right), \left(v_i, e_{i+(2n/3)}\right), \left(e_{i+(2n/3)}, u_{i+2n/3}\right)\right\}$$

Here, $\langle Q^{(1)}\rangle \cong (n(n-3)/3)P_4, \langle Q^{(2)}\rangle \cong (n(n-2)/2)P_4$, $\langle Q^{(3)}\rangle \cong (n/2)P_4$ $\langle Q^{(4)}\rangle \cong (n(n-1))P_4, \langle Q^{(5)}\rangle \cong nP_4, \langle Q^{(6)}\rangle \cong nP_4$.
Hence, $B_3\left(K_{1,n}^+\right) - nK_2$ is P_4- decomposable.
Case 2. n is odd, $n \geq 9$
Then the edge set of $B_3\left(K_{1,n}+\right)$ can be decomposed into $\left(\left(11n^2-3n\right)/6\right)P_4$ and nK_2. The edge set of nK_2 is given by the set $\bigcup_{i=1}^{n}\left\{(v, f_i)\right\}$. The edge set of $\left(\left(11n^2-3n\right)/6\right)P_4$ is given by the edge sets, $Q^{(4)}, Q^{(5)}, Q^{(6)}$ as in Case 1 and the sets $Q^{(7)}$ and $Q^{(8)}$, where

$$Q^{(7)} = U_{j=1}^{(n-3)/3}\left\{U_{i=1}^{n} Q_{ji}^{(7)}\right\},$$
$$Q_{ji}(7) = \left\{\left(e_{i+j}, v_i\right), \left(v_i, e_{i+j+1}\right), \left(e_{i+j+1}, v_{i+2j-2}\right)\right\}$$
$$Q^{(8)} = \bigcup_{j=1}^{(n-1)/2}\left\{U_{i=1}^{n} Q_{ji}^{(8)}\right\},$$
$$Q_{ji}^{(8)} = \left\{\left(v_i, f_{i+j}\right), \left(f_{i+j,} f_{ij}\right), \left(f_j, v_{i+i}\right)\right\}$$

Here, $\langle Q^{(7)}\rangle \cong (n(n-3)/3)P_4, \langle Q^{(8)}\rangle \cong ((n(n-1)/2)P_4)$.
Hence, $B_3\left(K_{1,n}^+\right) - nK_2$ is P_4-decomposable. □

Theorem 3.7. *For $n \geq 4$ and $n \equiv 1(\bmod 3)$, the graph*

$$B_3\left(K_{1,n}{}^+\right) - 2nK_2$$

is P_4- decomposable.

Proof. Then the edge set of $B_3\left(K_{1,n}^+\right)$ can be decomposed into $\left(\left(11n^2-5n\right)/6\right)P_4$ and $2nK_2$. The edge set of $2nK_2$

is given by the set $U_{i=1}^{n}\{(v,f_i),(u_i,e_i)\}$. The edge set of $\left(\left(11n^2-5n\right)/6\right)P_4$ is given by the edge sets $Q^{(2)},Q^{(3)},Q^{(4)}$, $Q^{(5)}$ as in Theorem 3.6 and the set $Q^{(9)}$ where,

$$Q^{(9)} = U_{j=1}^{(n-1)/3}\left\{U_{i=1}^{n}Q_{ji}^{(9)}\right\},$$
$$Q_{ji}^{(9)} = \left\{(e_{i+j},v_i),(v_i,e_{i+j+1}),(e_{i+j+1},v_{i+3j})\right\}.$$

Here, $\langle Q^{(9)}\rangle \cong (n(n-1)/3)P_4$. Hence, $B_3\left(K_{1,n}^{+}\right)-2nK_2$ is P_4- decomposable. □

Theorem 3.8. *For $n \geq 5$ and $n \equiv 2(\bmod 3)$, the graph*

$$B_3\left(K_{1,n}^{+}\right)-3nK_2$$

is P_4- decomposable.

Proof. Then the edge set of $B_3\left(K_{1,n}^{+}\right)$ can be decomposed into $\left(\left(11n^2-7n\right)/6\right)P_4$ and $3nK_2$. The edge set of $3nK_2$ is given by the set $U_{i=1}^{n}\left\{(v,f_i),(u_i,e_i),\left(v_i,e_{i+(2n-3)/3}\right)\right\}$

The edge set of $\left(\left(11n^2-7n\right)/6\right)P_4$ is given by the edge sets $Q^{(4)},Q^{(5)},Q^{(8)}$ as in Theorem 3.6 and the set $Q^{(10)}$, where

$$Q^{(10)} = U_{j=1}^{(n-2)/3}\left\{U_{i=1}^{n}Q_{ji}^{(10)}\right\},$$
$$Q_{ji}(10) = \left\{(e_{i+j},v_i),(v_i,e_{i+j+1}),(e_{i+j+1},v_{i+3j})\right\}.$$

Here, $\langle Q^{(10)}\rangle \cong (n(n-2)/3)P_4$ Hence, $B_3\left(K_{1,n}^{+}\right)-3nK_2$ is P_4- decomposable. □

4. Conclusion

In this paper, P_4-Decomposition of Boolean Function Graph $B(\overline{Kp},L(\bar{G}),NINC)$ of path, cycle, stars and corona graphs are obtained.

References

[1] P.Chithra Devi and J. Paulraj Joseph, P4- Decomposition of Total Graphs, *Journal of Discrete Mathematical Sciences & Cryptography*, 17(5&6)(2014), 473-498.

[2] Harary F, *Graph Theory,* Addison- Wesley Reading Mass., 1969.

[3] K. Heinrich, J. Liu and M.Yu, P4- Decomposition of regular Graphs, *Journal of Graph Theory*, 31(2)(1999), 135-143.

[4] T. N. Janakiraman, S. Muthammai, M. Bhanumathi, Domination Numbers on the Boolean Function Graph of a Graph, *Mathematica Bohemica*, 130(2)(2005), 135-151.

[5] T. N. Janakiraman, S. Muthammai, M. Bhanumathi, Domination Numbers on the Complement of the Boolean Function Graph of a Graph, *Mathematica Bohemica*, 130(3)(2005), 247-263.

[6] T. N. Janakiraman, S. Muthammai, M. Bhanumathi, On the Boolean Function Graph of a Graph and on its Complement, *Mathematica Bohemica*, 130(2)(2005), 113-134.

[7] S.Muthammai and S.Dhanalakshmi, Edge Domination in Boolean Function Graph B(G, L(G), NINC) of a Graph, *IJIRSET Journal*, 4(12)(2015), 12346-12350.

[8] S.Muthammai and S.Dhanalakshmi, Edge Domination in Boolean Function Graph B(G, L(G), NINC) of Corona of Some Standard Graphs, *Global Journal of Pure and Applied Mathematics*, 13(1)(2017), 152-155.

[9] S.Muthammai and S.Dhanalakshmi, Connected and total edge Domination in Boolean Function Graph B(G, L(G), NINC) of a graph, *International Journal of Engineering, Science and Mathematics,* 6(6)(2017), ISSN: 2320-0294.

[10] S.Muthammai and S.Dhanalakshmi, Domatic edge Domination in Boolean Function Graph B(G, L(G), NINC) of a graph, *Journal of Emerging Technologies and Innovative Research(JETIR)*, 5(3)(2018), ISSN: 2349-5162.

[11] C.Sunil Kumar, On P4- Decomposition of Graphs, *Taiwanese Journal of Mathematics,* 7(4)(2003), 657-664.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆