



Certain topological indices and their polynomials of some cutting number nanostar Dendrimer $NS[n]$

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Abstract

Some theoretical results about nanostar Dendrimer by topological indices are explained in this research paper. This paper details with 1st 2nd and 3rd cutting number Zagreb Index, Multiplicative and polynomial and also reverse hyper-Zagreb cutting number index multiplicative and polynomial of nanostar Dendrimer.

Keywords

Cutting number, Zagreb Polynomial, Index of Hyper-Zagreb, Reverse Index of Hyper-Zagreb, Reverse Hyper-Zagreb polynomial.

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1. Introduction

The topological index characterized on the structure of these compound can equip to surmise physical attributes, chemical reactivity and biological action to the scientists [11, 3].

In [10], Gutman introduced the 1st and 2nd Zagreb indices are

$$M_1(G) = \sum_{uv \in E(G)} (d(u) + d(v))$$

$$M_2(G) = \sum_{uv \in E(G)} (d(u) \times d(v))$$

Fath-Tabar [1, 2&8] defined 1st and 2nd Zagreb polynomials as

$$M_1(G, x) = \sum_{uv \in E(G)} x^{d(u)+d(v)}$$

$$M_2(G, x) = \sum_{uv \in E(G)} x^{d(u) \cdot d(v)}$$

In [2], G. H. Fath-Tabar et al. defined the 3rd Zagreb

index as

$$M_3(G) = \sum_{uv \in E(G)} |d(u) - d(v)|$$

and the polynomial as

$$M_3(G, x) = \sum_{uv \in E(G)} x^{|d(u)-d(v)|}$$

The 1st and 2nd Reverse hyper-Zagreb indices was also defined in the samp paper as

$$HCM_1(G) = \sum_{uv \in E(G)} (c(u) + c(v))^2$$

$$HCM_2(G) = \sum_{uv \in E(G)} (c(u)c(v))^2$$

The 1st and 2nd Reverse hyper-Zagreb polynomials are defined as

$$HCM_1(G, x) = \sum_{uv \in E(G)} x^{(c(u)+c(v))^2}$$

$$HCM_2(G, x) = \sum_{uv \in E(G)} x^{(c(u) \cdot c(v))^2}$$

A cutting number [7], $c(v)$ of a vertex $v \in V(G)$ in a connected graph G is the number of pairs of vertices $\{v, w\}$ such that v and w are in different components of G .

We now consider the Nanostar Dendrimers $NS[n]$, where n is steps of growth see Fig. 1& Fig. 2.

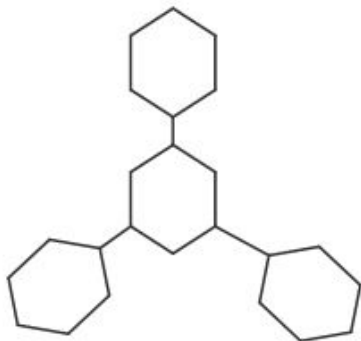


Figure 1. Nanostar Dendrimer NS[1]

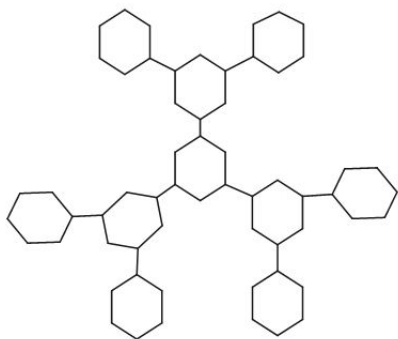


Figure 2. Nanostar Dendrimer NS[2]

In [7], we have found number of edges uv with cutting number of end vertices $(c(u), c(v))$ which is given in the Table 1.

2. Some cutting number Topological Indices and polynomials of Nanostar Dendrimer $NS[n]$

In this paper, we introduce some other cutting number based topological indices of graphs. For two-connected graphs, cutting number of each vertex is zero. So, we define these indices for graphs with cut vertices only.

We define the 1st, 2nd and 3rd cutting number Zagreb indices of G as

$$M_{1C}(G) = \sum_{uv \in E[G]} [c(u) + c(v)]$$

$$M_{2C}(G) = \sum_{uv \in E[G]} [c(u)c(v)]$$

$$M_{3C}(G) = \sum_{uv \in E[G]} |c(u) - c(v)|$$

We define the 1st, 2nd and 3rd cutting number Zagreb poly-

mial of the graph G as

$$M_{1C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)+c(v)]}$$

$$M_{2C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)c(v)]}$$

$$M_{3C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)-c(v)]},$$

where $c(u)$ and $c(v)$ are the cutting number of u and v .

We define the 1st 2nd and 3rd cutting number Reverse hyper-Zagreb indices of G as

$$RHM_{1C}(G) = \sum_{uv \in E[G]} [c(u) + c(v)]^2$$

$$RHM_{2C}(G) = \sum_{uv \in E[G]} [c(u)c(v)]^2$$

$$RHM_{3C}(G) = \sum_{uv \in E[G]} |c(u) - c(v)|^2$$

We define the 1st 2nd and 3rd cutting number Reverse hyper-Zagreb polynomial of the graph G as

$$RHM_{1C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)+c(v)]^2}$$

$$RHM_{2C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)c(v)]^2}$$

$$RHM_{3C}[G, x] = \sum_{uv \in E[G]} x^{[c(u)-c(v)]^2}$$

Theorem 2.1. Consider $NS[n]$ be the Nanostar Dendrimer. Then,

- (i) $M_{1C}[NS(n)] = 1944n \times 2^{2n} - 2106 \times 2^{2n} - 648 \times 2^{2n+1} + 648 \times 2^{n+1} + 2862 \times 2^n - 756.$
- (ii) $M_{2C}[NS(n)] = 15552 \times 2^{4n+1} - 9720 \times 2^{3n+1} - 34452 \times 2^{2n+1} + 9180 \times 2^{n+1} - 3888 \times 2^{n+3} + 23328 \times 2^{2n+2} + 40824 \times 2^{3n} - 70308 \times 2^{2n} + 34776 \times 2^n - 75816n \times 2^{3n} + 27540n \times 2^{2n} - 5292$
- (iii) $M_{3C}[NS(n)] = 1296n \times 2^n - 1458 \times 2^{2n} - 432 \times 2^{2n+1} + 432 \times 2^{n+1} + 36n \times 2^n + 1962 \times 2^n - 504.$

Proof.

$$(i) M_{1C}[NS(n)] = \sum_{uv \in E[NS(n)]} [c(u) + c(v)]$$

$$= \sum_{i=0}^{n-1} (6 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7) + 0]$$

$$+ \sum_{i=0}^{n-1} (3 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)]$$

$$+ \{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}$$

$$+ \sum_{i=0}^{n-1} (6 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6) + 0]$$

$$= 1944n \times 2^{2n} - 2106 \times 2^{2n} - 648 \times 2^{2n+1} + 648 \times 2^{n+1} + 2862 \times 2^n - 756$$



Table 1. The Nanostar Dendrimer $NS[n]$

Number of edges $e = uv$	Cutting number of end vertices $(c(u), c(v))$
$12 \times 2^{n-1}$	$(0,0)$
$6 \times 2^{n-1}$	$[\{ ((18 \times 2^n) - 12) - 6 \} \times 5, 0]$
$3 \times 2^{n-1}$	$[\{ ((18 \times 2^n) - 12) - 6 \} \times 5, \{ ((18 \times 2^n) - 12) - 7 \} \times 6]$
$6 \times 2^{n-1}$	$[\{ ((18 \times 2^n) - 12) - 7 \} \times 6, 0]$
$6 \times 2^{n-2}$	$[\{ ((18 \times 2^n) - 12) - 18 \} \times 17, 0]$
$3 \times 2^{n-2}$	$[\{ ((18 \times 2^n) - 12) - 18 \} \times 17, \{ ((18 \times 2^n) - 12) - 19 \} \times 18]$
$6 \times 2^{n-2}$	$[\{ ((18 \times 2^n) - 12) - 19 \} \times 18, 0]$
\vdots	
6×2^i	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-i}) - 6) \} \times ((6 \times 2^{n-i}) - 7), 0]$
3×2^i	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-i}) - 6) \} \times ((6 \times 2^{n-i}) - 7), \{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-i}) - 5) \} \times ((6 \times 2^{n-i}) - 6)]$
6×2^i	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-i}) - 5) \} \times ((6 \times 2^{n-i}) - 6), 0]$
\vdots	
6×2^2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-2}) - 6) \} \times ((6 \times 2^{n-2}) - 7), 0]$
3×2^2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-2}) - 6) \} \times ((6 \times 2^{n-2}) - 7), \{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-2}) - 5) \} \times ((6 \times 2^{n-2}) - 6)]$
6×2^2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-2}) - 5) \} \times ((6 \times 2^{n-2}) - 6), 0]$
6×2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-1}) - 6) \} \times ((6 \times 2^{n-1}) - 7), 0]$
3×2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-1}) - 6) \} \times ((6 \times 2^{n-1}) - 7), \{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-1}) - 5) \} \times ((6 \times 2^{n-1}) - 6)]$
6×2	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^{n-1}) - 5) \} \times ((6 \times 2^{n-1}) - 6), 0]$
6×1	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^n) - 6) \} \times ((6 \times 2^n) - 7), 0]$
3×1	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^n) - 6) \} \times ((6 \times 2^n) - 7), \{ ((18 \times 2^n) - 12) - ((6 \times 2^n) - 5) \} \times ((6 \times 2^n) - 6)]$
6×1	$[\{ ((18 \times 2^n) - 12) - ((6 \times 2^n) - 5) \} \times ((6 \times 2^n) - 6), 0]$
Total number of edges	$21 \times 2^n - 15$

(ii) $M_2C[NS(n)]$

$$= \sum_{uv \in E[NS(n)]} [c(u)c(v)]$$

$$= \sum_{i=0}^{n-1} (3 \times 2^i) [\{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \}$$

$$\times \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6)]$$

$$= 15552 \times 2^{4n+1} - 9720 \times 2^{3n+1} - 34452 \times 2^{2n+1} + 9180$$

$$\times 2^{n+1} - 3888 \times 2^{n+3} + 23328 \times 2^{2n+2} + 40824 \times 2^{3n} - 70308$$

$$\times 22n + 34776 \times 2^n - 75816n \times 2^{3n} + 27540n \times 2^{2n} - 5292$$

(iii) $M_3C[NS(n)] = \sum_{uv \in E[NS(n)]} |c(u) - c(v)|$

$$= \sum_{i=0}^{n-1} (6 \times 2^i) [\{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} - 0]$$

$$+ \sum_{i=0}^{n-1} (3 \times 2^i) [\{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \}$$

$$- \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \}]$$

$$+ \sum_{i=0}^{n-1} (6 \times 2^i) [\{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} - 0]$$

$$= 1296n \times 2^{2n} - 1458 \times 2^{2n} - 432 \times 2^{2n+1} + 432 \times 2^{n+1} + 36n$$

$$\times 2^n + 1962 \times 2^n - 504$$

□

$NS[n]$ are computed as follows:

(i) $M_{1C}[NS(n), x]$

$$= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]}$$

$$+ 3 \sum_{i=0}^{n-1} 2^i x^{[216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84]}$$

$$+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-1} - 108 \times 2^n + 42]}$$

(ii) $M_{2C}[NS(n), x] = 3 \sum_{i=0}^{n-1} 2^i$

$$[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} -$$

$$25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} -$$

$$x \{ 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764 \}]$$

(iii) $M_{3C}[NS(n), x]$

$$= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]}$$

$$+ 3 \sum_{i=0}^{n-1} 2^i x^{[12 \times 2^{2n-i} - 18 \times 2^n]}$$

$$+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-1} - 108 \times 2^n + 42]}$$

Theorem 2.2. The Zagreb polynomial of Nanostar Dendrimer



Proof.

$$\begin{aligned}
 \text{(i) } M_1C[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{[c(u)+c(v)]} \\
 &= \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} + 0 \}} \\
 &+ \sum_{i=0}^{n-1} (3 \times 2^i) \\
 &\quad \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} \\
 &\times x + \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} \}} \\
 &+ \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} + 0 \}} \\
 &= 6 \sum_{i=0}^{n-1} 2^i x^{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42} \\
 &+ 3 \sum_{i=0}^{n-1} 2^i x^{216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84} \\
 &+ 6 \sum_{i=0}^{n-1} 2^i x^{108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42} \\
 \text{(ii) } M_{2C}[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{[c(u)c(v)]} \\
 &= \sum_{i=0}^{n-1} (3 \times 2^i) \\
 &\quad \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} \\
 &\times x + \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} \}} \\
 &= 3 \sum_{i=0}^{n-1} 2^i \\
 &\quad [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} \\
 &\quad - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} \\
 &\quad - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764] \\
 \text{(iii) } M_{3C}[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{[c(u)-c(v)]} \\
 &= \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} - 0 \}} \\
 &+ \sum_{i=0}^{n-1} (3 \times 2^i) \\
 &\quad \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} \\
 &\times x - \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} \}} \\
 &\sum_{i=0}^{n-1} (6 \times 2^i) x^{\{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} - 0 \}} \\
 &= 6 \sum_{i=0}^{n-1} 2^i x^{(108 \times 2^{2n-1} - 36 \times 2^{2n-2i} + 6 \times 2^{n-1} - 126 \times 2^n + 42)} \\
 &+ 3 \sum_{i=0}^{n-1} 2^i x^{(12 \times 2^{n-1} - 18 \times 2^n)} \\
 &+ 6 \sum_{i=0}^{n-1} 2^i x^{(108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42)}
 \end{aligned}$$

Theorem 2.3. Let $NS[n]$ be the molecular graph of Nanostar Dendrimer. Then

$$\begin{aligned}
 \text{(i) } RHM_{1C}[NS(n)] &= 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2 \\
 &+ 3 \sum_{i=0}^{n-1} 2^i [216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84]^2 \\
 &+ 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2 \\
 \text{(ii) } RHM_{2C}[NS(n)] &= 3 \sum_{i=0}^{n-1} 2^i [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} \\
 &\quad - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} \\
 &\quad - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764]^2 \\
 \text{(iii) } RHM_{3C}[NS(n)] &= 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2 \\
 &+ 3 \sum_{i=0}^{n-1} 2^i [12 \times 2^{n-1} - 18 \times 2^n]^2 \\
 &+ 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2 \\
 \text{Proof.} & \\
 \text{(i) } RHM_{1C}[NS(n)] &= \sum_{uv \in E[NS(n)]} [c(u) + c(v)]^2 \\
 &= \sum_{i=0}^{n-1} (6 \times 2^i) \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} + 0 \}^2 \\
 &+ \sum_{i=0}^{n-1} (3 \times 2^i) \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} \\
 &+ \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} \}^2 \\
 &+ \sum_{i=0}^{n-1} (6 \times 2^i) \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} + 0 \}^2 \\
 &= 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2 \\
 &+ 3 \sum_{i=0}^{n-1} 2^i [216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84]^2 \\
 &+ 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2 \\
 \text{(ii) } RHM_{2C}[NS(n)] &= \sum_{uv \in E[NS(n)]} [c(u)c(v)]^2 \\
 &= \sum_{i=0}^{n-1} (3 \times 2^i) \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6) \} \times (6 \times 2^{n-i} - 7) \} \\
 &\times \{ \{ ((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5) \} \times (6 \times 2^{n-i} - 6) \} \}^2
 \end{aligned}$$



$$= 3 \sum_{i=0}^{n-1} 2^i [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} - 25272 \times 2^{3n-i} + 9180 \times 2^{2n-1} + 1296 \times 2^{4n-4i} - 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764]_n^2$$

$$\begin{aligned} \text{(iii) } RHM_{3C}[NS(n)] &= \sum_{uv \in E[NS(n)]} |c(u) - c(v)|^2 \\ &= \sum_{i=0}^{n-1} (6 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} - 0]^2 \\ &+ \sum_{i=0}^{n-1} (3 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} \\ &- \{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}^2 \\ &+ \sum_{i=0}^{n-1} (6 \times 2^i) [\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\} - 0]^2 \\ &= 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-1} - 126 \times 2^n + 42]^2 \\ &+ 3 \sum_{i=0}^{n-1} 2^i [12 \times 2^{n-1} - 18 \times 2^n]^2 \\ &+ 6 \sum_{i=0}^{n-1} 2^i [108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2 \end{aligned}$$

Theorem 2.4. Let $NS[n]$ be the Nanostar Dendrimer. The following topological indices can be Calculated:

$$\begin{aligned} \text{(i) } RHM_{1C}[NS(n), x] &= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2} \\ &+ 3 \sum_{i=0}^{n-1} 2^i x^{[216 \times 2^{2n-i} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84]^2} \\ &+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2} \\ &+ [11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} \\ &- 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} \\ &- 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764]_n^2 \end{aligned}$$

$$\begin{aligned} \text{(ii) } RHM_{2C}[NS(n), x] &= 3 \sum_{i=0}^{n-1} 2^i \\ &[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} \\ &- 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} \\ &- 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764]_n^2 \end{aligned}$$

$$\begin{aligned} \text{(iii) } RHM_{3C}[NS(n), x] &= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} + 6 \times 2^{n-i} - 126 \times 2^n + 42]^2} \\ &+ 3 \sum_{i=0}^{n-1} 2^i x^{[12 \times 2^{n-1} - 18 \times 2^n]^2} \\ &+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-i} - 108 \times 2^n + 42]^2} \end{aligned}$$

Proof.

$$\begin{aligned} \text{(i) } RHM_{1C}[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{[c(u)+c(v)]^2} \\ &= \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6) + 0\}^2} \\ &+ \sum_{i=0}^{n-1} (3 \times 2^i) \\ &[\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} \\ &\times x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-1} - 5)\} \times (6 \times 2^{n-i} - 6)\}^2} \\ &+ \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-1} - 5)\} \times (6 \times 2^{n-1} - 6) + 0\}^2} \\ &= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-1} - 36 \times 2^{2n-2i} + 6 \times 2^{n-1} - 126 \times 2^n + 42]^2} \\ &+ 3 \sum_{i=0}^{n-1} 2^i x^{[216 \times 2^{2n-1} - 72 \times 2^{2n-2i} - 234 \times 2^n + 84]^2} \\ &+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-i} - 36 \times 2^{2n-2i} - 6 \times 2^{n-1} - 108 \times 2^n + 42]^2} \end{aligned}$$

$$\begin{aligned} \text{(ii) } RHM_{2C}[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{[c(u)c(v)]^2} \\ &= \sum_{i=0}^{n-1} (3 \times 2^i) \\ &[\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} \times \\ &x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 5)\} \times (6 \times 2^{n-i} - 6)\}^2} \\ &[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i} \\ &- 25272 \times 2^{3n-i} + 9180 \times 2^{2n-i} + 1296 \times 2^{4n-4i} \\ &- 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764]_n^2 \end{aligned}$$

$$\begin{aligned} \text{(iii) } RHM_{3C}[NS(n), x] &= \sum_{uv \in E[NS(n)]} x^{|c(u)-c(v)|^2} \\ &= \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-i} - 6)\} \times (6 \times 2^{n-i} - 7)\} - 0\}^2} \\ &+ \sum_{i=0}^{n-1} (3 \times 2^i) \\ &[\{((18 \times 2^n) - 12) - (6 \times 2^{n-1} - 6)\} \times (6 \times 2^{n-1} - 7)\} \\ &\times x^{-\{((18 \times 2^n) - 12) - (6 \times 2^{n-1} - 5)\} \times (6 \times 2^{n-1} - 6)\}^2} \\ &+ \sum_{i=0}^{n-1} (6 \times 2^i) x^{\{((18 \times 2^n) - 12) - (6 \times 2^{n-1} - 5)\} \times (6 \times 2^{n-1} - 6) - 0\}^2} \\ &= 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-1} - 36 \times 2^{2n-2i} + 6 \times 2^{n-1} - 126 \times 2^n + 42]^2} \\ &+ 3 \sum_{i=0}^{n-1} 2^i x^{[12 \times 2^{n-1} - 18 \times 2^n]^2} \\ &+ 6 \sum_{i=0}^{n-1} 2^i x^{[108 \times 2^{2n-1} - 36 \times 2^{2n-2i} - 6 \times 2^{n-1} - 108 \times 2^n + 42]^2} \end{aligned}$$



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3. Conclusion

In this paper we determined cutting number Zagreb indices, Multiplicative Zagreb indices, Zagreb polynomial, Reverse hyper-Zagreb indices, Multiplicative Reverse hyper-Zagreb indices and Reverse hyper-Zagreb polynomial for Nanostar Dendrimer. It would be interesting for future study to investigate other topological indices of these Nanostar Dendrimer.

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