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Some connectivity eccentric indices and modified eccentric indices of Benzenoid H_k system

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Abstract

The topological indices correlate certain physicochemical properties such as boiling point, stability of compounds. In this paper, we define the eccentricity based connectivity eccentric index,the product connectivity eccentric index, sum connectivity eccentric index, sum line connectivity eccentric index and product line connectivity eccentric index and Modified eccentric indices, we evaluate these indices for benzenoid H_k system.

Keywords

Connectivity Eccentric Index, Sum Connectivity Eccentric Index, Product Connectivity Eccentric Index, Modified Eccentric Index.

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Contents

1. Introduction

In this paper, we consider finite simple undirected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $e(G)$. The $e = uv \in E(G)$. Let eL $_{XG}$ (e) denote the eccentricity of an edge ein $L(G)$, where $L(G)$ is the line graph of G. The vertices and edges of a graph are called the elements of G.

One of the best known and widely used topological index is the product connectivity index or Randić index introduced by Randicin [8] . The product connectivity index of a graph G is defined in [5] as

$$(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

Motivated by the definition of the product connectivity index, the multiplicative product connectivity index, multiplicative sum connectivity index were very recently proposed in [7]. They are defined as follows:
The multiplicative product connectivity index of a graph G is defined as

$$\chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

The multiplicative sum connectivity index of a graph G is defined as

$$\chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$$

The multiplicative atom bond connectivity index of a graph G is defined as

$$ABCII = \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$

The multiplicative geometric-arithmetic index of a graph G is defined as

$$\text{GAII } (G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

In [7], Kulli introduced the general multiplicative geometric-arithmetic index of a graph G. This topological index is defined as follows:

$$GA^aII(G) = \left(\prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}\right)a$$

The modified first and second Zagreb indices are respectively defined by Kulli [5] as

$$^mM_1(G) = \sum_{u\in VG)} \frac{1}{d_c(u)^2},\ ^mM_2(G) = \sum_{uv\in EG)} \frac{1}{d_c(u)d_-(v)}$$

The modified first and second K eccentric indices of a graph are defined as

$$^mB_1(G) = \sum_{ue} \frac{1}{d_G(u)+d_G(e)},\ ^mB_2(G) = \sum_{ue} \frac{1}{d_G(u)d_G(e)}$$

The harmonic index of a graph G is defined as

$$H(G) = \sum_{uv\in EG} \frac{2}{d_G(u)+d_G(v)}$$

this index was studied by Favaron et. al. [5].

Kulli introduced the harmonic K-Banhatti index of a graph G as

$$H_b(G) = \sum_{ue} \frac{2}{d_G(u)+d_G(e)}$$

where in all the cases ue means that the vertex u and edge e are incident with u in G.

2. Some Connectivity Eccentric indices and Modified Eccentric indices of a graph G

In this paper,we define the eccentricity based connectivity indices and modified indices of a graph G.

First, we define connectivity eccentric index of a graph G as

$$\chi E(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}$$

We define the product connectivity eccentric index of a graph G as

$$\chi_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}.$$

We define the sum connectivity eccentric index of a graph G as

$$XE(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}$$

We define the multiplicative sum connectivity eccentric index of a graph G as

$$X_pE(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}$$

We define the sum line connectivity eccentric index of a graph G as

$$SLCEII = \sum_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$

We define the product line connectivity eccentric index of a graph G as

$$PLCEII = \prod_{ue} \sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$

We define the modified eccentric first and second K eccentric indices of a graph G as

$$^mB_1(G) = \sum_{ue} \frac{1}{e_{(G)}(u)+e_{L(G)}(e)},$$
$$^mB_2(G) = \sum_{ue} \frac{1}{e_{(G)}(u)e_{L(G)}(e)}.$$

We define the harmonic index and harmonic K-eccentric index of a graph G as

$$H(G) = \sum_{uv\in EG)} \frac{2}{e_{(G)}(u)+e_{(G)}(v)},$$
$$H_b(G) = \sum_{ue} \frac{2}{e_{(G)}(u)+e_{L(G)}(e)}$$

We define the modified Multiplicative eccentric first and second K-eccentric indices of a graph G as

$$^mB_1(G) = \prod_{ue} \frac{1}{e_{(G)}(u)+e_{L(G)}(e)},$$
$$^mB_2(G) = \prod_{ue} \frac{1}{e_{(G)}(u)e_{L(G)}(e)}$$

We define the harmonic eccentric index and harmonic K-eccentric index of a graph G as

$$H(G) = \prod_{ue} \frac{2}{e_{(G)}(u)+e_{(G)(v)}},$$
$$H_b(G) = \prod_{ue} \frac{2}{e_{(G)}(u)+e_{L(G)}(e)}$$

where in all the cases ue means that the vertex u and edge e are incident in G and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G.Here, we evaluate these indices for benzenoid H_k system.

2.1 Connectivity Eccentric indices and Modified eccentric indices of Benzenoid H_K system:

The circumcoronene homologous series of benzenoid also belongs to the family of molecular graphs that has several copy of benzene C_6 on its circumference. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The eccentricities of $u, v \in V(G)$ are denoted by $e(u), e(v)$ respectively and for $e = uv \in E(G)$, denote the eccentricities of the end vertices of the edgee by $(e(u), e(v))$.

The Circumcoronene homologous Series of Benzenoid $H_k(k \geq 1)$ with edges (Fig. 1)

Let V be the vertex set of H_k and E be the edge set in H_k, then $|V| = 6k^2$ and $|E| = 9k^2 - 3k$ for the structure of H_k. First, we shall determine the number of edges $e = uv$ with the eccentricity of the end vertices $e(u), e(v)$ and eccentricity of the edge e in $L(G)$. We give these values in the following Table 2.1.

Using MATLAB programme, we have calculated these all indices for H_1, H_2 and H_3. Those values are given below as corollaries. The values of eccentricity based connectivity indices of H_k are given in the following theorems.

Theorem 2.1. *Let G be a Circumcoronene Series of Benzenoid $H_k(k \geq 1)$. Theconnectivity eccentric index of G is given by*

$$\begin{aligned}\chi E(G) = & 6\sum_{r=1}^{k}\left[\frac{2}{2k+2r-1}\right]\\ & +6\sum_{r=1}^{k-1}\left[\frac{r}{2k+2r-1}+\frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]\\ & +12\sum_{r=1}^{k-1}\left[\frac{r}{2k+2r}+\frac{r}{\sqrt{(2k+2r+1)(2k+2r)}}\right]\end{aligned}$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k By using the definition of connectivity eccentric index, we have,

$$\chi E(G) = \sum_{ue}\frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}$$
$$= \sum_{e=uv\in E(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}\right]$$
$$= \sum_{e=uv\in E1(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}\right]+\ldots\ldots$$
$$+\sum_{e=uv\in E_{3(k-1)+1}(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}\right]$$
$$=\left[6\left(\frac{1}{\sqrt{(2k+1)(2k+1)}}+\frac{1}{\sqrt{(2k+1)(2k+1)}}\right)\right.$$
$$+6\left(\frac{1}{\sqrt{(2k+1)(2k+1)}}+\frac{1}{\sqrt{(2k+2)(2k+1)}}\right)+\ldots$$
$$+6\left(\frac{1}{\sqrt{(2k+2(k-1)+1)(2k+2(k-1)+1)}}\right.$$
$$\left.\left.+\frac{1}{\sqrt{(2k+2(k-1)+1)(2k+2(k-1)+1)}}\right)\right]$$

$$\begin{aligned}\chi E(G) = & 6\sum_{r=1}^{k}\left[\frac{2}{2k+2r-1}\right]\\ & +6\sum_{r=1}^{k-1}\left[\frac{r}{2k+2r-1}+\frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]\\ & +12\sum_{r=1}^{k-1}\left[\frac{r}{2k+2r}+\frac{r}{\sqrt{(2k+2r+1)(2k+2r)}}\right]\end{aligned}$$

□

Corollary 2.2. *Eccentric based connectivity index of H_1, H_2 and H_3 are given by*

$$\chi E(H_1) = 4, \chi E(H_2) = 10.2614, \chi E(H_3) = 15.998$$

Theorem 2.3. *Let G be a Circumcoronene Series of Benzenoid $H_k(k \geq 1)$. The product connectivity eccentric indexof G is given by*

$$\begin{aligned}\chi_p E(G) = & 6\left[\prod_{r=1}^{k}\left[\frac{1}{(2k+1)^2}\right]\right.\\ & \times\prod_{r=1}^{k-1}\left[\frac{r}{(4k+4r-1)}\times\frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]\\ & \left.\times 2\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r)}\times\frac{r}{\sqrt{(2k+2r+1)(2k+2r1)}}\right]\right]\end{aligned}$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of multiplicative product connectivity eccentric index, we have,

$$\chi_p E(G) = \prod_{ue}\frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}$$
$$= \prod_{e=uv\in E(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}\times\frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}\right]$$
$$= \prod_{e=uv\in E_1(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}\times\frac{1}{\sqrt{e_{H_k}(v)e_{L(H_k)}(e)}}\right]\times\ldots$$
$$\times\prod_{e=uv\in E_{3(k-1)+1}(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)e_{L(H_k)}(e)}}\times\frac{1}{e_{H_k}(v)e_{L(H_k)}(e)}\right]$$
$$= 6\left[\prod_{r=1}^{k}\left[\frac{1}{(2k+1)}\right]^2\right.$$
$$\times\prod_{r=1}^{k-1}\left[\frac{r}{(4k+4r-1)}\times\frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]$$
$$\left.\times 2\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r)}\times\frac{r}{\sqrt{(2k+2r+1)(2k+2r1)}}\right]\right]$$

□

Corollary 2.4. *Eccentric based connectivity index of H_1, H_2 and H_3 are given by* $\chi_p E(H_1) = 0.6667$, $\chi_p E(H_2) = 0.0020$, $\chi_p E(H_3) = 2.9289e^{-007}$.

Table 1

Edge set	No. of edges $e=uv$	Eccentricity of end vertices $(e(u),e(v))$	Eccentricity of e in $L(G)e_{L(G)}(e)$
E_1	6	$(2k+1,2k+1)$	$2k+1$
E_2	6	$(2k+1,2k+2)$	$2k+1$
E_3	12	$(2k+2,2k+3)$	$2k+2$
E_4	6	$(2k+3,2k+3)$	$2k+3$
E_5	12	$(2k+3,2k+4)$	$2k+3$
E_6	24	$(2k+4,2k+5)$	$2k+4$
E_7	6	$(2k+5,2k+5)$	$2k+5$
E_8	18	$(2k+5,2k+6)$	$2k+5$
E_9	36	$(2k+6,2k+7)$	$2k+6$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$E_{3(k-2)-2}$	6	$(2k+2(k-2)-1,2k+2(k-2)-1)$	$2k+2(k-2)-1$
$E_{3(k-2)-1}$	$6(k-2)$	$(2k+2(k-2)-1,2k+2(k-2))$	$2k+2(k-2)-1$
$E_{3(k-2)}$	$12(k-2)$	$(2k+2(k-2),2k+2(k-1)-1)$	$2k+2(k-2)$
$E_{3(k-1)-2}$	6	$(2k+2(k-1)-1,2k+2(k-1)-1)$	$2k+2(k-1)-1$
$E_{3(k-1)-1}$	$6(k-1)$	$(2k+2(k-1)-1,2k+2(k-1))$	$2k+2(k-1)-1$
$E_{3(k-1)}$	$12(k-1)$	$(2k+2(k-1),2k+2(k-1)+1)$	$2k+2(k-1)$
$E_{3(k-1)+1}$	6	$(2k+2(k-1)+1,2k+2(k-1)+1)$	$2k+2(k-1)+1$

Theorem 2.5. *Let G be a Circumcoronene Series of Benzenoid $H_k (k\geq 1)$. The sum connectivity eccentric index of G is given by*

$$
\begin{aligned}
XE(G) =& 6\sum_{r=1}^{k}\left[\frac{1}{2\sqrt{(2k+2r-1)}}\right]\\
&+6\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r-2}}+\frac{r}{\sqrt{4k+4r-1}}\right]\\
&+12\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r}}+\frac{r}{\sqrt{4k+4r+1}}\right]
\end{aligned}
$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k, By using the definition of sum connectivity eccentric index, we have,

$$
\begin{aligned}
XE(G) &= \sum_{ue}\frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}\\
&= \sum_{e=uv\in E(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]\\
&= \sum_{e=uv\in E1(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]+\cdots\\
&+ \sum_{e=uv\in E_{3(k-1)+1}(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}}+\frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]\\
&= 6\sum_{r=1}^{k}\left[\frac{1}{2\sqrt{(2k+2r-1)}}\right]\\
&+6\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r-2}}+\frac{r}{\sqrt{4k+4r-1}}\right]\\
&+12\sum_{r=1}^{k-1}\left[\frac{r}{\sqrt{4k+4r}}+\frac{r}{\sqrt{4k+4r+1}}\right]
\end{aligned}
$$

□

Corollary 2.6. *Eccentric based connectivity index of H_1, H_2 and H_3 are given by $XE(H_1) = 4.8990$, $XE(H_2) = 17.5006$, $XE(H_3) = 30.1729$.*

Theorem 2.7. *Let G be a Circumcoronene Series of Benzenoid $H_k (k\geq 1)$. The sum connectivity eccentric index of G is given by*

$$
\begin{aligned}
X_pE(G) = 6&\left[\prod_{r=1}^{k}\left[\frac{2}{(2k+2r-1)}\right]\right.\\
&\times\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r-1)}+\frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]\\
&\left.\times 2\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r)}+\frac{r}{\sqrt{(2k+2r+1)(2k+2r1)}}\right]\right]
\end{aligned}
$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k. By using the definition ofmultiplicative sum connectivity eccentric index, we have,

$$
\begin{aligned}
X_pE(G) &= \prod_{ue}\frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}\\
&= \prod_{e=uv\in E(G)}\left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}}\times\frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}}\right]
\end{aligned}
$$

$$= \prod_{e=uv\in E_1(G)} \left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}} \right] \times \ldots$$
$$\times \prod_{e=uv\in E_{3(k-1)+1}(G)} \left[\frac{1}{\sqrt{e_{H_k}(u)+e_{L(H_k)}(e)}} \times \frac{1}{\sqrt{e_{H_k}(v)+e_{L(H_k)}(e)}} \right]$$
$$= 6\left[\prod_{r=1}^{k}\left[\frac{2}{(2k+2r-1)}\right]\right.$$
$$\times \prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r-1)} + \frac{r}{\sqrt{(2k+2r)(2k+2r-1)}}\right]$$
$$\left.\times 2\prod_{r=1}^{k-1}\left[\frac{r}{(2k+2r)} + \frac{r}{\sqrt{(2k+2r+1)(2k+2r1)}}\right]\right]$$

□

Corollary 2.8. *Eccentric based multiplicative sum connectivity index of H_1, H_2 and H_3 are given* by $X_pE(H_1) = 4$, $X_pE(H_2) = 36.3753$, $X_pE(H_3) = 146.8437$.

Theorem 2.9. *Let G be a Circumcoronene Series of Benzenoid $H_k(k \geq 1)$.The sum line connectivity eccentric index of G is given by*

$$SLCEII = 6\sum_{r=1}^{k}\left[\frac{1}{2}\right] + 6\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] + 12\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k. By using the definition of sum line connectivity eccentric index, we have,

$$SLCEII = \sum e = uv \in E(G)\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$
$$= 6\sqrt{\frac{(2k+1)}{(2k+1)+(2k+1)}} + 6\sqrt{\frac{(2k+1)}{(2k+1)+(2k+2)}}$$
$$+ \ldots\ldots + 6\sqrt{\frac{(2k+2(k-1)+1)}{(2k+2(k-1)+1)+(2k+2(k-1)+1)}}$$
$$= 6\sum_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] + 6\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right]$$
$$+ 12\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]$$

□

Corollary 2.10. *Eccentric based connectivity index of H_1, H_2 and H_3 are given by* SLCEII$(H_1) = 4.2426$, $SLCEII(H_2) = 20.6829$, $SLCEII(H_3) = 49.8749$.

Theorem 2.11. *Let G be a Circumcoronene Series of Benzenoid $H_k(k \geq 1)$. The product line connectivity eccentric index of G is given by*

$$PLCEII = 6\left[\prod_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] \times \prod_{r=1}^{k-1}\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right] \times 2\prod_{r=1}^{k-1}\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]\right]$$

Proof. Let G be a Circumcoronene Series of Benzenoid H_k. By using the definition of product line connectivity eccentric index, we have,

$$PLCEII = \prod_{e=uv\in E(G)}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$
$$= 6\sqrt{\frac{(2k+1)}{(2k+1)+(2k+1)}} \times 6\sqrt{\frac{(2k+1)}{(2k+1)+(2k+2)}}$$
$$\times \ldots\ldots \times 6\sqrt{\frac{(2k+2(k-1)+1)}{(2k+2(k-1)+1)+(2k+2(k-1)+1)}}$$
$$= 6\left[\prod_{r=1}^{k}\left[\sqrt{\frac{1}{2}}\right] \times \prod_{r=1}^{k-1}\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r-1}{4k+4r-1}}\right]\right.$$
$$\left.\times 2\prod_{r=1}^{k-1}\sum_{r=1}^{k-1} r\left[\sqrt{\frac{2k+2r}{4k+4r-1}}\right]\right]$$

□

Corollary 2.12. *Eccentric based connectivity index of H_1, H_2 and H_3 are given by $PLCEII(H_1) = 4.2426$,* PLCEII$(H_2) = 593.6051$, $PLCEII(H_3) = 3.5245e^{+005}$.

3. Modified eccentric first and second K-eccentric index of Benzenoid $\mathrm{H_k}$, k, system

Theorem 3.1. *For any positive integer number k, let H_k be the general form of circumcoronene series of benzenoid system, then*

$$(i)\quad {}^{m}B_1(H_k) = 6\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right] + 6\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2} + \frac{r}{4k+4r-1}\right] + 12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r} + \frac{r}{4k+4r+1}\right]$$

$$(ii)\quad {}^{m}B_2(H_k) = 6\sum_{r=1}^{k}\left[\frac{2}{(2k+2(r-1)+1)^2}\right] + 6\sum_{r=1}^{k-1}\left[\frac{r}{(2k+2(r-1)+1)^2}\right.$$

$$+\frac{r}{(2k+2r)(2k+2(r-1)+1}\Bigg]$$

$$+12\sum_{r=1}^{k-1}\left[\frac{r}{(2k+2r)^2}+\frac{r}{(2k+(2r+1)(2k+2r)}\right]$$

(iii) $$H_b(H_k)=12\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right]$$

$$+12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2}+\frac{r}{4k+4r-1}\right]$$

$$+24\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r}+\frac{r}{4k+4r+1}\right]$$

Proof. Consider the General form of H_k - Circumcoronene graph, Using Table 2.1 , we obtain the following:

(ii) $^mB_1(H_k)$

$$=\sum_{uv\in E(G)}\frac{1}{e_{Hk}(u)+e_{L(Hk)}(e)}$$

$$=\sum_{uv\in E1(G)}\left[\frac{1}{e_G(u)+e_{L(G)}(e)}+\frac{1}{e_G(v)+e_{L(G)}(e)}\right]+\dots$$

$$+\sum_{uv\in E3(k-1)+1(G)}\left[\frac{1}{e_G(u)+e_{L(G)}(e)}+\frac{1}{e_G(v)+e_{L(G)}(e)}\right]$$

$$=6\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right]$$

$$+6\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2}+\frac{r}{4k+4r-1}\right]$$

$$+12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r}+\frac{r}{4k+4r+1}\right]$$

(ii) $$^mB_2(H_k)=\sum_{uv\in EG)}\frac{1}{e_{Hk}(u)e_{L(Hk)}(e)}$$

$$=6\sum_{r=1}^{k}\left[\frac{2}{(2k+2(r-1)+1)^2}\right]$$

$$+6\sum_{r=1}^{k-1}\Bigg[\frac{r}{(2k+2(r-1)+1)^2}$$

$$+\frac{r}{(2k+2r)(2k+2(r-1)+1)}\Bigg]$$

$$+12\sum_{r=1}^{k-1}\left[\frac{r}{(2k+2r)^2}+\frac{r}{(2k+(2r+1)(2k+2r)}\right]$$

(iii) $$H_b(H_k)=\sum_{uv\in EG)}\frac{2}{e_{Hk}(u)+e_{L(Hk)}(e)}$$

$$=12\sum_{r=1}^{k}\left[\frac{1}{2k+2(r-1)+1}\right]$$

$$+12\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r-2}+\frac{r}{4k+4r-1}\right]$$

$$+24\sum_{r=1}^{k-1}\left[\frac{r}{4k+4r}+\frac{r}{4k+4r+1}\right]$$

□

Corollary 3.2. *H_1 be the first terms of this Circumcoronene series of Coronene H_k. Then $^mB_1(H_1)=2$, $m\,B_2(H_1)=1.3333, H_b(H_1)=4$.*

Corollary 3.3. *H_2 be the second terms of this Circumcoronene series of Coronene H_k. Then $^mB_1(H_2)=5.1257$, $m\,B_2(H_2)=1.7839, H_b(H_2)=10.2513$*

Corollary 3.4. *H_3 be the third terms of this Circumcoronene series of coronene H_k. Then $^mB_1(H_3)=7.9948,^mB_2(H_3)=1.8156, H_b(H_3)=15.9896$.*

4. Modified Multiplicative eccentric first and second K-eccentric indices of Benzenoid H_k system

Theorem 4.1. *For any positive integer number k, let H_k be the general form of circumcoronene series of benzenoid system, then*

(i) $$^mB_1\prod(H_k)=6\prod_{r=1}^{k}\left[\frac{1}{4(2k+2r-1)^2}\right]$$

$$\times 6\prod_{r=1}^{k-1}r^2\left[\frac{1}{(4k+4r-1)^2}\right]$$

$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}\right]$$

(ii) $$^mB_2\prod(H_k)=6\prod_{r=1}^{k}\left[\frac{1}{(2k+2(r-1)+1)^4}\right]$$

$$\times 6\prod_{r=1}^{k-1}r^2\left[\frac{1}{(2k+2(r-1)+1)^2}\times\frac{1}{(2k+2r)(2k+2r-1)}\right]$$

$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{(2k+2r)^2}\times\frac{1}{(2k+(2r+1))(2k+2r)}\right]$$

(iii) $$H_b\prod(H_k)=12\prod_{r=1}^{k}\left[\frac{1}{4(2k+2r-1)^2}\right]$$

$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{(4k+4r-1)^2}\right]$$

$$\times 24\prod_{r=1}^{k-1}r^2\left[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}\right]$$

Proof.

(i) $$^mB_1\prod(H_k)=\prod_{uv\in EG)}\left[\frac{1}{e_{Hk}(u)+e_{L(Hk)}(e)}\right]$$

$$=\prod_{(uv\in E1G)}\left[\frac{1}{e_G(u)e_{L(G)}(e)}+\frac{1}{e_G(v)e_{L(G)}(e)}\right]+\dots$$

$$+\prod_{uv\in E(3(k-1)+1)(G)}\left[\frac{1}{e_G(u)e_{L(G)}(e)}+\frac{1}{e_G(v)e_{L(G)}(e)}\right]$$
$$=6\prod_{r=1}^{k}\left[\frac{1}{4(2k+2r-1)^2}\right]\times 6\prod_{r=1}^{k-1}r^2\left[\frac{1}{(4k+4r-1)^2}\right]$$
$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}\right]$$

(ii) $$^mB_2\prod(H_k)=\prod_{uv\in EG)}\left[\frac{1}{e_{Hk}(u)\times e_{L(Hk)}(e)}\right]$$
$$=\prod_{(uv\in E1G)}\left[\frac{1}{e_G(u)e_{L(G)}(e)}\times\frac{1}{e_G(v)e_{L(G)}(e)}\right]\times\ldots\ldots$$
$$\times\prod_{uv\in E(3(k-1)+1)(G)}\left[\frac{1}{e_G(u)e_{L(G)}(e)}\times\frac{1}{e_G(v)e_{L(G)}(e)}\right]$$
$$=6\prod_{r=1}^{k}\left[\frac{1}{(2k+2(r-1)+1)^4}\right]$$
$$\times 6\prod_{r=1}^{k-1}r^2\left[\frac{1}{(2k+2(r-1)+1)^2}\times\frac{1}{(2k+2r)(2k+2r-1)}\right]$$
$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{(2k+2r)^2}\times\frac{1}{(2k+(2r+1))(2k+2r)}\right]$$

(iii) $$H_b\prod(H_k)=\prod_{uv\in BG)}\left[\frac{2}{e_{Hk}(u)xe_{L(Hk)}(e)}\right]$$
$$=2\prod_{uv\in E1G)}\left[\frac{1}{e_G(u)e_{L(G)}(e)}+\frac{1}{e_G(v)e_{L(G)}(e)}\right]+\ldots$$
$$+2\prod_{uv\in E(3(k-1)+1)(G)}\left[\frac{1}{e_G(u)e_L(G)(e)}+\frac{1}{e_G(v)e_L(G)(e)}\right]$$
$$=12\prod_{r=1}^{k}\left[\frac{1}{4(2k+2r-1)^2}\right]$$
$$\times 12\prod_{r=1}^{k-1}r^2\left[\frac{1}{(4k+4r-1)^2}\right]$$
$$\times 24\prod_{r=1}^{k-1}r^2\left[\frac{1}{2(2k+2r)^2}\times\frac{1}{4k+4r+1}\right]$$

□

Corollary 4.2. *H_1 be the first terms of this Circumcoronene series of Benzene H_k. Then $^mB_1\prod(H_1)=0.1667$, $^mB_2\prod(H_1)=0.0741$, $H_b\prod(H_1)=0.3333$.*

Corollary 4.3. *H_2 be the second terms of this Circumcoronene series of Coronene H_k. Then $^mB_1\prod(H_2)=7.7066e^{-006}$, $^mB_2\prod(H_2)=1.5232e^{-009}$, $H_b\prod(H_2)=1.2331e^{-004}$.*

Corollary 4.4. *H_3 be the third terms of this Circumcoronene series of Circumcoronene H_k. Then $^mB_1\prod(H_3)=1.7761e^{-011}$, $^mB_2\prod(H_3)=1.9153e^{-020}$, $H_b\prod(H_3)=2.2734e^{-009}$.*

5. Conclusion

The purpose of this paper is to discuss the eccentric indices of chemical structure H_k. We have determined the connectivity eccentric index, sum connectivity eccentric index and product connectivity eccentric index, sum and product line connectivity eccentric index and Modified eccentric first and second K -eccentric index and Modified Multiplicative eccentric first and second K -eccentric indices of Benzenoid H_k system.This can be extended for other structures also.

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