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An EPQ model with two-level trade credit and multivariate demand incorporating the effect of system improvement and preservation technology

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Abstract

Nowadays every decision-maker from production industries is more concern with the system improvement. They also focused in adopting different marketing strategies to make their production system as successful system. In the current study, a production quantity model is explored for deteriorating item by considering volume flexibility. Multivariate demand is considered in the model which is the function of promotional efforts, selling price, and credit period. An additional investment is made by the decision-maker to improve the system quality and reduce the deterioration rate. Whole of the analysis is carried out under the effect of inflation. Profit function of the current study is nonlinear so a search algorithm is suggested to obtain the optimal values of selling price, production time, and production rate. At the end, numerical analysis along with sensitivity is executed to validate the model and to observe the effect of different parameters on the optimal solution of the problem.

Keywords

Economic production quantity, promotional efforts, preservation technology, two level trade credit, inflation, deterioration, multivariate demand.

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Contents

1. Introduction

Decision-makers of the industries are conscious about their requirement and day to day expenditure. Everybody explores different scientific tools in their business to optimize their resources in order to get financial benefits. Stiff competition and changing scenario of business, compels the decision-makers of inventory management to explore different market situations and mechanism so that they can sustain in the market. By chaining the policies and adopting the latest mechanism make their business model as successful model. This research work intends to develop a production inventory model with different realistic assumptions to make the manufacturing system as financially sustainable system.

Most of the researchers dealing with manufacturing sector assumed that produced items are of good quality. But in manufacturing industries, due to erroneous production and handling produced items are often not of perfect quality. Therefore, fraction of imperfect items is produced during the production process. This issues was pointed out by the Taft [33] and after that explored by Rosenblatt and Lee [27] and Lee and Rosenblatt [21]. These researchers are the pioneer researchers who laid down the foundation stone in this vast research area. A note was suggested by Goyal and Cardenas-Barron [12] for the EPQ model where the production process is imperfect. With help of that note, they suggested that we can easily found the optimal solution. Chiu [3] presented the impact of rework for the imperfect production process. They assumed that imperfect items which cannot be repaired are discarded. Further, a production system which may shift from an 'in control' stage to an 'out of control' stage was investigated by Sana [28]. They assumed that due to this production process not remain perfect. Kumar and Goswami [20] proposed a particle swarm optimization technique to obtain the optimal value of production run time for imperfect production system. They considered demand rate as random variable to incorporate the stochastic variability in it. Hsu and Hsu [16] obtained the optimal backorder and production lot size for the manufacturer by considering imperfect production system. They illustrated two different models by considering fraction of imperfect production as crisp or random variable. An EPQ model was derived by Taleizadeh [34] by considering imperfect production process. Screening process was also carried out at the manufacturer end. They assumed that imperfect items are repaired at workshop. Cheng et al. [2] presented an integrated inventory model with imperfect production system. They assumed that imperfect items are disposes in multiple batches. They presented the numerical examples to illustrate the effect of imperfect production process on the optimal solutions. Malik and Sarkar [22] presented disruption problem in production system where production process is not perfect. They obtained the optimal manufacturing lot size such that total profit of the system is maximum. De et al. [4] explored an economic production quantity model under non-random uncertain environment. They assumed that production process was not perfect. Screening and rework process was carried out by the manufacturer. Guchhait et al. [13] modelled an economic production quantity by considering setup cost reduction. They also considered investment in processes improvement of imperfect production system.

Initially, researchers dealing with manufacturing industries assumed that either demand is constant or time-dependent. Practically, various factors such as selling price, promotional efforts, trade credit, etc., influenced the demand of customers. Due to this, nowadays researchers are making efforts to analyze the effect of various factors on the demand while developing the inventory model for production system. Yang [36] explored the pricing strategies for inventory problem for deteriorating items by considering price sensitive demand. Teng and Chang [35] illustrated an economic production quantity models. They assumed that demand depends on price and stock dependent and deterioration occurs for the products. How the demand was affected by trade credit is explored by Jaggi et al. [18] and also obtained the replenishment policy for economic order quantity problem. Palanivel and Uthayakumar [25] designed an economic order quantity model for non-instantaneous deteriorating items. They considered demand as the function of advertisement and price. Geetha and Udayakumar [8] explored the necessary and sufficient to obtain the optimal solution of economic order quantity model with partial backlogging under the effect of advertisement and price dependent demand. Heydari et al. [14] proposed the coordination scheme for two-echelon supply chain by considering stochastic demand which depends on credit period. Bhunia et al. [1] presented a production-inventory model by considering the demand as the function of marketing cost and selling price. They considered that different costs associated with inventory as inter-valued function. Johari et al. [19] presented an integrated inventory model by considering price and credit dependent demand. They observed that credit financing and pricing can enhanced the overall performance of the integrated system. Integrated model was presented by Dey et al. [5] by considering demand as the function of selling price. Further, they obtained the optimal value of shipment size, shipment number, and selling price. Panja and Mondal [26] investigated two-echelon supply chain by considering credit period as well as selling price dependent demand. Inventory model was proposed by Shaikh et al. [30] for non-instantaneous deteriorating items by considering that demand is sensitive with price and demand.

Utility or quality of different products such as seafood, medicines, foods, battery, chips, etc., is affected due to poor storage facility, improper handling etc. This phenomenon in inventory termed as deterioration. Due to the deterioration process, usefulness of the products gradually decreases. Ghare and Schrader [11] were the first researchers who investigated the inventory problem by considering deterioration. Initially, researchers who modelled the problem of manufacturing industries assumed that rate of deterioration are either constant or a random variable. Many decades, researchers assumed that rate of deterioration as uncontrollable variable. But in practical situation, rate of deterioration depends on the preservation facility and environmental conditions. Deterioration process can be avoided in the system but rate of deterioration can be slow down by adopting proper equipment or processes. For example, chemical deterioration and microbial spoilage can be slow down by storing the products in refrigerator. Hsu et al. [17] adopted the preservation technology (PT) in inventory model to reduce the rate of deterioration. Hsu et al. [17] model was extended by Dye and Hsieh [7] by considering preservation technology. They incorporated the preservation technology cost by considering equivalent cost per replenishment period. Dye [6] investigated the inventory model for non-instantaneous items by considering preservation technology. Mathematically, they found that economic losses can be reduced and customer's satisfaction can be increased by adopting preservation technology. Dye and Yang [8] designed a pricing strategy and investment policy in preservation technology for inventory problem by considering time and price sensitive demand. Dye et al. [5] investigated an integrated model to determine the dynamic pricing and preservation technology (PT) investment in system. Pal et al. [24] explored an inventory model for deteriorating products with constant demand rate. In order to reduce the deterioration rate, they considered preservation technology (PT). Partial backlogging was considered model. Sheikh et al. [31] investigated an inventory model with trade credit policy and partial backlogging.

They considered preservation technology (PT) to control the rate of deterioration and assumed that demand depends on time. Namdeo et al. [23] explored the inventory model for deteriorating items by considering preservation technology. They considered price and stock sensitive demand. They observed that discount policy helps the business organization for smooth running. Hishamuddin et al. [15] presented an improved preservation technology for integrated inventory model for deteriorating items. Shah et al. [29] presented an inventory problem by considering preservation technology cost (PTC) to reduce the rate of deterioration. They considered shortages that were partially backlogged. They obtained the optimal value of positive inventory time, preservation technology cost, and cycle time.

Present research investigates the economic production quantity model by considering the investment in system improvement programme. Investment is also made to control the rate of deterioration. Two level trade credit policies are adopted in the present model. Customers' demand is considered as the function of credit period, selling price, and promotional efforts made by the sale's team. Effect of inflation is also considered in the present research.

Remaining part of the study is organized as follows: Assumptions and notations are illustrated in section 2. Mathematical formulation is illustrated in section 3 and whole of study is carried out under the effect of inflation. Section 4 contains the solution algorithm to obtain the optimal values of the decision variables. Section 5 presents the numerical analysis with sensitivity analysis are presented. Section 6 contains the concluding remarks with future extension of the present work.

2. Assumptions and Notations

In this section, we present the basic assumptions and notations which are used to develop the mathematical model for production system.

2.1 Notations

P	Production rate	units/unit time
M	Credit period offered by the supplier	unit time
N	Credit period offered by the manufacturer	unit time
I_e	Interest earned by the manufacturer	\$/\$/ unit time
I_p	Interest paid by the manufacturer	\$/\$/ unit time
$\theta(P,\varphi)$	$(=(a+bP)e^{-c\varphi})$ where $0 \le \theta(P,\varphi) \le 1$ where a,b, and c are constant	
φ	Capital invested to reduce deterioration rate	\$/ unit time
$\varepsilon(\omega)$	Fraction of defective production rate	
ω	Capital invested to reduce fraction of defective production rate	\$/ unit time
C_s	Setup cost	\$/ unit time
C_h	Holding cost	\$/ unit / unit time
$C_m(P)$	$\left(=m_1+\frac{m_2}{p}+m_3P\right)$ Manufacturing cost, Where m_1 is the fixed material cost per unit item, $\frac{m_2}{p}$ is the labor/energy cost, m_3P is tool/die cost	\$/ unit / unit time
$C_p(d_1,\vartheta)$	$(=K(\vartheta-1)^2 d_1^{\varepsilon})$ Investment for promotional activity where K and ε are constant	\$ frequency/unit time
s	Selling price	\$/ unit
t_p	Production period	unit time
	T Cycle length	unit time
$D(s,\rho,M)$	$(=d_1(\vartheta+\alpha_1 M)-\alpha_2 s)$ Demand d_1,α_1, and α_2 are constant	units/unit time
ϑ	Coefficient of enhancement in basic demand due to promotional activity made by the decision-maker	
k	Inflation rate	%

2.2 Assumptions

1. Production process is imperfect and the fraction of defective production rate can be reduced with the help of investment in system.
2. Deterioration rate depends on production rate can be reduced by additional amount.
3. Planning horizon of production system is infinite.
4. Two-level of trade credit is considered.
5. Shortages are not permitted during the planning horizon.
6. Consumers demand depends on selling price, promo-

tional efforts made by decision-maker and credit period offered by supplier.

7. Investment in promotional activity is an increases function of basic demand and promotional activity.

3. Formulation of Mathematical Model of Manufacturer

Production process starts at $'t = o$ ' at the rate ^4P ' upto the time t_p inventory level of the manufacturer rises at the rate $P - D(s,\rho,M)$. At the point $t = t_p$, production process stops and afterward inventory level of manufacturer decreases at the rate $D(s,\rho,M)$. At $t = T$, inventory level becomes zero. Further, it is considered that manufacturing rate is greater than the demand rate. Fig.1 illustrate the inventory level of manufacturer. Thus, inventory level of manufacturer at any

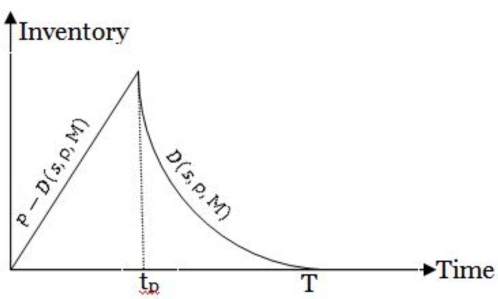

Figure 1. Graphical Representations of Manufacturer's Inventory Level

time $t\varepsilon\,(0,t_p]$ is

$$\frac{dI(t)}{dt} + \theta(P,\varphi)I(t) = \varepsilon(\omega)P - D(s,\rho,M), t\varepsilon\,(0,t_p] \tag{3.1}$$

subjected to the condition $I(0) = 0$. During the time interval $[t_p,T]$, inventory level of the manufacturer is

$$\frac{dI(t)}{dt} + \theta(P,\varphi)I(t) = -D(s,\rho,M), t\varepsilon\,[t_p,T] \tag{3.2}$$

subjected to the condition $I(0) = 0$.
From equations (3.1) and (3.2) we have

$$I(t) = \begin{cases} \frac{p\in(\omega)-D(s,\rho,M)}{\theta(p,\varphi)}\left(1-e^{-\theta(p,\varphi)t}\right), t\varepsilon\,(0,t_p] \\ \frac{D(s,p,M)}{\theta(p,\varphi)}\left(e^{-\theta(p,\varphi)(T-t)}-1\right), t\in[t_p,T] \end{cases} \tag{3.3}$$

Due to the equation of continuity at t_p, we have

$$\frac{\varepsilon(\omega)-D(s,\rho,M)}{\theta(P,\varphi)}\left(1-e^{-\theta(p,\varphi)t_p}\right) = \frac{D(s,p,M)}{\theta(p,\varphi)}\left(e^{-\theta(p,\varphi)(T-t_p)}-1\right) \Rightarrow T = t_p + \frac{1}{\theta(p,\varphi)} \log\left[1+\frac{\varepsilon(\omega)P-D(s,\rho,M)}{D(s,\rho,M)}\left(1-e^{-\theta(p,\varphi)t_p}\right)\right] \tag{3.4}$$

Now, we evaluate different costs associated with manufacturer step by step under the inflationary environment.
Revenue: $D(s,\rho,M)$ is the demand during the period $[0,T]$. Thus, total revenue generated by the manufacturer is

$$s\int_0^T D(s,\rho,M)e^{-kt}dt = \frac{sD(s,p,M)}{k}\left(1-e^{-kT}\right)$$

Setup Cost: It is paid lump sum so setup cost under the inflationary environment is $C_s e^{-kT}$.
Manufacturing Cost: Production process is performed upto the time to at the rate $\varepsilon(\omega)$. So, manufacturing cost of the item under the inflationary environment is

$$C_m(P)\int_0^{t_p} Pe^{-kt}dt = \frac{PC_m(P)}{k}\left(1-e^{-kt_p}\right)$$

Holding Cost: Item is hold by the manufacturer upto the time T. So, holding cost of the stock under the inflationary environment is

$$\begin{aligned} &C_h\int_0^{t_p} I(t)e^{-kt}dt + C_h\int_{t_p}^T I(t)e^{-kt}dt \\ &= C_h\left[\frac{\varepsilon(\omega)P-D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right. \\ &\left.+\frac{1}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p}-1\right)\right\} \\ &+\frac{D(s,p,M)}{\theta(p,\varphi)}\left\{\frac{e^{kT}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi)t_p)}-e^{-(k+\theta(p,\varphi)T)}\right)\right. \\ &\left.\left.+\frac{1}{k}\left(e^{-kT}-e^{-kt_p}\right)\right\}\right] \end{aligned}$$

Promotional Cost: Promotional cost is made by the manufacturer one time at the end of the cycle. So the investment in promotional activity is $C_p\,(d_1,\vartheta)\,e^{-kT}$.
System Improvement Cost: With some extra investment, fraction of defective production rate can be reduced. So, investment in system improvement cost is ωe^{-kT}.
Preservation Technology Cost: By investing some extra amount in preservation technology, rate of deterioration can be effectively reduced. So, investment in adopting the preservation technology mechanism under the inflationary environment is φe^{-kT}.

In the present model, it is assumed that manufacturer received trade credit period 'M' from supplier and in turn they provide the trade credit period ' N ' to his/her customers. Thus, following two conditions arises according the values of M and N :

(i) $M > N$ i.e., manufacturer trade credit (MTC) is greater than the customer trade credit (CTC)

(ii) $M < N$ i.e., manufacturer trade credit (MTC) is shorter than the customer trade credit (CTC)

Scenario-1: $M > N$
Under this scenario, following cases arises:
Case-1: $M \leq t_p$
In this situation, manufacturer trade credit (MTC) period is

shorter than the production period. In this situation, manufacturer paid the interest to the supplier and received the interest from the customers. These, can be calculated as follows: Interest paid by the manufacturer under the inflationary environment is

$$
\begin{aligned}
&= C_m(P)I_p\left[\int_M^{t_p} I(T)e^{-kt}dt + \int_{t_p}^{T} I(T)e^{-kt}dt\right] \\
&= C_m(P)I_p\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(p,\varphi)}\left\{\frac{1}{k}\left(e^{-kM} - e^{-kt_p}\right)\right.\right. \\
&+ \left.\frac{1}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi))M}\right)\right\} \\
&+ \frac{D(s,\rho,M)}{\theta(p,\varphi)}\left\{\frac{\varepsilon^{\theta(P,\varphi)T}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi))T}\right)\right. \\
&+ \left.\left.\frac{1}{r}\left(e^{-kT} - e^{-kt_p}\right)\right\}\right]
\end{aligned}
$$

Interest earned by the manufacturer under the inflationary environment is

$$
\begin{aligned}
&= sI_e\left[\int_0^{N} D(s,\rho,M)te^{-kt}dt + \int_N^{M} D(s,\rho,M)te^{-kt}dt\right] \\
&= \frac{sl_\varepsilon D(s,\rho,M)}{k^2}\left[1-(1+Mk)e^{-Mk}\right]
\end{aligned}
$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPT), interest paid from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$
\begin{aligned}
TP_{11} &= \frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right) + \frac{sI_\varepsilon D(s,p,M)}{k^2} \\
&\times\left[1-(1+Mk)e^{-Mk}\right] - C_s e^{-kT} - \frac{PC_m(P)}{k}(1-e^{-kt_p}) \\
&- C_h\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right. \\
&+ \left.\frac{1}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - 1\right)\right\} \\
&+ \frac{D(s,p,M)}{\theta(p,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi T)}\right)\right. \\
&+ \left.\left.\frac{1}{k}\left(e^{-k\,T} - e^{-kt_p}\right)\right\}\right] - C_p\left(d_1,\vartheta\right)e^{-kT} - \omega e^{-kT} - \varphi e^{-kT} \\
&- C_m(P)I_p\left[\frac{\varepsilon(\omega)P - D(s,p,M)}{\theta(p,\varphi)}\left\{\frac{1}{k}\left(e^{-kM} - e^{-kt_p}\right)\right.\right. \\
&+ \left.\frac{1}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi))M}\right)\right\} \\
&\times\frac{D(s,\rho,M)}{\theta(p,\varphi)}\left\{\frac{e^{kT}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi))T}\right)\right. \\
&+ \left.\left.\frac{1}{r}\left(e^{-kT} - e^{-kt_p}\right)\right\}\right] \qquad (3.5)
\end{aligned}
$$

Problem-1: Objective of the present problem is to maximize the objective function i.e., total profit per unit time of the manufacturer. Thus,

$$
\text{Maximize } \pi_{11}\left(s,t_p,P\right) = \frac{TP_{11}\left(s,t_p,p\right)}{T} \qquad (3.6)
$$

under the condition $0 < t_p \le T$ and $0 < M \le t_p$.

Case -2: $t_p \le M \le T$

In this situation, manufacturer trade credit (MTC) period is greater than the production period. In this situation, manufacturer paid the interest to the supplier and received the interest from the customers. These, can be determined as follows: Interest paid by the manufacturer under the inflationary environment is

$$
\begin{aligned}
&= C_m(P)I_p\int_M^{T} I(T)e^{-kt}dt \\
&= C_m(P)I_p\left[\frac{D(s,p,M)}{\theta(p,\varphi)}\left\{\frac{\theta^{\theta(P,\varphi)T}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))M}\right.\right.\right. \\
&\left.\left.\left. - e^{-(k+\theta(p,\varphi))T}\right) + \frac{1}{r}\left(e^{-kT} - e^{-kM}\right)\right\}\right]
\end{aligned}
$$

Interest earned by the manufacturer under the inflationary environment is

$$
\begin{aligned}
&= sI_e\left[\int_0^{N} D(s,\rho,M)te^{-kt}dt + \int_N^{M} D(s,\rho,M)te^{-kt}dt\right] \\
&= \frac{sI_\varepsilon D(s,\rho,M)}{u^2}\left[1-(1+Mk)e^{-Mk}\right]
\end{aligned}
$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPTC), interest paid from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$
\begin{aligned}
TP_{12} &= \frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right) + \frac{sI_\varepsilon D(s,p,M)}{k^2} \\
&\times\left[1-(1+Mk)e^{-Mk}\right] - C_s e^{-kT} - \frac{PC_m(p)}{k}(1-e^{-kt_p}) \\
&- C_h\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(p,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right. \\
&+ \left.\frac{1}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - 1\right)\right\} \\
&+ \frac{D(s,\rho,M)}{\theta(p,\varphi)}\left\{\frac{e^{kT}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))t_p} - e^{-(k+\theta(p,\varphi T)}\right)\right. \\
&+ \left.\left.\frac{1}{k}\left(e^{-k\,T} - e^{-kt_p}\right)\right\}\right] - C_p\left(d_1,\vartheta\right)e^{-kT} - \omega e^{-kT} - \varphi e^{-kT} \\
&- C_m(P)I_p\left[\frac{D(s,p,M)}{\theta(p,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(p,\varphi)}\left(e^{-(k+\theta(p,\varphi))M}\right.\right.\right. \\
&\left.\left.\left. - e^{-(k+\theta(p,\varphi))T}\right) + \frac{1}{r}\left(e^{-kT} - e^{-kM}\right)\right\}\right] \qquad (3.7)
\end{aligned}
$$

Problem-2: Objective of the present problem is to maximize the objective function. i.e., total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{12}(s,t_p,P)=\frac{TP_{12}(s,t_p,p)}{\tau} \tag{3.8}$$

under the condition $0<t_p\leq T$ and $0<t_p<M\leq T$.

Case -3: $N<T\leq M$

In this situtation, manufacturer's trade credit (MTC) period is more than the cycle length (T). In this situation, producer need not to pay any interest to the supplier and but received the interest from the customers. These, can be derived as follows: Interest paid by the manufacturer under the inflationary environment is zero. Interest earned by the manufacturer under the inflationary environment is

$$\begin{aligned}&=sI_e\left[\int_0^N D(s,\rho,M)te^{-kt}dt+\int_N^T D(s,\rho,M)te^{-kt}dt\right.\\&\quad\left.+\int_T^M D(s,\rho,M)Te^{-kt}\right]\\&=\frac{sI_\varepsilon D(s,\rho,M)}{k^2}\left[1-kTe^{-Mk}-e^{-Tk}\right]\end{aligned}$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPTC), interest paid from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$\begin{aligned}TP_{13}&=\frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right)+\frac{sI_eD(s,\rho,M)}{k^2}\\&\times\left[1-kTe^{-Mk}-e^{-Tk}\right]-C_se^{-kT}-\frac{PC_m(P)}{k}(1-e^{-kt_p})\\&-C_h\left[\frac{\varepsilon(\omega)P-D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right.\\&\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p}-1\right)\right\}\\&+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{kT}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p)}-e^{-(k+\theta(P,\varphi)T)}\right)\right.\\&\left.\left.+\frac{1}{k}\left(e^{-k\,T}-e^{-kt_p}\right)\right\}\right]-C_p(d_1,\vartheta)e^{-kT}-\omega e^{-kT}-\varphi e^{-kT}\end{aligned} \tag{3.9}$$

Problem-3: Objective of the present problem is to maximize the total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{13}(s,t_p,P)=\frac{TP_{13}(s,t_p,P)}{T} \tag{3.10}$$

under the condition $0<t_p\leq T$ and $0<N<T\leq M$ Case $-4: T\leq N<M$.

In this situation, customer as well as manufacturer trade credit (MTC) period is greater than the cycle length (T). In this situation, producer need not to pay any interest to the supplier and but they received the interest from the customers. These, can be derived as follows:

Interest paid by the manufacturer under the inflationary environment is zero. Interest earned by the manufacturer under the inflationary environment is

$$\begin{aligned}&=SI_e\left[\int_0^T D(s,\rho,M)te^{-kt}dt+\int_T^N D(s,\rho,M)Te^{-kt}dt\right.\\&\quad\left.+\int_N^M D(s,\rho,M)Te^{-kt}\right]\\&=\frac{sI_eD(s,\rho,M)}{k^2}\left[1-N(k+1)e^{-Nk}+kT\left(e^{-Tk}-e^{-Nk}\right)\right.\\&\quad\left.+kT\left(e^{-Nk}-e^{-Mk}\right)\right]\end{aligned}$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPTC), interest paid (IP) from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$\begin{aligned}TP_{14}&=\frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right)\\&+\frac{sI_eD(s,\rho,M)}{k^2}\left[1-N(k+1)e^{-Nk}+kT\left(e^{-Tk}-e^{-Nk}\right)\right.\\&\left.+kT\left(e^{-Nk}-e^{-Mk}\right)\right]-C_se^{-kT}-\frac{PC_m(P)}{k}\left(1-e^{-kt_p}\right)\\&-C_h\left[\frac{\varepsilon(\omega)P-D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right.\\&\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p}-1\right)\right\}\\&+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{kT}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p)}-e^{-(k+\theta(P,\varphi)T)}\right)\right.\\&\left.\left.+\frac{1}{k}\left(e^{-kT}-e^{-kt_p}\right)\right\}\right]\\&-C_p(d_1,\vartheta)e^{-kT}-\omega e^{-kT}-\varphi e^{-kT}\end{aligned} \tag{3.11}$$

Problem-4: Objective of the present problem is to maximize the objective function i.e., total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{14}(s,t_p,P)=\frac{TP_{14}(s,t_p,P)}{T} \tag{3.12}$$

under the condition $0<t_p\leq T$ and $T\leq M<N$.

Scenario-2: $M<N$

Under this scenario, following cases arises:

Case-5: $M<N\leq t_p$

In this situation, customer trade credit (CTC) period is shorter than the production period. In this situation, manufacturer paid the interest to the supplier and received the interest from the customers. These, can be calculated as follows: Interest paid by the manufacturer under the inflationary environment is

$$= C_m(P)I_p\left[\int_M^{t_p} I(T)e^{-kt}dt + \int_{t_p}^{T} I(T)e^{-kt}dt\right]$$
$$= C_m(P)I_p\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(e^{-kM} - e^{-kt_p}\right)\right.\right.$$
$$\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - e^{-(k+\theta(p,\varphi))M}\right)\right\}$$
$$+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - e^{-(k+\theta(P,\varphi))T}\right)\right.$$
$$\left.\left.+\frac{1}{r}\left(e^{-kT} - e^{-kt_p}\right)\right\}\right]$$

Interest earned by the manufacturer under the inflationary environment is

$$= SI_e\int_0^M D(s,\rho,M)te^{-kt}dt$$
$$= \frac{sI_eD(s,\rho,M)}{k^2}\left[1-(1+Mk)e^{-Mk}\right]$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPTC), interest paid (IP) from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$TP_{25} = \frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right) + \frac{sI_eD(s,\rho,M)}{k^2}$$
$$\times\left[1-(1+Mk)e^{-Mk}\right] - C_se^{-kT} - \frac{PC_m(P)}{k}(1-e^{-kt_p})$$
$$-C_h\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right.$$
$$\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - 1\right)\right\}$$
$$+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p)} - e^{-(k+\theta(P,\varphi)T)}\right)\right.$$
$$\left.\left.+\frac{1}{k}\left(e^{-k\,T} - e^{-kt_p}\right)\right\}\right] - C_p(d_1,\vartheta)e^{-kT} - \omega e^{-kT} - \varphi e^{-kT}$$
$$-C_m(P)I_p\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(e^{-kM} - e^{-kt_p}\right)\right.\right.$$
$$\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - e^{-(k+\theta(P,\varphi))M}\right)\right\}$$
$$+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{kT}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - e^{-(k+\theta(P,\varphi))T}\right)\right.$$
$$\left.\left.+\frac{1}{r}\left(e^{-kT} - e^{-kt_p}\right)\right\}\right] \tag{3.13}$$

Problem-5: Objective of the present problem is to maximize the objective function i.e., total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{25}(s,t_p,P) = \frac{TP_{25}(s,t_p,P)}{T} \tag{3.14}$$

under the condition $0 < t_p \le T$ and $0 < M < N \le t_p$.

Case-6: $t_p < M < N \le T$.

In this situation, manufacturer as well as customer trade credit period is greater than the production period. In this situation, manufacturer paid the interest to the supplier and received the interest from the customers. These, can be derived as follows:

Interest paid by the manufacturer under the inflationary environment is

$$= C_m(P)I_p\int_M^T I(T)e^{-kt}dt$$
$$= C_m(P)I_p\left[\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))M}\right.\right.\right.$$
$$\left.\left.\left.-e^{-(k+\theta(P,\varphi))T}\right) + \frac{1}{r}\left(e^{-kT} - e^{-kM}\right)\right\}\right]$$

Interest earned by the manufacturer under the inflationary environment is

$$= SI_e\int_0^M D(s,\rho,M)te^{-kt}dt$$
$$= \frac{sI_eD(s,\rho,M)}{k^2}\left[1-(1+Mk)e^{-Mk}\right]$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPA), investment in system improvement cost (ISIC), investment in preservation technology cost (IPT), interest paid (IP)from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$TP_{26} = \frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right) + \frac{sI_eD(s,\rho,M)}{k^2}$$
$$\times\left[1-(1+Mk)e^{-Mk}\right] - C_se^{-kT} - \frac{PC_m(P)}{k}(1-e^{-kt_p})$$
$$-C_h\left[\frac{\varepsilon(\omega)P - D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right.$$
$$\left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p} - 1\right)\right\}$$
$$+\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{kT}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p)} - e^{-(k+\theta(P,\varphi)T)}\right)\right.$$
$$\left.\left.+\frac{1}{k}\left(e^{-kT} - e^{-kt_p}\right)\right\}\right] - C_p(d_1,\vartheta)e^{-kT} - \omega e^{-kT} - \varphi e^{-kT}$$
$$-C_m(P)I_p\left[\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{\theta(P,\varphi)T}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))M}\right.\right.\right.$$
$$\left.\left.\left.-e^{-(k+\theta(P,\varphi))T}\right) + \frac{1}{r}\left(e^{-kT} - e^{-kM}\right)\right\}\right] \tag{3.15}$$

Problem-6: Objective of the present problem is to maximize the objective function i.e., total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{26}(s,t_p,P) = \frac{TP_{26}(s,t_p,P)}{T} \tag{3.16}$$

under the condition $0 < t_p \leq T$ and $0 < t_p < M < N \leq T$.

Case-7: $T \leq M < N$.

In this situation, manufacturer as well customer trade credit period is more than the cycle length (T). In this scenario, manufacturer nped not to pay any interest to the supplier and but received the interest from the customers. These, can be derived as follows:

Interest paid by the manufacturer under the inflationary environment is zero. Interest earned by the manufacturer under the inflationarv environment is

$$= SI_e\left[\int_0^T D(s,\rho,M)te^{-kt}dt + \int_T^M D(s,\rho,M)Te^{-kt}\right]$$
$$= \frac{sI_eD(s,\rho,M)}{k^2}\left[1-e^{-Tk}-kTe^{-Mk}\right]$$

We subtract the sum of setup cost (SC), manufacturing cost (MC), holding cost (HC), investment in promotional activity (IPT), investment in system improvement cost (ISIC), investment in preservation technology cost (IPTC), interest paid(IP) from the sum of revenue (R) and interest earn (IE) to get the total profit of the manufacturer. Thus, profit of the manufacturer is

$$\begin{aligned} TP_{13} = {} & \frac{sD(s,\rho,M)}{k}\left(1-e^{-kT}\right)+\frac{sI_eD(s,\rho,M)}{k^2} \\ & \times\left[1-e^{-Tk}-kTe^{-Mk}\right]-C_se^{-kT}-\frac{PC_m(P)}{k}(1-e^{-kt_p}) \\ & -C_h\left[\frac{\varepsilon(\omega)P-D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{1}{k}\left(1-e^{-kt_p}\right)\right.\right. \\ & \left.+\frac{1}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p}-1\right)\right\} \\ & +\frac{D(s,\rho,M)}{\theta(P,\varphi)}\left\{\frac{e^{kT}}{k+\theta(P,\varphi)}\left(e^{-(k+\theta(P,\varphi))t_p)}-e^{-(k+\theta(P,\varphi)T)}\right)\right. \\ & \left.\left.+\frac{1}{k}\left(e^{-k\,T}-e^{-ktp}\right)\right\}\right]-C_p(d_1,\vartheta)e^{-kT}-\omega e^{-kT}-\varphi e^{-kT} \end{aligned} \tag{3.17}$$

Problem-7: Objective of the present problem is to maximize the objective function i.e., total profit per unit time of the manufacturer. Thus,

$$\text{Maximize } \pi_{27}(s,t_p,P) = \frac{TP_{27}(s,t_p,P)}{T} \tag{3.18}$$

under the condition $0 < t_p \leq T$ and $0 < T \leq M < N$.

4. Solution Methodology

In all the cases, decision variables are s, tp, and P. In each case, objective function is the non linear function of s, tp, and P. So, closed form solution is not possible. We adopted the following calculus based search algorithm to optimize the objective function. It is assumed that the objective function satisfies the first order condition, i.e., the necessary conditions, which are as follows:

$$\frac{\partial \pi_{ij}}{\partial s} = 0;\ \frac{\partial \pi_{ij}}{\partial t_p} = 0;\ \frac{\partial \pi_{ij}}{\partial P} = 0. \tag{4.1}$$

Step 1. $t_p = 0$, and $P = 0$, and find s^* from equation (4.1).
Step 2. Find t_p^* from equation (4.2), using the revised values of Step 1.
Step 3. Find P^* from equation (4.3), using the revised values of Step 2.
Step 4. Apply all the above mentioned steps using the revised values of s^*, t_p^*, and P^*. Repeat this process until the values of s^*, t_p^*, and P^* remain unchanged.
Step 5. Evaluate the different principal minors of Hessian matix of π_{ij} i.e.,

$$\begin{bmatrix} \frac{\partial^2\pi_{ij}}{\partial s^2} & \frac{\partial^2\pi_{ij}}{\partial s\partial t_p} & \frac{\partial^2\pi_{ij}}{\partial s\partial P} \\ \frac{\partial^2\pi_{ij}}{\partial t_p\partial s} & \frac{\partial^2\pi_{ij}}{\partial t_p^2} & \frac{\partial^2\pi_{ij}}{\partial t_p\partial P} \\ \frac{\partial^2\pi_{ij}}{\partial P\partial s} & \frac{\partial^2\pi_{ij}}{\partial P\partial t_p} & \frac{\partial^2\pi_{ij}}{\partial P^2} \end{bmatrix}$$

at the point $\left(s^*, t_p^*, P^*\right)$ and if it is found to be negative semi-definite. Then, $\pi_{ij}\left(s^*, t_p^*, P^*\right)$ is the optimal value of the objective function. Whole of the process is carried out with the help of software MATHEMATICA 5.2.

5. Numerical Example with Sensitivity Analysis

In this section, first we perform the numerical analysis for the different cases explained in section 3. For this, data has been taken from the literature which is as follows:

$C_s = 2OO, Ch = 4, I_e = 0.1O, I_p = 0.14, a = 0.04,\ b = 0.0001,$
$c = 0.0075, \varphi = 30, \sigma = 0.06, \tau = 0.0079, \omega = 33,$
$m_2 = 400, m_1 = 47, m_3 = 0.1, \vartheta = 1.43, K = 4,$
$d_1 = 150, \varepsilon = 1.21., \alpha_1 = 1.2, \alpha_2 = 0.1, k = 0.06$

5.1 Sensitivity Analysis (SA)

In this section, sensitivity analysis is carried out which is helpful to analyze the effect of key parameters on the optimal value of production time, cycle length, production rate, selling price, and total profit of the system.

a. Sensitivity analysis (SA) with respect to material cost:

Fig.2 shows the effect of material cost (m_1) on the optimal solution and on the profit of the system. Increase in the material, increases the production cost and hence the profit of the system decreases significantly. This suggests that production run time decreases should be decrease due to the high material cost.

Table 1. Optimal Solution for Different Cases

	Case 1 ($i=1$, $j=1$) M=0.10 N=0.04	Case 2 ($i=1$, $j=2$) M=0.32 N=0.29	Case 3 ($i=1$, $j=3$) M=0.62 . N=0.49	Case 4 ($i=1$, $j=4$) M=0.71 N=0.65	Case 5 ($i=2$, $j=5$) M=0.22 N=0.28	Case 6 ($i=2$, $j=6$) M=0.32 N=0.39	Case 7 ($i=2$, $j=6$) M=0.38 N=0.43
Production Time (t (t_p)	0.2712	0.2589	0.2621	0.2891	0.2701	0.2689	0.1769
Cycle Length (T)	0.5895	0.4989	0.5589	0.6231	0.6092	0.5825	0.3891
Production Rate (P)	128.234	128.92	128.02	128.45	128.39	128.90	128.07
Selling Price (s)	154.98	154.09	154.48	154.78	154.91	154.49	154.78
Total Profit $(\pi_{ij}(t_p, P, s))$	6882.48	6974.76	6989.21	6998.23	6823.47	6892.01	6936.92

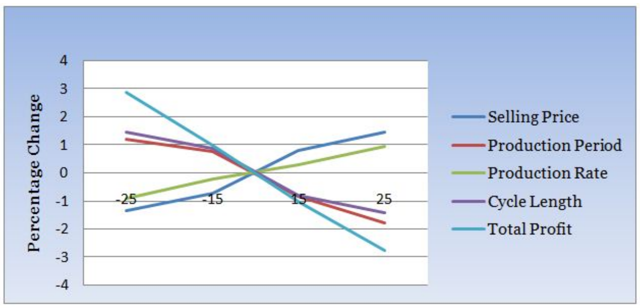

Figure 2. Effect of Material Cost

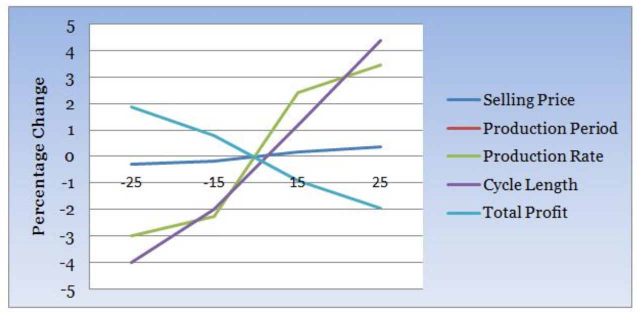

Figure 4. Effect of Deterioration Rate

b. Sensitivity analysis (SA) with respect to holding cost:

It is observed that holding cost is positively correlated with selling price where negatively correlated with production time, cycle length, production rate, and total profit of the system. Due to higher holding cost, it is advisable to decrease the production rate and production time. Further, it is also suggested that increase the selling price as the holding cost increases.

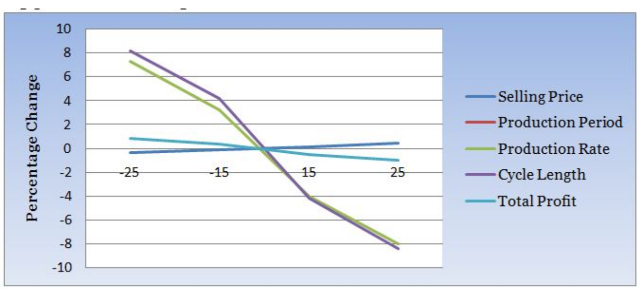

Figure 3. Effect of Holding Cost

c. Sensitivity analysis (SA) with respect to deterioration rate:

We observe that rate of deterioration is positively correlated with production period, production rate, and cycle length where it is negatively correlated with selling price and profit of the system. Thus as the rate of deterioration increases, more production is required to meet the demand and hence the production run time increase.

d. Sensitive analysis (SA) with respect to inflation:

Fig.5 shows that there is observable effect of inflation on the total profit of system and on the decision variables. Total profit of the system increases as the rate of inflation increases. All of the decision variables are negatively correlated.

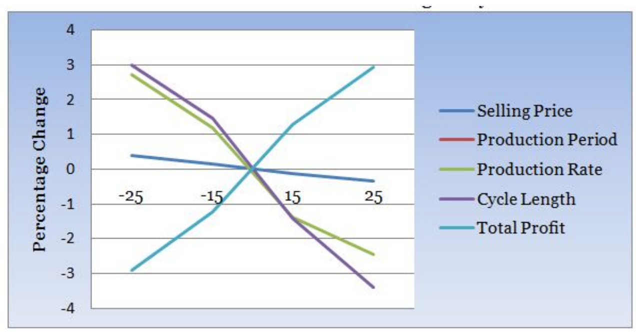

Figure 5. Effect of Inflation

e. Sensitivity analysis (SA) with respect to setup cost

Fig.6 shows the effect of setup cost on the optimal values of crucial i.e., decision variables and on the total profit (TP) of the system. Results indicate that decision variables are positively correlated to setup cost while total profit of the system is negatively correlated.

6. Concluding Remark and Future Extension

Flexible economic production quantity (EPQ) model is developed here for deteriorating products. Combined effect of trade credit, selling price and promotional activity is considered on

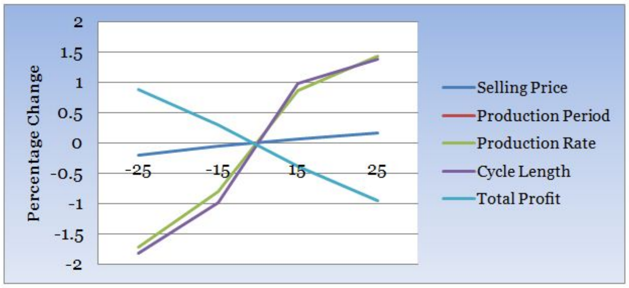

Figure 6. Effect of Setup Cost

the ccustomers' demand. Investment is made to reduce the rate of deterioration and improve the production process. Study is carried out under the inflationary environment. Sensitivity analysis is performed with respect to important parameters. Obtained result indicates that profit of the production system increases as the trade credit period increases. Also it is found that total profit of the manufacturing system is highly influenced by material cost and inflation. But this effect is in opposite direction. High inflation results high profit while profit decreases due to increase in material cost. In the situation of high material cost, it is advisable for the decision maker to lower down the production rate and decrease the production run period. For future research, present work can be investigated in fuzzy environment by considering imprecise costs. Considering shortages and partial backlogging is also one of the fruitful extensions of the present work.

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