Malaya Journal of Matematik, Vol. 9, No. 1, 461-464, 2021

https://doi.org/10.26637/MJM0901/0077

Some cordial labeling for commuting graph of a subset of the dihedral group

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Abstract

The commuting graph of a group G, denoted by $\Gamma(G)$, is a graph whose vertices are all the elements of G and two distinct vertices are joined by an edge whenever they commute. For an abelian group, the commuting graph becomes complete, so we shall consider the non abelian group for cordial labeling of commuting graph. In this paper, we examine sum cordial graph, signed product cordial graph, divisor cordial graph for the commuting graph of a subset H of dihedral group D_{2n}.

Keywords

Commuting graph, Dihedral group, Sum cordial, Signed product cordial, Divisor cordial.

AMS Subject Classification

05Cxx, 05C78, 05C90, 20Dxx, 94C15.

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Article History: Received **11** November **2020**; Accepted **09** February **2021**

Contents

1. Introduction

The exploration of algebraic structures, using the properties of graphs has been ongoing investigation topics for the last twenty years. The field of graph theory play a vital role in various fields such as graph labeling [5], graph coloring, topological indices [6–8], fuzzy graph theory [10], graph networks. Literally thousands of research articles have appeared in graph theory and the most relevant ones are graph labeling. Labeled graphs utilized in the models for a broad range of applications such as: coding theory, X-ray crystallography, radar, database management, astronomy, circuit design, communication networks addressing and models for constraint programming over finite domain. For all other terminology and notations we follow Harary [3]. Cordial graphs were first introduced by Cahit [1] as a weeker version of both graceful graphs and harmonious graphs.

J. Shiama, established an idea of Sum cordial [9] in 2012. Jayapal Baskar Babujee, Shobana Loganathan, introduced the Signed product cordial [4] in 2011. R. Varatharajan, S. Navaneethakrishnan, K. Nagarajan, introduced the Divisor cordial [13] in 2011. In this paper, we investigate sum cordial, signed product cordial, divisor cordial labeling for commuting graph of the subset of dihedral group D_{2n}.

In Section 2, we recall the basic definitions of cordial labeling, sum cordial labeling, signed product cordial, divisor cordial labeing, dihedral group, commuting graph etc.,

In Section 3, we proved that the commuting graph of a subset of dihedral group are sum cordial, signed product cordial and divisor cordial labeling.

2. Preliminaries

In this section, we discuss the preliminary definition of commuting graph, dihedral group, cordial labeling, sum cordial labeling, signed product labeling, divisor cordial labeling.

Definition 2.1. *[12] Let G be a finite group. The **commuting graph** $\Gamma(G)$ whose vertices are all elements of G and the vertices are joined by an edge whenever they commute.*

Definition 2.2. *Let G be a finite group and H is a subset of G. The commuting graph of the subset H of the group G is* *the graph $C(G,H)$ whose vertices are the elements of H and the vertices are joined by edge iff $xy = yx$ for all $x, y \in H$.*

Definition 2.3. *[2] The symmetry group of regular n polygon with rotation and reflection is called* ***dihedral group*** *which is denoted by D_{2n}. It contains $2n$ elements. The group representation is*

$$D_{2n} = \{r, s | r^n = s^2 = (sr)^2 = 1\}$$

Definition 2.4. *[5] Let $G = (V,E)$ be a graph. A mapping $f : V(G) \to \{0,1\}$ is called* ***binary vertex labeling*** *of G and $f(v)$ is called the label of the vertex v of G under f.*

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ be the number of vertices of G having label 0 and 1 respectively under f and $e_f(0)$, $e_f(1)$ be the number of edges having label 0 and 1 respectively under f^*.

Definition 2.5. *[1] A vertex labeling $f : V(G) \to \{0,1\}$ and the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(uv) = |f(u) - f(v)|$. Such labeling is called* ***cordial labeling*** *if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.*

Definition 2.6. *[9] A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \to \{0,1\}$ is defined by $f^*(uv) = (f(u) + f(v))(mod 2)$ is named* ***sum cordial labeling*** *if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.A graph G is sum cordial if it admits sum cordial labeling.*

Definition 2.7. *[4] A vertex labeling of graph G, $f : V(G) \to \{-1,1\}$ with induced edge labeling $f^* : E(G) \to \{-1,1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a* ***signed product cordial labeling*** *if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$. A graph G is signed product cordial if it admits signed product cordial labeling.*

Definition 2.8. *[5] A* ***Divisor cordial labeling*** *of a graph G with the vertex set V is a bijection $f : V \to \{1,2,...,|V|\}$ such that if each edge uv is assigned the label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and 0 otherwise. A graph G is divisor cordial if it admits divisor cordial labeling.*

Theorem 2.9. *[11] Let G be a dihedral group D_{2n} for $n \geq 3$ and $C(D_{2n},H)$ be a commuting graph of a subset H of D_{2n}, then $C(D_{2n},H) = K_n$ iff $H = \{r, r^2, ..., r^{n-1}\}$*

Theorem 2.10. *[11] Let $C(D_{2n},H)$ be a commuting graph of a subset H of dihedral group D_{2n}. Let $n \geq 3$ be an odd integer then $C(D_{2n},H) = K_{1,n}$ iff $H = \{e, s, sr, sr^2, ..., sr^{n-1}\}$*

3. Main Results

In this section, we discuss some cordial labeling of the commuting graph of the subset of the dihedral group $D_{2n} = \{r, s | r^n = s^2 = (sr)^2 = 1\}$ for $n \geq 3$ is an integer. Let $H = \{e, s, sr, sr^2 ..., sr^{n-1}\}$ are the subset of D_{2n}.

Theorem 3.1. *$C(D_{2n},H)$ is the sum cordial if n is odd for every $n \geq 3$.*

Proof. Let $G = D_{2n}$ be a dihedral group and $H = \{e, s, sr, sr^2, ..., sr^{n-1}\}$ be the subset of the dihedral group.

Let $V(C(D_{2n},H)) = \{v_0, v_1, v_2, ..., v_n\}$ $= \{e, s, sr, sr^2, ..., sr^{n-1}\}$

$$E(C(D_{2n},H)) = \{v_0 v_i | 1 \leq i \leq n\}$$

Now, Define $f : V(C(D_{2n},H)) \to \{0,1\}$ as

$$f(v_i) = \begin{cases} 1, & i \text{ is odd} \\ 0, & otherwise \end{cases} \quad \text{where } 0 \leq i \leq n$$

Clearly, $v_f(0) = \frac{n+1}{2} = v_f(1)$

The induced edge labeling $f^* : E(C(D_{2n},H)) \to \{0,1\}$ is given by $f^*(v_0 v_i) = [f(v_0) + f(v_i)](mod 2)$ where $1 \leq i \leq n$

we have, $e_f(0) = \frac{n-1}{2}$ and $e_f(1) = \frac{n+1}{2}$

Thus, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Therefore, $C(D_{2n},H)$ satisfy the sum cordial labeling.

$\therefore C(D_{2n},H)$ is the sum cordial.

□

Example 3.2. *$C(D_{14},H)$ is the sum cordial.*

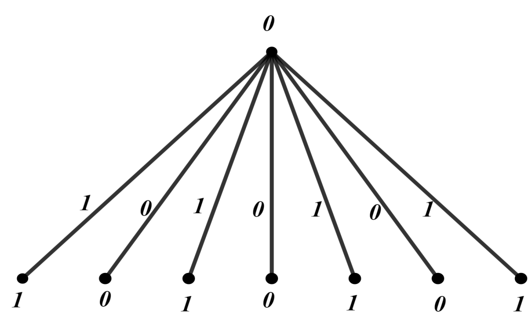

Figure 1. $C(D_{14},H)$

Clearly, $v_f(0) = 4 = v_f(1)$ and $e_f(0) = 3, e_f(1) = 4$

Thus, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Therfore, $C(D_{14},H)$ satisfy the sum cordial labeling.

$\therefore C(D_{14},H)$ is the sum cordial.

Theorem 3.3. *$C(D_{2n},H)$ is the signed product cordial if n is odd for every $n \geq 3$.*

Proof. Let $G = D_{2n}$ be a dihedral group and $H = \{e, s, sr, sr^2, ..., sr^{n-1}\}$ be the subset of the dihedral group.

Let $V(C(D_{2n},H)) = \{v_0, v_1, v_2, ..., v_n\}$
$= \{e, s, sr, sr^2, ..., sr^{n-1}\}$

$$E(C(D_{2n},H)) = \{v_0v_i \mid 1 \le i \le n\}$$

Now, Define $f : V(C(D_{2n},H)) \to \{-1,1\}$ as

$$f(v_i) = \begin{cases} 1, & i \text{ is odd} \\ -1, & otherwise \end{cases} \text{ where } 0 \le i \le n$$

Clearly, $v_f(-1) = \frac{n+1}{2} = v_f(1)$

The induced edge labeling $f^* : E(C(D_{2n},H)) \to \{-1,1\}$ is given by $f^*(v_0v_i) = f(v_0)f(v_i)$ where $1 \le i \le n$

we have, $e_f(-1) = \frac{n+1}{2}$ and $e_f(1) = \frac{n-1}{2}$

Thus, $|v_f(-1) - v_f(1)| \le 1$ and $|e_f(-1) - e_f(1)| \le 1$

Hence, $C(D_{2n},H)$ satisfy the signed product cordial labeling.

$\therefore C(D_{2n},H)$ is the signed product cordial.

□

Example 3.4. *$C(D_{10},H)$ is the signed product cordial.*

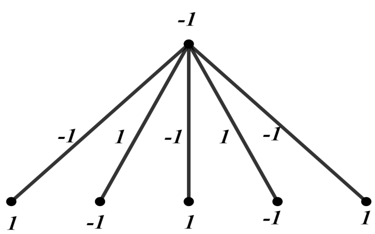

Figure 2. $C(D_{10},H)$

Clearly, $v_f(-1) = 3 = v_f(1)$ and $e_f(-1) = 3, e_f(1) = 2$

Thus, $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$

Hence, $C(D_{14},H)$ is the signed product cordial labeling.

$\therefore C(D_{14},H)$ is the signed product cordial.

Theorem 3.5. *$C(D_{2n},H)$ is a divisor cordial labeling if n is odd for every $n \ge 3$.*

Proof. Let $G = D_{2n}$ be a dihedral group and $H = \{e, s, sr, sr^2, ..., sr^{n-1}\}$ be the subset of the dihedral group.

Let $V(C(D_{2n},H)) = \{v_0, v_1, v_2, ..., v_n\}$
$= \{e, s, sr, sr^2, ..., sr^{n-1}\}$

$$E(C(D_{2n},H)) = \{v_0v_i \mid 1 \le i \le n\}.$$

Now define $f : V \to \{2, 3, ..., n+2\}$ us $f(v_i) = i+2$ where $0 \le i \le n$.

Then the induced edge labeling

$$f^*(v_0v_i) = \begin{cases} 1, & f(v_0) \ / \ f(v_i) \ or \ f(v_i) \ / \ f(v_0) \\ 0, & \text{otherwise} \end{cases}$$
where $1 \le i \le n$

Clearly, $e_f(0) = \frac{n+1}{2}$ and $e_f(1) = \frac{n-1}{2}$

Thus, $|e_f(0) - e_f(1)| \le 1$

Hence, $C(D_{2n},H)$ is a divisor cordial labeling.

$\therefore C(D_{2n},H)$ is a divisor cordial.

□

Example 3.6. *$C(D_{18},H)$ is the divisor cordial graph*

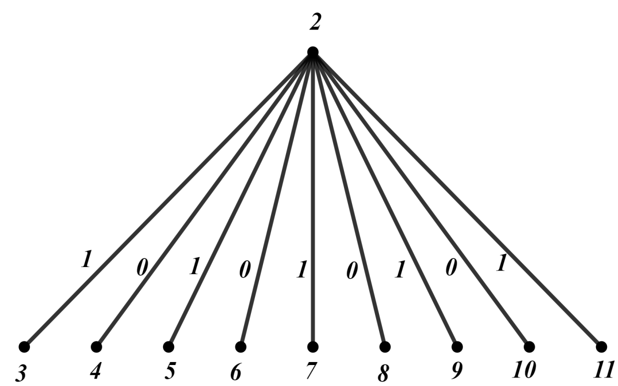

Figure 3. $C(D_{18},H)$

In the Figure 3,

Clearly, $e_f(0) = 4, e_f(1) = 5$. Thus,$|e_f(0) - e_f(1)| \le 1$

Hence, $C(D_{18},H)$ is the divisor cordial labeling.

$\therefore C(D_{18},H)$ is the divisor cordial.

4. Conclusion

In this paper sum cordial, signed product cordial and divisor cordial for the commuting graph of the subset of the dihedral group are obtained.

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