Malaya Journal of Matematik, Vol. 9, No. 1, 474-479, 2021

https://doi.org/10.26637/MJM0901/0080

Connected vertex–Edge dominating sets and connected vertex–Edge domination polynomials of triangular ladder

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Abstract

Let G be a simple connected graph of order n. Let $D_{\text{cye}}\ (G,i)$ be the family of connected vertex - edge dominating sets of G with cardinality i. The polynomial

$$D_{cve}(G,x) = \sum_{i=\gamma_{\text{cve}}\ (G)}^{n} d_{\text{cve}}\ (G,i)x^i$$

is called the connected vertex $-$ edge domination polynomial of G where $d_{\text{cve}}\ (G,i)$ is the number of vertex edge dominating sets of G. In this paper, we study some properties of connected vertex - edge domination polynomials of the Triangular Ladder TL_n. We obtain a recursive formula for $d_{\text{cve}}\ (TL_{n,i})$. Using this recursive formula, we construct the connected vertex - edge domination polynomial

$$D_{cve}\,(TL_{n,x}) = \sum_{i=n-2}^{2n} d_{cve}\,(TL_{n,i})\,x^i$$

of TL_n, where $D_{\text{cve}}\ (TL_{n,i})$ is the number of connected vertex - edge dominating sets of TL_n with cardinality i and some properties of this polynomial have been studied.

Keywords

Triangular ladder, Connected vertex – edge dominating set, connected vertex – edge domination number, connected vertex – edge domination polynomial.

AMS Subject Classification

05C38, 05C78.

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Article History: Received **09** November **2020**; Accepted **10** February **2021**

Contents

1. Introduction

Let $G = (V,E)$ be a simple graph of order n. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = U_{v \in S} N(V)$ and the closed neighbourhood of S is $N[S] = N(s) \cup S.A$ vertex $u \in V(G)$ vertex-edge dominates (ve-dominates) an edge $vw \in E(G)$ if

1. $u = v$ or $u = w(u$ is incident to $vw)$ or

2. uv or uw is an edge in $G(u$ is incident to an edge that is

adjacent to vw).

A vertex - edge dominating set S of G is called a connected vertex - edge dominating set if the induced subgraph $< S >$ is connected. The minimum cardinality of a connected vertex - edge dominating set of G is called the connected vertex - edge domination number of G and is denoted by $\gamma_{\text{cve}}(G)$. A connected vertex - edge dominating set with cardinality $\gamma_{\text{cve}}(G)$ is called γ_{cve} - set. We denote the set $\{1,2,\ldots,2n-1,2n\}$ by $[2n]$, throughout this chapter. Also we used the notation $\lceil x \rceil$ for the smallest integer greater than or equal to $\lceil x \rceil$ and $\lfloor x \rfloor$ for the largest integer less than or equal to x.

2. Connected vertex –Edge dominating sets of Triangular Ladders

Consider two paths $u_1u_2\ldots u_n$ and $v_1v_2\ldots v_n$. Join each pair of vertices u_iv_i and $u_{i+1}v_i, i = 1,2,\ldots,n$. The resulting graph is a Triangular Ladder. Let TL_n be a triangular ladder with $2n$ vertices. Label the vertices of TL_n as given in the following figure.

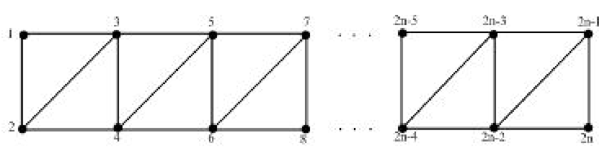

Figure 1

$$V(TL_n) = \{1,2,3,\ldots,2n-2,2n-1,2n\} \text{ and}$$
$$E(TL_n) = \{(1,3),(3,5),(5,7)\ldots(2n-5,2n-3),(2,4),(4,6),(6,8),\ldots(2n-4,2n-2),(2n-2,2n),(1,2),(3,4),(5,6),\ldots(2n-3,2n-2)(2n-1,2n),(2,3),(4,5),(6,7),\ldots(2n-4,2n-3),(2n-2,2n-1)\}$$

For the construction of the vertex- edge dominating sets of the Triangular Ladder TL_n, we need to investigate the connected vertex - edge dominating sets of $TL_n - \{2n\}$. In this section, we investigate the connected vertex- edge dominating sets of TL_n with cardinality i. We shall find the recursive formula for $d_{\text{cve}}(TL_n, i)$.

Lemma 2.1. *(i) For every $n \in N$ and $n \geq 4, \gamma_{cve}(TL_n) = n-1$.*

(ii) For every $n \in N$ and $n \geq 5, \gamma_{cve}(TL_n - \{2n\}) = n-2$.

(iii) $D_{cve}(TL_n, i) = \varphi$ iff $i < n-10$ or $i > 2n$.

(iv) $D_{ove}(TL_{mu} - \{2n\}, i) = \varphi$ iff $i < n-2$ or $i > 2n-1$.

Proof. (i) Clearly $\{3,5,7,9,\ldots,2n-1\}$ is a minimum connected vertex - edge dominating set for TL_n. If n is even or odd it contains $n-1$ elements. Hence $_*y_{\text{cve}}(TL_n) = n-10$.

(ii) Clearly, $\{3,5,7,9,\ldots,2n-3\}$ is a minimum connected vertex - edge dominating set for $TL_n - \{2n\}$. If n is even or odd, it contains $n-2$ elements. Hence, $\gamma_{\text{cve}}(TL_n - \{2n\}) = n-2$.

(iii) Follows from (i) and the definition of connected vertex - edge dominating set.

(iv) Follows from (ii) and the definition of connected vertex - edge dominating set.

□

Lemma 2.2. *(i) If $D_{cve}(TL_n - \{2n\}, i-1) = \varphi$, $D_{cve}(TL_{n-1} - \{2n-2\}, i-1) = \varphi$ and $D_{cve}(TL_{n-2,}i-1) = \varphi$, then $D_{cve}(TL_{n-1}, i-1) = \varphi$*

(ii) If $D_{cve}(TL_n - \{2n\}, i-1) \neq \varphi$, $D_{cve}(TL_{n-1} - \{2n-2\}, i-1) = \varphi$ and $D_{cve}(TL_{n-2,}i-1) \neq \varphi$, then $D_{cve}(TL_{n-1}, i-1) \neq \varphi$

(iii) If $D_{cve}(TL_n - \{2n\}, i-1) = \varphi$, and $D_{cve}(TL_{n-1}, i-1) \neq \varphi$ then $D_{cve}(TL_n, i) = \varphi$

(iv) If $D_{cve}(TL_n - \{2n\}, i-1) \neq \varphi$, and $D_{cve}(TL_{n-1}, i-1) \neq \varphi$, then $D_{cve}(TL_n, i) \neq \varphi$.

(v) If $D_{cve}(TL_n - \{2n\}, i-1) \neq \varphi$, and $D_{cve}(TL_{n-1}, i-1) = \varphi$, then $D_{cve}(TL_n, i) \neq \varphi$.

Proof. (i) Since, $D_{\text{cve}}(TL_n - \{2n\}, i-1) = \varphi$, $D_{cve}(TL_{n-1} - \{2n-2\}, i-1) = \varphi$ and $D_{\text{cve}}(TL_{n-2,}i-1) = \varphi$, by Lemma 2.1 (iii) and (iv) We have, $i-1 < n-2$ or $i-1 > 2n-1$
$i-1 < n-3$ or $i-1 > 2n-3$
and $i-1 < n-3$ or $i-1 < 2n-4$.
Therefore $i-1 < n-3$ or $i-1 > 2n-1$.
Therefore, $i-1 < n-2$ or $i-1 > 2n-2$ holds. Hence, $D_{\text{cve}}(TL_{n-1}, i-1) = \varphi$.

(ii) Since, $D_{\text{cve}}(TL_n - \{2n\}, i-1) \neq \varphi$, $D_{\text{cve}}(TL_{n-1} - \{2n-2\}, i-1) \neq \varphi$, and $D_{\text{cve}}(TL_n - 3, i-1) \neq \varphi$, by lemma 2.1 (iii) and (iv), we have, $n-2 \leq i-1 \leq 2n-1$, $n-3 \leq i-1 \leq 2n-3$ and $n-3 \leq i-1 \leq 2n-4$.
Suppose $D_{\text{cve}}(TL_{n-1}, i-1) = \varphi$. Then by Lemma 2.1 (iii), we have $i-1 \leq n-2$ or $i-1 > 2n-2$.
If $i-1 < n-2$, then $D_{\text{cve}}(TL_n - \{2n\}, i-1) = \varphi$, a contradiction. If $i-1 > 2n-2$, then $i-1 > 2n-3$ holds, which implies $D_{\text{cve}}(TL_{n-1} - \{2n-2\}, i-1) = \varphi$, a contradiction. Therefore, $D_{\text{cve}}(TL_{n-1}, i-1) \neq \varphi$.

(iii) Since $D_{\text{cve}}(TL_n - \{2n\}, i-1) = \varphi$ and $D_{\text{cve}}(TL_{n-1}, i-1) = \varphi$ by Lemma 2.1 (iii) and (iv), we have, $i-1 < n-3$ or $i-1 > 2n-1$ and $i-1 < n-2$ or $i-1 > 2n-2$. Therefore, $i-1 < n-2$ or $i-1 > 2n-1$. Therefore, $i < n-1$ or $i > 2n$. Hence, $D_{\text{cve}}(TL_n, i) = \varphi$.

(iv) Since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{\text{cve}}\ (TL_{n-1}, i-1) \neq \varphi$, by lemma 2.1 (iii) and (iv), we have, $n-2 \leq i\ 1 \leq 2n-1$ and $n-2 \leq i-1 \leq 2n-2$. Suppose $D_{\text{cve}}\ (TL_n, i) = \varphi$, then by Lemma 2.1 (iii), we have $i < n-1$ or $i > 2n$. Therefore, $i-1 < n-2$ or $i-1 > 2n-1$ If $i-1 < n-2$, then $D_{\text{cve}}\ (TL_{n-1}, i-1) = \varphi$, a contraction. If $i-1 > 2n-1$, then $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) = \varphi$ a contraction. Therefore, $D_{\text{cve}}\ \left(TL_{n \neq} i\right) \neq \varphi$.

(v) Since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$ by lemma 2.1 (iv), we have $n-2 \leq i-1 \leq 2n-1$. Also, since $D_{\text{cve}}\ (TL_{n-1},\ i-1) = \varphi$, by Lemma 2.1 (iii), we have $i-1 < n-2$ or $i-1 > 2n-2$. If $i-1 < n-2$, then $D_{\text{cve}}\ (TL_n - \{2n\},\ i-1) = \varphi$ a contraction.

□

Lemma 2.3. *Suppose that $D_{cve}\ (TL_n, i) \neq \varphi$, then for every $n \in N$,*

(i) $D_{cve}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-1}, i-1) = \varphi$ iff $i = 2n$.

(ii) $D_{cve}\ (TL_n - \{2n\}, i-1) \neq \varphi, D_{cve}\ (TL_{n-1,} i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-1} - \{2n-2\}_{win}\ , 1) = \varphi$ iff $i = 2n-1$.

(iii) $D_{cve}\ (TL_n - \{2n\}, i-1) \neq \varphi, D_{cve}\ (TL_{n-1}, i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-1} - \{2n-2\}, i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-2,} i-1) = \varphi$ iff $i = 2n-2$.

Proof. Assume that $D_{\text{cve}}\ (TL_n, i) \neq \varphi$. Then $n-1 \leq i \leq 2n$.

(i) $\Leftrightarrow$) since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$, by lemma 2.1 (iv), we have $n-2 \leq i-1 \leq 2n-1$. Therefore, $n-1 \leq i \leq 2$.
Also since $D_{\text{cve}}\ (TL_{n-1}, i-1) = \varphi$, by lemma 2.1 we have, $i-1 < n-2$ or $i-1 > 2n-2$. If $i-1 < n-2$, then $i < n-1$ which implies $D_{\text{cve}}\ (TL_n, i) = \varphi$, a contradiction. Therefore $i-1 > 2n-2$. Therefore $i-1 \geq 2n-1$. This implies $i \geq 2n$. Therefore, $i = 2n$. ($\Leftrightarrow$) follows from Lemma 2.1 (iii) and (iv)
($\Leftrightarrow$) since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$, and $D_{\text{cve}}\ (TL_{n-1}, i-1) \neq \varphi$ bylemma 2.1 (iii) and (iv), we have $n-2 \leq i-1 \leq 2n-1$ and $n-2 \leq i-1 \leq 2n-1$. Therefore, $n-2 \leq i-1 \leq 2n-2$. Therefore, $n-1 \leq i \leq 2n-2$. Also, since $D_{\text{cve}}\ (TL_{n-1} - \{2n-2\}, i-1) = \varphi$, by lemma 2.1 (iv), we have $i-1 < n-3$ or $i-1 > 2n-3$. Therefore, $i < n-2$ or $i > 2n-2$ If $i < n-2$, then $i < n-1$ holds which implies $D_{\text{cve}}\ (TL_n, i) = \varphi$, a contradiction. Therefore, $i < 2n-2$ Ther efore $\geq 2n-1$. Combining together, we have $= 2n-1$ ($\Leftrightarrow$) follows from Lemma 2.1 (iii) and (iv).

(ii) Since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi, D_{\text{cve}}\ (TL_{n-1}, i-1) \neq \varphi$ and $D_{\text{cve}}\ (TL_n - \{2n-2\}, i-1) \neq \varphi$ by lemma 2.1 (iii) and (iv), we have $n-2 \leq i-1 \leq 2n-1, n-2 \leq i-1 \leq 2n-2$ and $n-3 \leq i-1 \leq 2n-3$. Therefore, $n-2 \leq i-1 \leq 2n-3$. Therefore, $n-1 \leq i \leq 2n-2$. Also, since $D_{\text{cve}}\ (TL_{n-2}, i-1) = \varphi$, by lemma 2.1 (iii), we have, $i-1 < n-3$ or $i-1 > 2n-4$. If $i-1 < n-3$, then $i < n-2$. Therefore, $i < n-2$ holds, which implies $D_{\text{cve}}\ (TL_n, i) = \varphi_n$ a contradiction.

Therefore, $i-1 > 2n-4$ Therefore $i > 2n-3$ Therefore $\geq 2n-2$. Together we have $i = 2n-2$ ($\Leftarrow$) follows from Lemma 2.1 (iii) and (iv).

□

Theorem 2.4. *For every $n \geq 4$*

(i) If $D_{cve}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-1}, i-1) = \varphi$ then $D_{cve}\ (TL_n, i) = \{[2n]\}$.

(ii) If $D_{cve}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{cve}\ (TL_{n-1}, i-1) \neq \varphi$, then $D_{cve}\ (TL_n, i) = \{X_1 \cup \{2n\}$, if $2n-1 \in X_1/X_1 \in D_{cve}\ (TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}$, if $2n-4$ or

$$2n-3 \in X_2/X_2 \in D_{cve}\ (TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\},$$

$$\text{if } 2n-2 \in X_2/X_2 \in D_{cve}\ (TL_{n-1}, i-1)\}.$$

Proof. (i) Since $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{\text{cve}}\ (TL_{n-1}, i-1) = \varphi$, by theorem 2.3 (i), $i = 2n$. Therefore $D_{\text{cve}}\ (TL_n, i) = \{[2n]\}$.

(ii) Let $Y_1 = \{X_1 \cup \{2n\},$ if

$$2n-1 \in X_1/X_1 \in D_{\text{cve}}\ (TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\},$$

if $2n-2$ or $2n-1 \in X_1/X_1 \in D_{cve}(TL_n - \{2n\}, i-1)\}$ and $Y_2 = \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1}, i-1)\} \cup X_2 \cup \{2n-1\}$, if $2n-2 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1}, i-1)\}$

Obviously,

$$Y_1 \cup Y_2 \subseteq D_{\text{cve}}\ (TL_n, i) \tag{2.1}$$

Now, let $Y \in D_{\text{cve}}\ (TL_n, i)$. If $2n \in Y$, then atleast one of the vertices labeled $2n-2$ or $2n-1$ is in Y. In either cases $Y = \{X_1 \cup \{2n\}\}$ for some $X_1 \in D_{\text{cve}}\ (TL_n - \{2n\}, i-1)$. Therefore $Y \in Y_1$. Suppose that $2n-1 \in Y, 2n \notin Y$, them atleast one of the vertices labeled $2n-3$ or $2n-2$ is in Y. If $2n-3 \in Y$, then $Y = \{X_1 \cup \{2n-1\}\}$ for some $X_1 \in D_{\text{cve}}\ (TL_n - \{2n\},\ i-1)$ If $2n-1 \in Y$, then $Y = \{X_2 \cup \{2n-1\}\}$ for some $X_2 \in D_{\text{cve}}\ (TL_{n-1}, i-1)$ Therefore $Y \in Y_1$ or $Y \in Y_2$.

Now, suppose that, $2n-2 \in Y, 2n-1 \notin Y, 2n \notin Y$ then atleast one of the vertices labeled $2n-4$ or $2n-3$ is in Y. In either cases, $Y = \{X_2 \cup \{2n-2\}\}$ for some $X_2 \in D_{\text{cve}}\ (TL_{n-1},\ i-1)$. Therefore $Y \in Y_2$. Hence

$$D_{\text{ove}}\ (TL_{\text{nu}}\ i) \subseteq Y_1 \cup Y_2. \tag{2.2}$$

From (2.1) and (2.2), we have $D_{cve}(TL_n, i) = \{X_1 \cup \{2n-1\}$, if $2n-3 \in X_1/X_1 \in D_{cve}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}$, if $2n-2$ or $2n-1 \in X_1/X_1 \in D_{\text{cve}}\ (TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1},\ i-1)\} \cup \{X_2 \cup \{2n-1\}$, if $2n-2 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1},\ i-1)\}$. □

Theorem 2.5. *If $D_{cve}\ (TL_n,i)$ is the family of connected vertex edge dominating sets of TL_n cardinality i, where $i \geq n-1$, then*

$$d_{cve}\ (TL_n,i) = d_{cve}\ (TL_n - \{2n\}, i-1) + d_{cve}\ (TL_{n-1}, i-1)$$

Proof. We consider the two cases given in theorem 2.4 Suppose $D_{\text{cve}}\ (TL_n - \{2n\}, i-1) \neq \varphi$ and $D_{\text{cve}}\ (TL_{n-1}, i-1) = \varphi$. Then $i = 2n$

$$\begin{aligned} d_{\text{cve}}\ (TL_{n,i}) &= d_{\text{cve}}\ (TL_{n,2}2n) = 1 \\ d_{\text{cve}}\ (TL_n - \{2n\}, i-1) &= d_{\text{cve}}\ (TL_n - \{2n\}\ 2n-1) = 1 \\ d_{cve} (TL_{n-1}, i-1) &= d_{\text{cve}}\ (TL_{n-1}, 2n-1) = 0. \end{aligned}$$

Therefore,

$$d_{\text{cve}}\ (TL_n - \{2n\}, i-1) + d_{\text{cve}}\ (TL_{n-1}, i-1) = 1 + 0 = 1$$

Therefore, in this case

$$d\,(TL_n,i) = d_{\text{cve}}\ (TL_n - \{2n\}, i-1) + d_{\text{cve}}\ (TL_{n-1}, i-1)$$

holds. By theorem 2.4 (ii) we have,
$D_{\text{cve}}\ (TL_n, i) = \{X_1 \cup \{2n-1\},$ if

$$2n-3 \in X_1/X_1 \in D_{\text{cve}}\ (TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\},$$

if $2n-2$ or

$$2n-1 \in X_1/X_1 \in D_{\text{cve}}\ (TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\},$$

if $2n-4$ or

$$2n-3 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1}, i-1)\} \cup X_2 \cup \{2n-1\},$$

if

$$2n-2 \in X_2/X_2 \in D_{\text{cve}}\ (TL_{n-1}, i-1)\}.$$

Therefore

$$_{\text{cve}}\, d_{\text{cve}}\ (TL_n, i) = d_{\text{cve}}\ (TL_n - \{2n\}, i-1) + d_{\text{cve}}\ (TL_{n-1}, i-1).$$

□

3. Connected total domination polynomials of triangular ladders

Theorem 3.1. *Let $D_{cve}\ (TL_n,i)$ be the family of connected vertex edge dominating sets of TL_n with cardinality i and let $d_{cve}\ (TL_n,i) = |D_{cve}\ (TL_n,i)|$. Then the connected vertex edge domination polynomial $D_{cve}\ (TL_n,x)$ of TL_n is defined as,*

$$D_{cve}\ (TL_n,x) = \sum_{i=\gamma_{cve}(TL_n)}^{2n} d_{cve}\ (TL_n,i)\,x^i.$$

Theorem 3.2. *For every $n \geq 4$,*

$$D_{cve}\ (TL_n,x) = x\,[D_{cve}\ (TL_n - \{2n\},x) + D_{cve}\ (TL_{n-1},x)]$$

with initial values

$$\begin{aligned} D_{cve}\ (TL_2 - \{4\},x) &= 3x^2 + x^3 \\ D_{cve}\ (TL_2,x) &= 5x^2 + 4x^3 + x^4 \\ D_{cve}\ (TL_3 - \{6\},x) &= 7x^2 + 8x^3 + 5x^4 + x^5 \\ D_{cue}\ (TL_3,x) &= 7x^2 + 12x^3 + 12x^4 + 6x^5 + x^6 \\ D_{cue}\ (TL_4 - \{8\},x) &= 5x^2 + 14x^3 + 20x^4 + 17x^5 + 7x^6 + x^7 \\ D_{cre}\ (TL_4,x) &= 3x^2 + 12x^3 + 26x^4 + 32x^5 + 23x^6 \\ &\quad + 8x^7 + x^8 \\ D_{cue}\ (TL_5 - \{10\},x) &= x^2 + 8x^3 + 26x^4 + 46x^5 + 49x^6 \\ &\quad + 30x^7 + 9x^8 + x^9 \end{aligned}$$

Proof. We have,

$$d_{\text{cve}}\ (TL_n,i) = d_{\text{cve}}\ (TL_n - \{2n\}, i-1) + d_{\text{cve}}\ (TL_{n-1}, i-1).$$

Therefore,

$$d_{cve}\,(TL_n,i)\,x^i = d_{cve}\,(TL_n - \{2n\}, i-1)\,x^i + d_{cve}\,(TL_{n-1}, i-1)\,x^i$$

$$\begin{aligned} \sum_{i=n-1}^{2n} d_{cve}\,(TL_n,i) &= \sum_{i=n-1}^{2n} d_{cve}\,(TL_n - \{2n\}, i-1)\,x^i \\ &\quad + \sum_{i=n-1}^{2n} d_{cve}\,(TL_{n-1}, i-1)\,x^i \\ &= x\sum_{i=n-2}^{2n} d_{cve}\,(TL_n - \{2n\}, i-1)\,x^{i-1} \\ &\quad + x\sum_{i=n-2}^{2n} d_{cve}\,(TL_{n-1}, i-1)\,x^{i-1} \\ &= xD_{cve}\,(TL_n - \{2n\},x) + xD_{cve}\,(TL_{n-1},x) \\ &\quad + x\,[D_{cve}\,(TL_n - \{2n\},x) + D_{cve}TL_{n-1},x] \end{aligned}$$

Therefore,

$$D_{\text{cve}}\ (TL_n,x) = x\,[D_{\text{cve}}\ (TL_n - \{2n\},x) + D_{\text{cve}}\ (TL_{n-1},x)].$$

With initial values,

$$\begin{aligned} D_{cv_\theta}\,(TL_2 - \{4\},x) &= 3x^2 + x^3 \\ D_{cve}\,(TL_2,x) &= 5x^2 + 4x^3 + x^4 \\ D_{cv\theta}\,(TL_3,\{6\},x) &= 7x^2 + 8x^3 + 5x^4 + x^5 \\ D_{cve}\,(TL_3,x) &= 7x^2 + 12x^3 + 12x^4 + 6x^5 + x^6 \\ D_{cve}\,(TL_4,\{8\},x) &= 5x^2 + 14x^3 + 20x^4 + 17x^5 + 7x^6 + x^7 \\ D_{cve}\,(TL_4,x) &= 3x^2 + 12x^3 + 26x^4 + 32x^5 + 23x^6 \\ &\quad + 8x^7 + x^8 \\ D_{cve}\,(TL_5 - \{10\},x) &= x^2 + 8x^3 + 26x^4 + 46x^5 + 49x^6 \\ &\quad + 30x^7 + 9x^8 + x^9 \end{aligned}$$

□

Table 1

$\downarrow n$ $\rightarrow i$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$TL_2-\{4\}$	3	1															
TL_2	5	4	1														
$TL_3-\{6\}$	7	8	5	1													
TL_3	7	12	12	6	1												
$TL_4-\{8\}$	5	14	20	17	7	1											
TL_4	3	12	26	32	23	8	1										
$TL_5-\{10\}$	1	8	26	46	49	30	9	1									
TL_5	0	4	20	52	78	72	38	10	1								
$TL_6-\{12\}$	0	1	12	46	98	127	102	47	11	1							
TL_6	0	0	5	32	98	176	199	140	57	12	1						
$TL_7-\{14\}$	0	0	1	17	78	196	303	301	187	68	13	1					
TL_7	0	0	0	6	49	176	372	502	441	244	80	14	1				
$TL_8-\{16\}$	0	0	0	1	23	127	372	675	803	628	312	93	15	1			
TL_8	0	0	0	0	7	72	303	744	1377	1244	872	392	107	16	1		
$TL_9-\{18\}$	0	0	0	0	1	30	199	675	1419	2180	1872	1191	485	122	17	1	
TL_9	0	0	0	0	0	8	102	502	1419	2796	3424	2744	1583	592	138	18	1

Example 3.3.

$$D_{cve}(TL_5,x) = 4x^3+20x^4+52x^5+78x^6+72x^7 + 38x^8+10x^9+x^{10}$$

$$D_{cve}(TL_6-\{12\},x) = x^3+12x^4+46x^5+98x^6+127x^9 + 102x^8+47x^9+11x^{10}+x^{11}$$

By theorem 3.2, *we have*

$$\begin{aligned} D_{cve}(TL_6,x) =& x\Big[4x^3+20x^4+52x^5+78x^6+72x^7 \\ &+38x^8+10x^9+x^{10}x^3+12x^4+46x^5+98x^6 \\ &+ +127x^7+102x^8+47x^9+11x^{10}+x^{11}\Big] \\ =& 5x^4+32x^5+98x^6+176x^7+199x^8+140x^9 \\ &+57x^{10}+12x^{11}+x^{12} \end{aligned}$$

We obtain, $d_{cve}(TL_n,i)$ and $d_{cve}(TL_n-\{2n\},i)$ for $2\le n\le 9$ as shown in table 1.

In the following theorem we obtain some properties of $d_{\text{cve}}(TL_n,i)$.

Theorem 3.4. *The following properties hold for the coefficients of $D_{cve}(TL_n,x)$ and $D_{cve}(TL_n-\{2n\},x)$ for all n.*

(i) $d_{cve}(TL_n,2n)=1$ *for all* $n\ge 2$.

(ii) $d_{cve}(TL_n,2n-1)=2n$ *for all* $n\ge 2$.

(iii) $d_{cve}(TL_n-\{2n\},2n-1)=1$ *for all* $n\ge 2$

(iv) $d_{cve}(TL_n-\{2n\},2n-2)=2n-1$ *for all* $n\ge 2$

(v) $d_{cve}(TL_n,2n-2)=2n^2-3n+3$ *for all* $n\ge 2$

(vi) $d_{cve}(TL_n-\{2n\},2n-3)=2n^2-5n+5$ *for all* $n\ge 3$

(vii) $d_{cve}(TL_n-\{2n\},n-3)=1$ *for all* $n\ge 5$

(viii) $d_{cve}(TL_n,n-2)=n-1$ *for all* $n\ge 4$

Proof. (i) Since $D_{\text{cve}}(TL_n,2n)=\{[2n]\}$, we have the result.

(ii) Since, $D_{cve}(TL_n,2n-1)=\{[2n]-\{x\}/x\in\{[2n]\}\}$, we have $d_{cve}(TL_n,2n-1)=2n$

(iii) Since, $D_{\text{cve}}(TL_n-\{2n\},2n-1)=\{[2n-1\}$, we have the result.

(iv) To prove $d_{\text{cve}}(TL_n-\{2n\},2n-2)=2n-1$, for every $n\ge 2$, we apply induction on n.

When $n=2$.
L.H.S $=d_{cve}(TL_2-\{4\},2)=3$ (From table : 1) and
R.H.S $=2\times 2-1=3$.
Therefore, the result is true for $n=2$ Now, suppose that the result is true for all natural numbers less than n and we prove it for n. By theorem 2.5, we have,

$$\begin{aligned} &d_{cve}(TL_n-\{2n\},2n-2) \\ &= d_{cve}(TL_{n-1},2n-3)+d_{cve}(TL_{n-1},\{2n-2\},2n-3) \\ &= 2(n-1)+1 \end{aligned}$$

That is, $d_{\text{cve}}(TL_n,\{2n\},2n-2)=2n-1$.
Hence, the result is true for all n.

(v) To prove $d_{\text{cve}}(TL_n,2n-2)=2n^2-3n+3$, for every $n\ge 2$, we apply induction on n.

When $n=2$, L.H.S $=d_{\text{cve}}(TL_2,2)=5$(from table 1) and R.H.S $=2\times 4-3\times 2+3=5$.
Therefore, the result is true for $n=2$. Now, suppose that the result is true for all numbers less than ' n ' and we prove it for

n. By theorem 2.5, we have

$$\begin{aligned} & d_{cve}(TL_n, 2n-2) \\ =& d_{cve}(TL_n - \{2n\}, 2n-3) + d_{cve}(TL_{n-1}, 2n-3) \\ =& d_{cve}(TL_{n-1}, 2n-4) + d_{cve}(TL_{n-1} - \{2n-2\}, 2n-4) \\ & + d_{cve}(TL_{n-1}, 2n-3) \\ =& 2(n-1)^2 - 3(n-1) + 3 + 2(n-1) - 1 + 2(n-1) \\ =& 2\left(n^2 - n + 1\right) - 3n + 3 + 3 + 2n - 2 - 1 + 2n - 2 \\ =& 2n^2 - 4n + 2 + n + 1 \\ =& 2n^2 - 3n + 3 \end{aligned}$$

Hence the result is true for all n.

(vi) To prove $d_{\text{cve}}(TL_n - \{2n\}, 2n-3) = 2n^2 - 5n + 5$, for every $n \geq 3$, we apply induction on.

When $n = 3$, L.H.S $= d_{cve}(TL_3 - \{6\}, 3) = 8$(From table 1)
R.H.S $= 2 \times 9 - 5 \times 3 + 5 = 8$.
Therefore, the result is true for $n = 3$. Now, suppose that the result is true for all numbers less than n and prove it for n. By theorem 2.5, we have

$$\begin{aligned} & d_{cve}(TL_3 - \{2n\}, 2n-3) \\ =& d_{cve}(TL_{n-1}, 2n-4) + d_{cv\theta}(TL_{n-1}, \{2n-2\}, 2n-4) \\ =& 2(n-1)^2 - 3(n-1) + 3 + 2(n-1) - 1 \\ =& 2\left(n^2 - 2n + 1\right) - 3n + 3 + 2n - 2 - 1 \\ =& 2n^2 - 4n + 2 - n + 3 \end{aligned}$$

$$d_{cve}(TL_n - \{2n\}, 2n-3) = 2n^2 - 5n + 5$$

Hence, the result is true for all n.

(vii) Since, $D_{\text{cve}}(TL_n - \{2n\}, n-3) = \{3, 5, 7, 9, \ldots, 2n-1\}$, we have the result.

(viii) To prove $d_{\text{cve}}(TL_n, n-2) = n-1$ for all $n \geq 4$, we apply induction on n.
When, $n = 4, d_{\text{cve}}(TL_4, 2) = 3$
R.H.S $= n - 1 = 4 - 1 = 3$.
Therefore, the result is true for $n = 4$ By theorem 2.5, we have,

$$\begin{aligned} d_{cve}(TL_n, n-2) =& d_{cve}(TL_n - \{2n\}, n-3) \\ & + d_{cve}(TL_{n-1}, n-3) \\ =& 1 + n - 2 \\ =& n - 1 \end{aligned}$$

Hence, the result is true for all n by mathematical induction. □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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