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Strongly perfect Plick and Lict graphs for some class of graphs

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Abstract

A graph G is said to be strongly perfect if each of its induced subgraphs H contains an independent set which meets all the cliques in H. In this paper, we develop results on strongly perfect graphs for plick and lict graphs of some class of graphs.

Keywords

Strongly perfect graph, plick graph, lict graph.

Mathematical Subject Classification 2010:

05C17, 05C38, 05C69.

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Contents

1. Introduction

In this paper we utilize finite, simple and undirected graphs. Let G be a graph. The vertex set of graph G is denoted as $V(G)$ and its edge set is denoted as $E(G)$. We refer [4] for undefined terminologies used in this paper. An **independent set** [2] in a graph is a set of vertices no two of which are adjacent. A **clique** [4] of a graph is a maximal complete subgraph. The **tadpole graph** $T_{m,n}$ [3] is the graph obtained by joining a cycle C_m to a path of length n, (m indicates number of vertices in cycle C_m and n indicates number of vertices in path P_n). The **friendship graph** F_x [3] is constructed by joining x copies of the cycle C_3 with a common vertex. A **wheel graph** [3] W_n is a graph with n vertices formed by connecting a single vertex to all vertices of a cycle. The **helm graph** H_y [3] , where y indicates the number of pendent edges, is the graph obtained from a wheel graph W_n by adjoining a pendant edge at each vertex of the cycle. The **plick graph** $P(G)$ [5] of a graph G is obtained from the line graph by adding a new vertex corresponding to each block of the original graph and joining this vertex to the vertices of the line graph which correspond to the edges of the block of the original graph. The **lict graph** $L_c(G)$ [6] of a graph G is one whose vertex set is the union of the edges and the set of cutpoints of G in which two vertices are adjacent if and only if the corresonding members of G are adjacent or the corresponding members of G are incident. The cutpoints and edges of a graph G are called its members. A graph is called **strongly perfect** [8], if each of its induced subgraphs H contains an independent set which meets all the cliques in H.

2. Preliminaries

In this section, we mention some standard results which will be used throughout this paper.

Theorem 2.1. *[1] Every bipartite graph is strongly perfect.*

Theorem 2.2. *[1] Let G be a graph with no induced P_4, then every maximal stable set (independent set) meets all the maximal cliques. Consequently, G is strongly perfect.*

Theorem 2.3. *[7] If every odd cycle of length at least five in a graph G has at least two chords, then G is strongly perfect.*

3. Results on plick graphs of some class of graphs to be strongly perfect

This section of paper develops results for plick graphs of some class of graphs to be strongly perfect.

Theorem 3.1. *Plick graph of every path graph P_n with $n \geq 2$ is strongly perfect.*

Proof. Let G be a path graph P_n, where n is the number of vertices with $n \geq 2$. Let $P(G)$ denote the plick graph of G. Then plick graph $P(G)$ of path graph P_n with $n \geq 2$ results into a bipartite graph. By Theorem 2.1, obtained plick graph is strongly perfect. □

Theorem 3.2. *If G is a cycle graph C_n with $n \geq 3$, then plick graph of G is*

$$P(G) = P(C_n) = \begin{cases} \text{strongly perfect} & \text{if } n = 3 \\ \text{strongly perfect} & \text{if } n \text{ is even} \\ \text{not strongly perfect} & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let G be a cycle graph C_n with $n \geq 3$, where n is the number of vertices in C_n.
Let $P(G)$ be the plick graph of G and H be any induced subgraph of $P(G)$. We consider two cases,
Case 1. If $n = 3$.
In this case the plick graph $P(G)$ of a cycle graph C_n with $n = 3$ is a complete graph K_4. The obtained plick graph $P(G)$ has no induced P_4. Thus by Theorem 2.2, plick graph $P(G)$ is strongly perfect.

Case 2. When $n > 3$.
We consider following subcases,
Subcase 2.1. If n is even with $n = 4$.
The plick graph $P(G)$ of a cycle graph C_4 produces a wheel graph W_5 and it has no induced P_4 in it. Thus by Theorem 2.2, plick graph of C_4 is strongly perfect.
Subcase 2.2. If n is even with $n = 2k$, where k = 3, 4, ...
The plick graph $P(G)$ of a cycle graph C_n when n is even with $n = 2k$, where k = 3, 4, ... is a wheel graph W_n with odd number of vertices. Each of this plick graph has odd cycle of length at least five with at least two chords. Hence by Theorem 2.3, the plick graph $P(G)$ of a cycle graph C_n is strongly perfect.
Subcase 2.3. If n is odd with $n = 2k+1$, where k = 2, 3, ...
The plick graph $P(G)$ of a cycle graph C_n when n is odd with $n = 2k+1$, k = 2, 3, ... is a wheel graph W_n with even number of vertices. The obtained plick graph $P(G)$ has one induced subgraph H as a cycle C_n, where $n = 5, 7, \ldots$ Let the vertex set of induced subgraph of plick graph $P(C_n)$ which is a cycle as shown in Figure 1 be $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set be $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_nv_1\}$.
Since the induced subgraph is a cycle C_n, it has clique as the complete graph K_2. Let $S = \{v_1, v_3, \ldots, v_{n-2}\}$ be the independent set of the induced subgraph, which is a cycle C_n, where

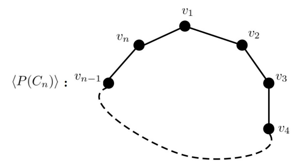

Figure 1. Induced subgraph of plick graph $P(C_n)$ with $n = 2k+1$, where k = 2, 3, ...

$n = 5, 7, \ldots$. We find that the independent set S meets all the cliques $\{v_1, v_2\}$, $\{v_2, v_3\}$,..., $\{v_n, v_1\}$. But the independent set S does not meet the clique $\{v_{n-1}, v_n\}$. Thus, definition of strongly perfect graph is not satisfied. Hence, plick graph $P(G)$ of a cycle graph C_n is not strongly perfect. □

Theorem 3.3. *If G is a tadpole graph $T_{m,n}$ with $m \geq 3$ and $n \geq 1$, then plick graph $P(G)$ of a graph G is*

$$P(G) = P(T_{m,n}) = \begin{cases} \text{strongly perfect} & \text{if } m = 3 \text{ and } n \geq 1 \\ \text{strongly perfect} & \text{if } m \text{ is even and } n \geq 1 \\ \text{not strongly perfect} & \text{if } m \text{ is odd and } n \geq 1. \end{cases}$$

Proof. Let $G = T_{m,n}$ with $m \geq 3$ and $n \geq 1$ be a tadpole graph. Let $P(G)$ be the plick graph of G and H be any induced subgraph of $P(G)$.
Consider two cases as follows,
Case 1. If $m = 3$ and $n \geq 1$.
The plick graph $P(G)$ of a tadpole graph $T_{m,n}$ for $m = 3$ and $n \geq 1$ contains one complete graph K_4, a cycle C_3, and a tree.

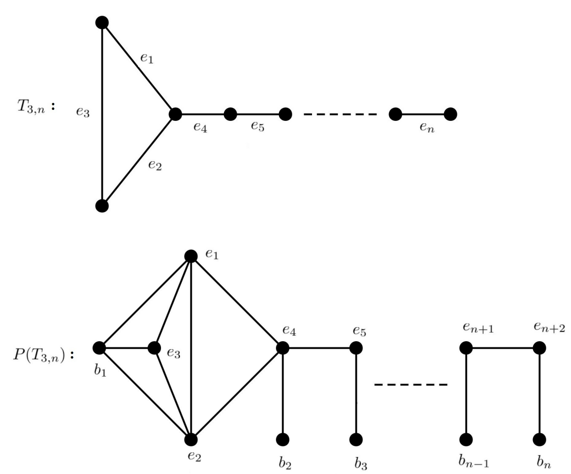

Figure 2. Tadpole graph $T_{3,n}$ and its plick graph $P(T_{3,n})$.

Let H be any induced subgraph of $P(G)$. For induced subgraph H, there exists an independent set which meets all the cliques of H. Further, consider graph $H - S_1$. The graph $H - S_1$ has an independent set S_2 which meets all cliques of $H - S_1$. Continuation of this process results into a trivial graph, for trivial graph the definition of strongly perfect graph holds good. Thus, each induced subgraph of $P(G)$ contains an independent set which meets all the cliques of H. Hence, plick graph $P(G)$ is strongly perfect.

Case 2. When $m > 3$ and $n \geq 1$.
This case consists of two subcases.

Subcase 2.1. If m is even and $n \geq 1$.
The plick graph $P(G)$ of a tadpole graph $T_{m,n}$, where m is even and $n \geq 1$ contains a wheel graph W_n with odd number of vertices, a cycle C_3 and a tree.

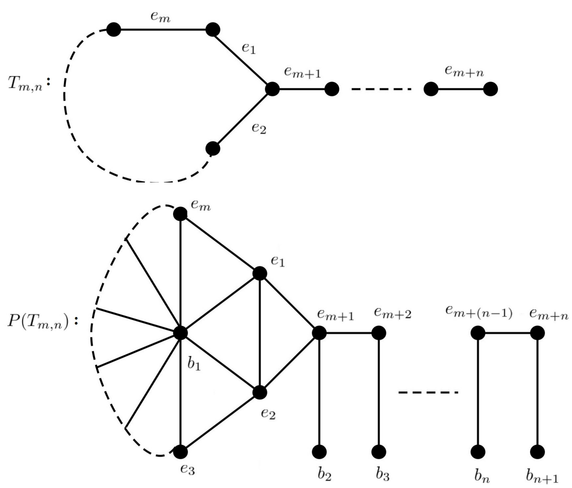

Figure 3. Tadpole graph $T_{m,n}$ where m has even values, $n \geq 1$ and its plick graph $P(T_{m,n})$.

Let H be any induced subgraph of plick graph $P(G)$ where G is a tadpole graph $T_{m,n}$. It has an independent set S_1 which meets all cliques of H. Now, consider the graph $H - S_1$. The independent set of $H - S_1$ is S_2 which meets all cliques of $H - S_1$. Continuation of this process results into a null graph. Since null graph is strongly perfect, hence each induced subgraph H of $P(G)$ contains an independent set which meets all the cliques of H. Thus, plick graph $P(G)$ is strongly perfect.

Subcase 2.2. If m is odd and $n \geq 1$.
The obtained plick graph $P(G)$ of a tadpole graph $T_{m,n}$ when m is odd and $n \geq 1$ contains a wheel graph W_n with even number of vertices, a cycle C_3 and a tree.
There exists one induced subgraph H of plick graph $P(G)$ as a cycle C_m with $m = 5, 7, ...$

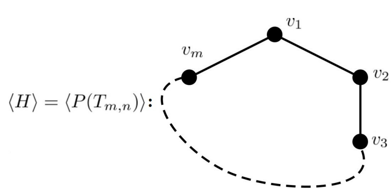

Figure 4. Induced subgraph of plick graph $P(T_{m,n})$, where m has odd values and $n \geq 1$.

Let vertex set of the induced subgraph which is a cycle as shown in Figure 4 be $V(C_m) = \{v_1, v_2, ..., v_m\}$ and edge set be $E(C_m) = \{v_1v_2, v_2v_3, ..., v_mv_1\}$. Since the induced subgraph is a cycle C_m, its clique is the complete graph K_2. Let $S = \{v_1, v_3, ..., v_{m-2}\}$ be the independent set of this induced subgraph which is a cycle C_m, where $m = 5, 7, ...$ From this it follows that the independent set S meets all the cliques $\{v_1, v_2\}$, $\{v_2, v_3\}$,..., $\{v_{m-2}, v_{m-1}\}$, $\{v_m, v_1\}$. But the independent set S does not meet the clique $\{v_{m-1}, v_m\}$. Thus, definition of strongly perfect graph fails in this case. Hence, plick graph $P(G)$ is not strongly perfect.

□

Theorem 3.4. *If G is a friendship graph F_x, where x is the number of copies of cycle C_3 with $x \geq 2$, then plick graph $P(G)$ of a graph G is strongly perfect.*

Proof. Let G be a friendship graph F_x, where x is the number of copies of cycle C_3 with $x \geq 2$.
Let $P(G)$ be the plick graph of G and H be any induced subgraph of $P(G)$.
The obtained plick graph $P(G)$ of friendship graph F_x contains one complete graph K_{2x} with $x \geq 2$ and x copies of complete graph K_4.
Consider two cases as follows,
Case 1. When $x = 2$.
The obtained plick graph contains three copies of complete graph K_4. Let H be any induced subgraph of $P(G)$. For in-

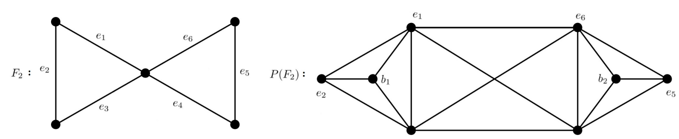

Figure 5. Friendship graph F_2 and its plick graph $P(F_2)$.

duced subgraph H there exists an independent set S_1 which meets all the cliques of H. Futher consider the graph $H - S_1$, it has an independent set S_2 which meets all the cliques of $H - S_1$. Continuing this process results into a trivial graph. Since the trival graph satisfies the definition of strongly perfect graph, it follows that each induced subgraph H of plick graph $P(G)$ contains an independent set which meets all the cliques of H. Hence, plick graph $P(G)$ is strongly perfect.

Case 2. When $x \geq 3$.
The obtained plick graph contains one complete graph K_{2x}, where $x \geq 3$ and x copies of complete graph K_4. This plick graph has an odd cycle of length five with at least two chords. From Theorem 2.3, plick graph $P(G)$ is strongly perfect.

□

Theorem 3.5. *Plick graph of every star graph $K_{1,n}$, where $n \geq 1$ is strongly perfect.*

Proof. Let G be a star graph $K_{1,n}$, where $n \geq 1$.
Let $P(G)$ be the plick graph of G and H be any induced subgraph of $P(G)$.
The plick graph $P(G)$ of a star graph $K_{1,n}$ is obtained in the form of a complete graph K_n, where $n \geq 1$ with one pendent edge at each vertex of K_n , these pendent edges are K_2 in nature.
Consider the following two cases,
Case 1. When $n = 1$ and $n = 2$.

For $n = 1$ and $n = 2$ the plick graph $P(G)$ are path graph P_2 and P_4 respectively. Since path graph is a bipartite graph, by Theorem 2.1, plick graph $P(G)$ is strongly perfect.

Case 2. When $n = 3$.
For $n = 3$ the plick graph $P(G)$ obtained contains a complete graph K_3 and one pendent edge K_2 at each vertex of K_3.

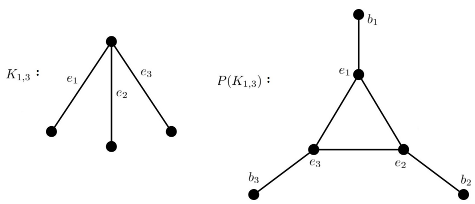

Figure 6. Star graph $K_{1,3}$ and its plick graph $P(K_{1,3})$.

Let H be any induced subgraph of plick graph $P(G)$. For induced subgraph H we find an independent set S_1 which meets all the cliques of H. Futher, consider graph $H - S_1$, for this graph we have S_2 as an independent set in $H - S_1$ such that S_2 meets all the cliques of $H - S_1$. Continuation of this process results into a trivial graph. Since trival graph satisfies the definition of strongly perfect graph, it follows that each induced subgraph H of plick graph $P(G)$ contains an independent set which meets all the cliques of H. Hence, plick graph $P(G)$ is strongly perfect.

Case 3. When $n \geq 4$.
For $n \geq 4$ the obtained plick graph $P(G)$ contains one complete graph K_n, where $n \geq 4$ with one pendent edge at each vertex of K_n, these pendent edges are K_2 in nature. In this case the plick graph obtained contains an odd cycle of length at least five with at least two chords. Thus, by Theorem 2.3, plick graph $P(G)$ is strongly perfect.

□

Theorem 3.6. *If G is a helm graph H_y with $y \geq 3$, then plick graph $P(G)$ of a graph G is*

$$P(G) = P(H_y) = \begin{cases} \textit{strongly perfect} & \textit{if } y = 3 \\ \textit{not strongly perfect} & \textit{if } y > 3. \end{cases}$$

Proof. Let $G = H_y$ with $n \geq 3$ be a helm graph.
Let $P(G)$ be the plick graph of G and H be any induced subgraph of $P(G)$.
We discuss the following two cases,
Case 1. If $y = 3$. The plick graph $P(G)$ of a helm graph H_3 contains an odd cycle of length at least five with at least two chords. By Theorem 2.3 plick graph $P(G)$ of H_3 is strongly perfect.

Case 2. If $y > 3$.
Let $y = 4$. The helm graph considered is H_4 and its plick graph is $P(H_4)$.

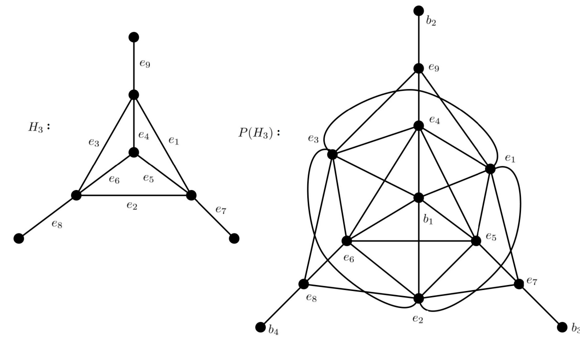

Figure 7. Helm graph H_3 and its plick graph $P(H_3)$.

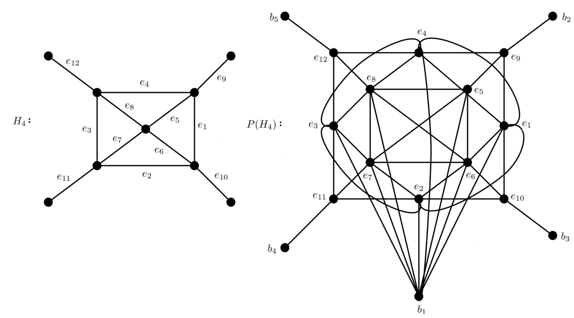

Figure 8. Helm graph H_4 and its plick graph $P(H_4)$.

Consider one of the induced subgraph of plick graph $P(H_4)$ which is as shown in the Figure 9, has an independent set $S = \{e_4, e_6\}$ which does not meet the cliques $\{b_1, e_5\}$, $\{b_1, e_8\}$, $\{b_1, e_7\}$ and some part of the clique $\{e_2, e_3, e_7\}$. Thus definition of strongly perfect graph is not satisfied. Hence plick graph $P(H_4)$ is not strongly perfect. For the remaining values of y (i.e., $y = 5, 6, ...$) of helm graph, the same result holds. Thus respective plick graphs $P(G)$ of helm graphs when $y > 3$ are not strongly perfect.

□

4. Results on lict graphs of some class of graphs to be strongly perfect

In this section, we prove lict graphs of some class of graphs to be strongly perfect.

Theorem 4.1. *Lict graph of every path graph P_n with $n \geq 3$ is strongly perfect.*

Proof. Let G be the path graph P_n with $n \geq 3$.
Let $L_c(G)$ denote the lict graph of a graph G and H be any induced subgraph of lict graph $L_c(G)$. The obtained lict graph $L_c(G)$ of path graph P_n with $n \geq 3$ contains cycles of length 3 which are $(n-2)$ in number, where n is the number of vertices in P_n.

For induced subgraph H we find an independent set S_1 which meets all cliques of H. Consider the graph $H - S_1$. It has an independent set S_2 which meets all the cliques of $H - S_1$. Continuation of this process leads to a trivial graph, for trivial graph the definition of strongly perfect graph holds

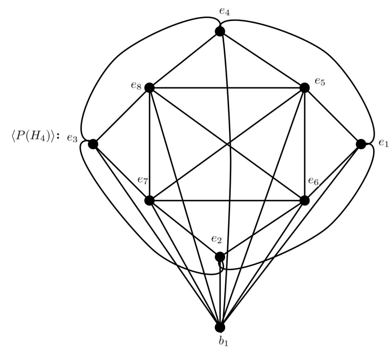

Figure 9. Induced subgraph of plick graph $P(H_4)$.

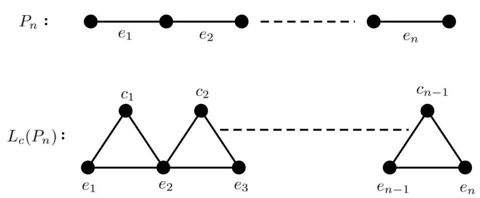

Figure 10. Path graph P_n and its lict graph $L_c(P_n)$.

good. Thus, each induced subgraph H of lict graph $L_c(G)$ contains an independent set which meets all the cliques of H. □

Theorem 4.2. *If G is a tadople graph $T_{m,n}$ with $m \geq 3$ and $n \geq 1$, then lict graph $L_c(G)$ of a graph G is*

$$L_c(G) = L_c(T_{m,n}) = \begin{cases} \text{strongly perfect} & \text{if } m = 3 \text{ and } n \geq 1 \\ \text{strongly perfect} & \text{if } m \text{ is even and } n \geq 1 \\ \text{not strongly perfect} & \text{if } m \text{ is odd and } n \geq 1. \end{cases}$$

Proof. Let $G = T_{m,n}$ with $m \geq 3$ and $n \geq 1$ be a tadpole graph. Let $L_c(G)$ be the lict graph of G and H be any induced subgraph of $L_c(G)$.
Consider two cases as follows,
Case 1. If $m = 3$ and $n \geq 1$.
The lict graph $L_c(G)$ of tadpole graph $T_{m,n}$ for $m = 3$ and $n \geq 1$ contains a cycle C_3, one complete graph K_4 and (n-1) number of cycles C_3 corresponding to path P_n, where n is the number of vertices in P_n.

In this case the induced subgraph H contains an independent set S_1 such that S_1 meets all the cliques of H. Further, consider the graph $H - S_1$. There exists an independent set S_2 in $H - S_1$, such that S_2 meets all the cliques of $H - S_1$. Continuation of this process leads to a trivial graph, this trivial graph is strongly perfect by the definition of strongly perfect graph. Thus, each induced subgraph of lict graph $L_c(G)$ contains an independent set which meets all the cliques of H. Hence, lict graph $L_c(G)$ is strongly perfect.

Case 2. When $m > 3$ and $n \geq 1$.
This case consists of two subcases.

Subcase 2.1. If m is even and $n \geq 1$.
The lict graph $L_c(G)$ of tadpole graph $T_{m,n}$, where m is even and $n \geq 1$ is made up of one even cycle C_m ($m = 4, 6, ...$), one complete graph K_4 and $(n-1)$ number of cycles C_3 corresponding to path P_n, where n is number of vertices in path P_n.

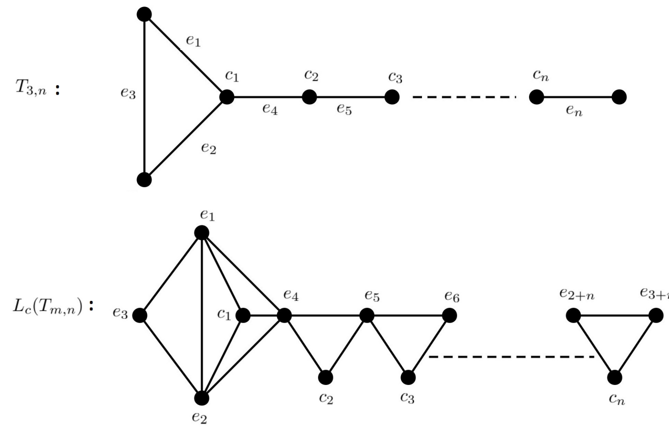

Figure 11. Tadpole graph $T_{3,n}$ and its lict graph $L_c(T_{3,n})$.

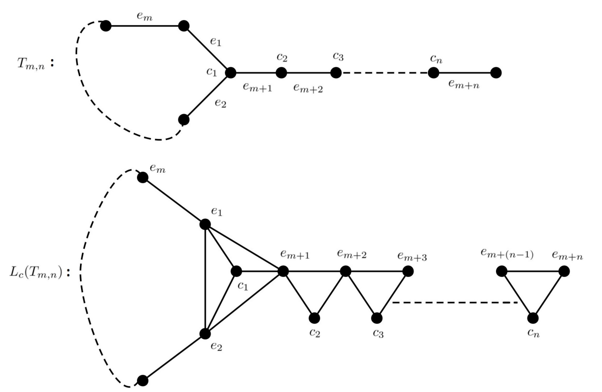

Figure 12. Tadpole graph $T_{m,n}$ where m has even values, $n \geq 1$ and its lict graph $L_c(T_{m,n})$.

Consider an induced subgraph H, there exists an independent set S_1 in H which meets all the cliques of H. Consider graph $H - S_1$, for this graph there exists an independent set S_2 which meets all the cliques of $H - S_1$. Continuation of this process results into a trivial graph which is strongly perfect. Thus, each induced subgraph of lict graph $L_c(G)$ contains an independent set which meets all the cliques of H. Hence, lict graph $L_c(G)$ is strongly perfect.

Subcase 2.2 If m is odd and $n \geq 1$.
The obtained lict graph $L_c(G)$ consists of an odd cycle C_m with $m = 5, 7, ...$, one complete graph K_4 and $(n-1)$ number of cycles C_3 corresponding to path P_n, where n is number of vertices in P_n. In lict graph $L_c(G)$ one induced subgraph H is a cycle C_m with $m = 5, 7, ...$

Let the vertex set of this induced subgraph be $V(C_m) = \{v_1, v_2, ..., v_m\}$ and edge set be $E(C_m) = \{v_1v_2, v_2v_3, ..., v_mv_1\}$. Since the induced subgraph is a cycle C_m, its clique is the complete graph K_2. Let $S = \{v_1, v_3, ..., v_{m-2}\}$ be the independent set of this induced subgraph which is a cycle C_m with

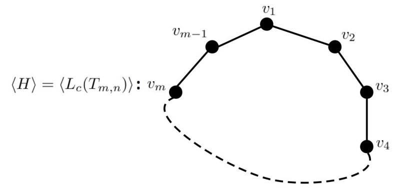

Figure 13. Induced subgraph of lict graph $L_c(T_{m,n})$, where m has odd values and $n \geq 1$.

$m = 5, 7, \ldots$. It is seen that the independent set S meets all the cliques $\{v_1, v_2\}$, $\{v_2, v_3\}$,..., $\{v_{m-2}, v_{m-1}\}$, $\{v_m, v_1\}$. But the independent set S does not meet the clique $\{v_{m-1}, v_m\}$. Thus, definition of strongly perfect graph fails. Hence, lict graph $L_c(G)$ is not strongly perfect.

□

Theorem 4.3. *Lict graph of every friendship graph F_x, where x is the number of copies of cycle C_3 with $x \geq 2$ is strongly perfect.*

Proof. Let the graph G be a friendship graph F_x, where x is the number of copies of cycle C_3 with $x \geq 3$. Let $L_c(G)$ be the lict graph of a graph G. The obtained lict graph $L_c(G)$ contains a complete graph K_{2x+1} and x number of cycles C_3. On observation, it is seen that every odd cycle of length at least five in the lict graph $L_c(G)$ has at least two chords. Thus, by Theorem 2.3, the obtained lict graph is strongly perfect.

□

Theorem 4.4. *Lict graph of every star graph $K_{1,n}$, where $n \geq 2$ is strongly perfect.*

Proof. Let G be a star graph $K_{1,n}$ where $n \geq 2$.
Let $L_c(G)$ be the lict graph of G. The obtained lict graph $L_c(G)$ of a star graph $K_{1,n}$ consists of a complete graph K_{n+1} with $n \geq 2$.
Consider the following two cases,
Case 1. When $n = 2$ and $n = 3$.
The lict graph of a graph G when $n = 2$ is K_3 and when $n = 3$ is K_4. The so obtained lict graphs $L_c(G)$ has no induced P_4. Thus by Theorem 2.2, the lict graph of a graph G when $n = 2$ and $n = 3$ are strongly perfect.

Case 2. When $n \geq 4$.
For $n \geq 4$ the obtained lict graph $L_c(G)$ contains complete graphs K_{n+1} where $n \geq 4$ respectively. These lict graphs have odd cycle of length at least five with at least two chords. By Theorem 2.3, the obtained lict graphs are strongly perfect.

□

Theorem 4.5. *If G is a helm graph H_y with $y \geq 3$, then lict graph $L_c(G)$ of a graph G is*

$$L_c(G) = L_c(H_y) = \begin{cases} \textit{strongly perfect} & \textit{if } y = 3 \\ \textit{not strongly perfect} & \textit{if } y > 3. \end{cases}$$

Proof. Let $G = H_y$ with $y \geq 3$ be a helm graph.
Let $L_c(G)$ be the lict graph of G and H be any induced subgraph of $L_c(G)$.

Consider the following two cases,
Case 1. If $y = 3$.
The lict graph $L_c(G)$ of a helm graph H_3 contains an odd cycle of length at least five with at least two chords. Thus by Theorem 2.3 lict graph $L_c(G)$ of H_3 is strongly perfect.

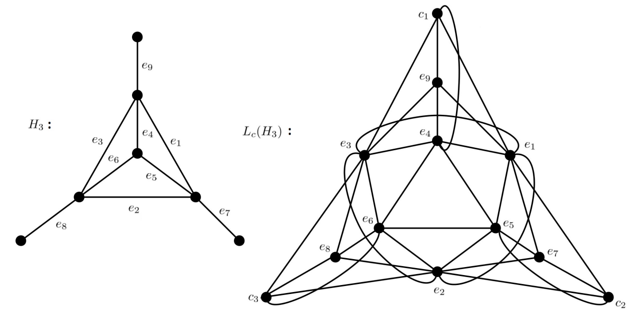

Figure 14. Helm graph H_3 and its lict graph $L_c(H_3)$.

Case 2. If $y > 3$.
This case consists of two subcases,
Subcase 2.1. When y is even.
Let $y = 4$. The helm graph considered is H_4 and its lict graph is $L_c(H_4)$.

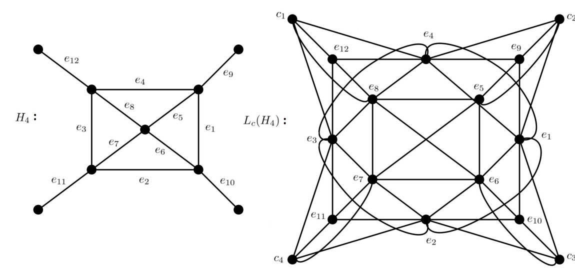

Figure 15. Helm graph H_4 and its lict graph $L_c(H_4)$.

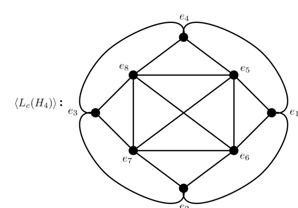

Figure 16. Induced subgraph of lict graph $L_c(H_4)$.

Consider one of the induced subgraph of lict graph $L_c(H_4)$ which is as shown in the Figure 16, has an independent set $S = \{e_1, e_8\}$ which does not meet some part of the clique $\{e_2, e_3, e_7\}$. Thus definition of strongly perfect graph is not satisfied. Hence lict graph $L_c(H_4)$ is not strongly perfect. For the remaining values of y (i.e., $y = 6, 8, \ldots$) of helm graph the same arugment holds. Thus respective lict graphs of helm graphs are not strongly perfect.
Subcase 2.2. When y is odd.
Let $y = 5$. The helm graph considered is H_5 and its lict graph is $L_c(H_5)$.

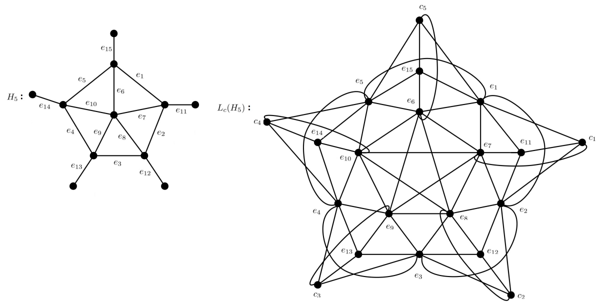

Figure 17. Helm graph H_5 and its lict graph $L_c(H_5)$.

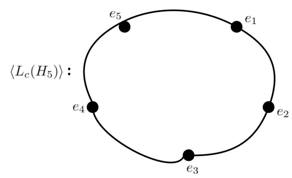

Figure 18. Induced subgraph of lict graph $L_c(H_5)$.

Consider one of the induced subgraph of lict graph $L_c(H_5)$ as shown in the Figure 18, which is an odd cycle of length 5. It has an independent set $S = \{e_1, e_3\}$ which doesnot meet the clique $\{e_4, e_5\}$. Thus definition of strongly perfect graph is not satisfied. Hence lict graph $L_c(H_5)$ is not strongly perfect. For the remaining values of y (i.e., $y = 7, 9, \ldots$) of helm graph the same arugment holds. Thus respective lict graphs of helm graphs are not strongly perfect. Hence for $y > 3$, lict graphs $L_c(G)$ of respective helm graphs are not strongly perfect.

□

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