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Price dependent demand model for deterioration and Weibull Amelioration

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Abstract

This paper presents a model of deterministic inventory with amelioration and deterioration. In this model, we have considered price dependent rate of demand, constant rate of deterioration, and varying holding cost. The objective of this model is to minimize the total cost. Numerical examples of the result have been given with sensitivity analysis.

Keywords

Amelioration, Deterioration, Total cost.

AMS Subject Classification

90B50.

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Contents

1. Introduction

Ameliorating items are those items whose economical value gets increased with time So far many models have been presented on deterioration but very less work is done on amelioration.Hwang [2] first introduced amelioration in the inventory model later Hwang[3] extended his work considering amelioration and deterioration both after that some other researchers presented models on the same considering different conditions. recently M. Valliathal et.al.[4],Minakshi Mallick et.al.[5],G.Santhi et.al.[1],P. D. Khatri et.al.[6],Yusuf I.Gwanda et.al.[7] established models on amelioration and deterioration with time-varying demand condition, Fully Backlogged Shortages, Price discount, Time-dependent holding Cost respectively. highbred fishes in pond and ducks, broiler, pigs, rabbits, chickens, etc. in the poultry farm are the examples of amelioration and deterioration whose value increase with time and decrease due to various ailment. In this paper we have developed an inventory model considering Price dependent demand rate,time-varying holding cost to minimize total cost.

2. Notations with Assumptions

2.1 Notations

$I(t)$ level of inventory at t

$\theta(t)$ Rate of Deterioration, θ(t)=θ, $0 < \theta < 1$

$A(t)$ Rate of Amelioration, $A(t) = \alpha\beta t^{\beta-1}, \alpha > 0, \beta > 0$

p Price of selling per unit item

$D(p)$ Rate of demand $D(p) = a + bp + cp^2$, $a \geq 0, b \neq 0, c \neq 0$.

T Cycle length

t_1 Period length when inventory in hand

Q_1 Initial level of Inventory

Q_2 Shortage of inventory

Q Order Quantity per cycle

A Ordering cost

$g+ht$ Holding cost

C_1 Shortage cost of inventory per unit time

C_2 Unit cost of an item

2.2 Assumptions

Rate of demand is the function of p.
Shortages are permitted and backlogged totally .
Replenishment is instantaneous.
Lead time is zero.

3. Mathematical Model Formulation

Level of $I(t)$ in $(0,T)$ are given by eq (3.1) and eq (3.2)

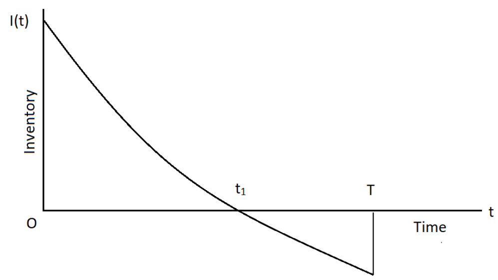

Figure 1. Inventory vs Time

$$\frac{dI(t)}{dt}+\theta(t)I(t)=A(t)-(a+bp+cp^2) \qquad 0\le t\le t_1 \tag{3.1}$$

$$\frac{dI(t)}{dt}=-(a+bp+cp^2) \qquad t_1\le t\le T \tag{3.2}$$

with $I(0)=Q_1$, $I(t_1)=0$ and $I(T)=-Q_2$.
On solving (3.1) and (3.2) with boundary conditions and neglecting higher powers of θ

$$I(t)=(a+bp+cp^2)\left[(t_1-t)+\frac{\theta}{2}(t_1^2-t^2)-\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{\alpha\theta t_1^{\beta+2}}{\beta+2}+\frac{\alpha\theta t^{\beta+2}}{\beta+2}\right], 0\le t\le t_1 \tag{3.3}$$

and

$$I(t)=(a+bp+cp^2)(t_1-T), \quad t_1\le t\le T \tag{3.4}$$

putting $t=0$ and $t=T$ in equations (3.3) and (3.4) respectively

$$Q_1=(a+bp+cp^2)\left[t_1+\frac{\theta t_1^2}{2}-\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta t_1^{\beta+2}}{\beta+2}\right] \tag{3.5}$$

$$Q_2=(a+bp+cp^2)(T-t_1) \tag{3.6}$$

Now $Q=Q_1+Q_2$, therefore

$$Q=(a+bp+cp^2)\left[T+\frac{\theta t_1^2}{2}-\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta t_1^{\beta+2}}{\beta+2}\right]. \tag{3.7}$$

Holding Cost

$$\begin{aligned}HC&=\int_0^{t_1}(g+ht)I(t)dt\\&=(a+bp+cp^2)\left[\frac{gt_1^2}{2}+\frac{g\theta t_1^3}{3}-\frac{g\alpha t_1^{\beta+2}}{\beta+2}-\frac{g\alpha\theta t_1^{\beta+3}}{\beta+3}\right.\\&\left.+\frac{ht_1^3}{6}+\frac{h\theta t_1^4}{8}-\frac{h\alpha t_1^{\beta+3}}{2(\beta+3)}-\frac{h\alpha\theta t_1^{\beta+4}}{2(\beta+4)}\right]\end{aligned} \tag{3.8}$$

Shortage Cost

$$\begin{aligned}SC&=-\int_{t_1}^{T}C_1I(t)dt\\&=C_1(a+bp+cp^2)\frac{(T-t_1)^2}{2}\end{aligned} \tag{3.9}$$

Purchase Cost

$$PC=C_2(a+bp+cp^2)\left[T+\frac{\theta t_1^2}{2}-\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta t_1^{\beta+2}}{\beta+2}\right] \tag{3.10}$$

Total cost per unit time is

$$\begin{aligned}&TC(T,t_1,p)\\&=\frac{1}{T}[OC+HC+SC+PC]\\&=\frac{1}{T}\left[A+(a+bp+cp^2)\left\{\left(\frac{gt_1^2}{2}+\frac{g\theta t_1^3}{3}-\frac{g\alpha t_1^{\beta+2}}{\beta+2}\right.\right.\right.\\&-\frac{g\alpha\theta t_1^{\beta+3}}{\beta+3}+\frac{ht_1^3}{6}+\frac{h\theta t_1^4}{8}-\frac{h\alpha t_1^{\beta+3}}{2(\beta+3)}-\left.\frac{h\alpha\theta t_1^{\beta+4}}{2(\beta+4)}\right)\\&\left.\left.+C_1\frac{(T-t_1)^2}{2}+C_2\left(T+\frac{\theta t_1^2}{2}-\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta t_1^{\beta+2}}{\beta+2}\right)\right\}\right]\end{aligned} \tag{3.11}$$

Let $t_1=\delta T, \quad 0<\delta<1$ then (3.11) becomes

$$\begin{aligned}&TC(T,p)\\&=\frac{1}{T}\left[A+(a+bp+cp^2)\left\{\left(\frac{g(\delta T)^2}{2}+\frac{g\theta(\delta T)^3}{3}\right.\right.\right.\\&-\frac{g\alpha(\delta T)^{\beta+2}}{\beta+2}-\frac{g\alpha\theta(\delta T)^{\beta+3}}{\beta+3}+\frac{h(\delta T)^3}{6}+\frac{h\theta(\delta T)^4}{8}\\&-\frac{h\alpha(\delta T)^{\beta+3}}{2(\beta+3)}-\left.\frac{h\alpha\theta(\delta T)^{\beta+4}}{2(\beta+4)}\right)+C_1\frac{(T-\delta T)^2}{2}\\&\left.\left.+C_2\left(T+\frac{\theta(\delta T)^2}{2}-\frac{\alpha(\delta T)^{\beta+1}}{\beta+1}-\frac{\alpha\theta(\delta T)^{\beta+2}}{\beta+2}\right)\right\}\right]\end{aligned} \tag{3.12}$$

The optimal values of $T = T^*$ and $p = p^*$ at which $TC(T,p)$ of eq (3.12) have minima can be evaluated by

$$\frac{\partial TC(T,p)}{\partial T} = 0 \text{ and } \frac{\partial TC(T,p)}{\partial p} = 0$$

Provided

$$\left[\frac{\partial^2 TC(T,p)}{\partial T^2}\right]\left[\frac{\partial^2 TC(T,p)}{\partial p^2}\right] - \left[\frac{\partial^2 TC(T,p)}{\partial T \partial p}\right]^2 > 0$$

and $\dfrac{\partial^2 TC(T,p)}{\partial T^2} > 0$, $\dfrac{\partial^2 TC(T,p)}{\partial p^2} > 0$

at $T = T^*$ and $p = p^*$

4. Example

Let parameters $A = 50, a = 300, b = -20, c = 2, g = 0.9, \alpha = 0.6, \beta = 2, \theta = 0.3, h = 0.6, C_1 = 0.9, C_2 = 1.5, , \delta = 0.1$ in appropriate units then using Mathematica 12.0 we get $T^* = 0.734165$, $p^* = 5$, $TC^*(T,p) = 511.228$.

5. Sensitivity Analysis

Table 1.

Parame -ters	Decision Variables	Percentage change in Parameters				
		-20	-10	0	10	20
A	T	0.656637	0.69648	0.734165	0.77011	0.804261
	p	5	5	5	5	5
	TC(T,p)	496.847	504.238	511.228	517.876	524.228
a	T	0.842184	0.782639	0.734165	0.69371	0.65928
	p	5	5	5	5	5
	TC(T,p)	403.758	457.792	511.228	564.171	616.699
b	T	0.709075	0.720598	0.734165	0.750091	0.768781
	p	4	4.5	5	5.5	6
	TC(T,p)	543.047	528.042	511.228	492.586	472.095
c	T	0.753244	0.742464	0.734165	0.727579	0.722223
	p	6.25	5.55556	5	4.54545	4.16667
	TC(T,p)	489.028	501.372	511.228	519.279	525.98
g	T	0.73508	0.734622	0.734165	0.73371	0.733255
	p	5	5	5	5	5
	TC(T,p)	511.06	511.144	511.228	511.311	511.395
C_1	T	0.81905	0.773133	0.734165	0.700554	0.671173
	p	5	5	4.40149	4.41511	4.4282
	TC(T,p)	1563.61	1519.73	1478.32	1439.02	1401.54
θ	T	0.734636	0.734401	0.734165	0.733931	0.733696
	p	5	5	5	5	5
	TC(T,p)	511.143	511.185	511.228	511.27	511.313
h	T	0.734195	0.73418	0.734165	0.734151	0.734136
	p	5	5	5	5	5
	TC(T,p)	511.225	511.126	511.228	511.229	511.23
C_2	T	0.734522	0.734343	0.734165	0.733988	0.73381
	p	5	5	5	5	5
	TC(T,p)	436.153	473.69	511.228	548.765	586.302
δ	T	0.72065	0.727416	0.734165	0.740893	0.747591
	p	5	5	5	5	5
	TC(T,p)	513.772	512.486	511.228	509.998	508.998

Table 2.

Parameter	Decision Variables	change in parameter		
		1	2	3
β	T	0.738595	0.734165	0.7333735
	p	5	5	5
	TC(T,p)	510.404	511.228	511.268

We have the above observations obtained by one parameter changing and all others fixed given in the example. The changes are displayed in table 1.

6. Conclusion

We have established the model with price dependent demand, varying holding cost, Weibull amelioration, and constant deterioration, totally backlogged shortage. This model is for those items where amelioration and deterioration take place at the same time e.g fish pond and poultry farm, applying various conditions we have minimized the total cost.

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