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Intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy prime ideals of near-rings

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Abstract

In this paper, we considered the concept of intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$- fuzzy ideals of a near-ring. Then we brought the concept of intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy prime ideals of near ring. We state and proved some theorems in intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy prime ideals of near ring.

Keywords

Near ring, $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideals of near-ring, intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$- fuzzy ideals of a near-ring, intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy prime ideals of a near-ring.

AMS Subject Classification

16Y30, 03E72, 03F55.

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Contents

1. Introduction

The concept of fuzzy was first introduced by Zadeh[12]. The idea of intuitionistic fuzzy set was introduced by Atanassov[2] as a generalization of notion of fuzzy sets. Abou-Zaid [1] introduced the concepts of fuzzy subnear-rings(ideals) and studied some of their related properties in near rings. The concept was discussed further by many researchers, see[4–8, 13, 15, 16]

A new type of fuzzy subgroup, that is, the $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy sub group, was introduced by Bhakat and Das[3] using the combined notions of the belongingness and quasicoincidence with the fuzzy points and fuzzy sets. It is now natural to investigate similar type of generalizations of the existing fuzzy subsystems with other algebraic structures, see[4, 5, 10, 11, 13, 14, 16]. In[4] Davvaz introduced the concepts of $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy subnear-rings (ideals) of near-rings and investigated some of their related properties. Zhan[13]considered the concept of $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy subnear-rings (ideals) of near-rings and obtained some of its related properties. Finally, some characterizations of $[\mu]_t$ by means of $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideals were also given. Zhan and Yin[16] redefined generalized fuzzy subnear-rings (ideals) of near-ring and investigated some of their related properties. Zhan and Yin[17] also introduce $(\in_\gamma, \in_\gamma \vee q_\delta)$- fuzzy subnear-rings (ideals) of a near-rings.

In this paper, the concept of intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy ideals of a near-rings is considered. We found the notion of intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy prime ideals of a near-ring and obtained some interesting results .

2. Preliminaries

Definition 2.1. *[3]* A fuzzy set μ of R of the form

$$\mu(y) = \begin{cases} t(\neq 0) & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t .A fuzzy point x_t is said to belong to (resp., be quasi-coincident with) a fuzzy set μ, written as $x_t \in \mu$ (resp., $x_t\ q\ \mu$) if $\mu(x) \geq t$ (resp.,$\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t\ q\ \mu$, then, we write $x_t \in \vee q\ \mu$. If $\mu(x) < t$ (resp., $\mu(x) + t \leq 1$) then, we call $x_t \overline{\in} \mu$ (resp., $x_t\ \overline{q}\ \mu$). We note that the symbol $(\overline{\in \vee q})$ means that $\in \vee\ q$ does not hold.

Result 2.2. *[17] Let $\gamma, \delta \in [0,1]$ be such that $\gamma < \delta$ .For a fuzzy point x_r and a fuzzy set μ of R, we say that*

1. $x_r \in_\gamma \mu$ *if* $\mu(x) \geq r > \gamma$.
2. $x_r\, q_\delta\, \mu$ *if* $\mu(x) + r > 2\delta$.
3. $x_r \in_\gamma \vee q_\delta\, \mu$ *if* $x_r \in_\gamma \mu$ *or* $x_r\, q_\delta\, \mu$.

Definition 2.3. *[17]* A fuzzy set μ of R is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subnear-ring of R if for all $t, r \in (\gamma, 1]$ and $x, y, a \in R$

1. a) $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu \Rightarrow (x+y)_{t\wedge r} \in_\gamma \vee q_\delta \mu$,
   b) $x_t \in_\gamma \mu \Rightarrow (-x)_t \in_\gamma \vee q_\delta \mu$,
2. $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu \Rightarrow (xy)_{t\wedge r} \in_\gamma \vee q_\delta \mu$,
   Moreover, μ is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideal of R if μ is$(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subnear-ring of R and
3. $x_r \in_\gamma \mu \Rightarrow (y+x-y)_r \in_\gamma \vee q_\delta \mu$,
4. $y_r \in_\gamma \mu$ and $x \in R \Rightarrow (xy)_r \in_\gamma \vee q_\delta \mu$,
5. $a_r \in_\gamma \mu \Rightarrow ((x+a)y - xy)_r \in_\gamma \vee q_\delta \mu$,

Definition 2.4. *[17]* An $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal μ of R is called prime if $\forall x, y \in R$ and $t \in (\gamma, 1]$. We have $(xy)_t \in_\gamma \mu \Rightarrow x_t \in_\gamma \vee q_\delta\, \mu$ (or) $y_t \in_\gamma \vee q_\delta\, \mu$

3. Main Results

In this section we are going to discuss about "INTUITIONISTIC $(\in_\gamma, \in_\gamma \vee q_\delta)$- FUZZY PRIME IDEALS OF NEAR-RINGS".

Definition 3.1. An Intuitionistic fuzzy set IFS $A = (\mu_A, \lambda_A)$ of a Near ring R,is called an Intuitionistic $(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal of R if for all $t, r \in (\gamma, 1]$ and $x, y, a \in R$.
(i)a) $x_t\ \varepsilon_\gamma\ \mu_A$ and $y_r\ \varepsilon_\gamma\ \mu_A\ \Rightarrow (x+y)_{t\wedge r}\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
b) $x_t\ \varepsilon_\gamma\ \mu_A\ \Rightarrow (-x)_t\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
(ii) $x_t\ \varepsilon_\gamma\ \mu_A$ and $y_r\ \varepsilon_\gamma\ \mu_A\ \Rightarrow (xy)_{t\wedge r}\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
(iii) $x_t\ \varepsilon_\gamma\ \mu_A\ \Rightarrow (y+x-y)_t\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
(iv) $y_t\ \varepsilon_\gamma\ \mu_A$ and $x \in R\ \Rightarrow (xy)_t\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
(v) $a_t\ \varepsilon_\gamma\ \mu_A\ \Rightarrow ((x+a)y - xy)_t\ \varepsilon_\gamma \vee\ q_\delta\ \mu_A$.
(vi)a)$x_t\ \varepsilon_\delta\ \lambda_A$ and $y_r\ \varepsilon_\delta\ \lambda_A\ \Rightarrow (x+y)_{t\vee r}\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.
b) $x_t\ \varepsilon_\delta\ \lambda_A\ \Rightarrow (-x)_t\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.
(vii) $x_t\ \varepsilon_\delta\ \lambda_A$ and $y_r\ \varepsilon_\delta\ \lambda_A\ \Rightarrow (xy)_{t\vee r}\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.
(viii)$x_t\ \varepsilon_\delta\ \lambda_A\ \Rightarrow (y+x-y)_t\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.
(ix) $y_t\ \varepsilon_\delta\ \lambda_A$ and $x \in R\ \Rightarrow (xy)_t\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.
(x) $a_t\ \varepsilon_\delta\ \lambda_A\ \Rightarrow ((x+a)y - xy)_t\ \varepsilon_\delta \vee\ q_\gamma\ \lambda_A$.

Definition 3.2. An Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal $A = (\mu_A, \lambda_A)$ of R is said to be an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R if $\forall\ x, y \in R$ and $t \in (\gamma, 1]$
(a)$(xy)_t \in_\gamma \mu_A \Rightarrow x_t \in_\gamma \vee q_\delta\ \mu_A$ (or) $y_t \in_\gamma \vee q_\delta\ \mu_A$
(b)$(xy)_t \in_\delta \lambda_A \Rightarrow x_t \in_\delta \vee q_\gamma\ \lambda_A$ (or) $y_t \in_\delta \vee q_\gamma\ \lambda_A$

Theorem 3.3. *An Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal $A = (\mu_A, \lambda_A)$ of R is prime if $\forall\ x, y \in R$ it satisfies*
$(a)^*\mu_A(x) \vee \mu_A(y) \vee \gamma \geq \mu_A(xy) \wedge \delta$
$(b)^*\lambda_A(x) \wedge \lambda_A(y) \wedge \delta \leq \lambda_A(xy) \vee \gamma$

Proof. $(a) \Rightarrow (a)^*$
Let $A = (\mu_A, \lambda_A)$ be an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal of R. If there exists $x, y \in R$ such that $\mu_A(x) \vee \mu_A(y) \vee \gamma < t = \mu_A(xy) \wedge \delta$
$(xy)_t \in_\gamma \mu_A$ but $x_t\ \overline{\in_\gamma}\ \mu_A$ and $y_t\ \overline{\in_\gamma}\ \mu_A$.
Since $\mu_A(x) + t < 2t \leq 2\delta$ and $\mu_A(y) + t < 2t \leq 2\delta$ then $x_t\ \overline{q_\delta}\ \mu_A$ and $y_t\ \overline{q_\delta}\ \mu_A$ and hence we have $x_t\ \overline{\varepsilon_\gamma \vee q_\delta}\ \mu_A$ and $y_t\ \overline{\varepsilon_\gamma \vee q_\delta}\ \mu_A$ which is a contradiction.Thus a^* holds.
$(a)^* \Rightarrow (a)$
Conversely, suppose that $\mu_A(x) \vee \mu_A(y) \vee \gamma \geq \mu_A(xy) \wedge \delta$.
Let $(xy)_t \in_\gamma \mu_A$. Then $\mu_A(xy) \geq t$. So $\mu_A(x) \vee \mu_A(y) \geq \mu_A(xy) \wedge \delta \geq t \wedge \delta$
We consider the following two cases.
(i)If $t \in (\gamma, \delta]$, then $\mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$ (i.e)$x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$.
Thus,$x_t \in_\gamma \vee q_\delta\ \mu_A$ (or) $y_t \in_\gamma \vee q_\delta\ \mu_A$
(ii)If $t \in (\delta, 1]$, then $\mu_A(x) \vee \mu_A(y) \geq \delta$ So $\mu_A(x) \geq \delta$ (or) $\mu_A(y) \geq \delta$
Hence,$\mu_A(x) + t > 2\delta$ (or) $\mu_A(y) + t > 2\delta$
(i.e)$x_t\ q_\delta\ \mu_A$ (or) $y_t\ q_\delta\ \mu_A$.
Thus, $x_t \in_\gamma \vee q_\delta\ \mu_A$ (or) $y_t \in_\gamma \vee q_\delta\ \mu_A$
$(b) \Rightarrow (b)^*$
Similarly, Let λ_A be an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal of R. If there exists $x, y \in R$ such that $\lambda_A(x) \wedge \lambda_A(y) \wedge \delta = t > \lambda_A(xy) \vee \gamma$, $(xy)_t \in_\delta \lambda_A$ but $x_t\ \overline{\in_\delta}\ \lambda_A$ and $y_t\ \overline{\in_\delta}\ \lambda_A$.
Since $\lambda_A(x) + t \geq 2t > 2\gamma$ and $\lambda_A(y) + t \geq 2t > 2\gamma$, then $x_t\ \overline{q_\gamma}\ \lambda_A$ and $y_t\ \overline{q_\gamma}\ \lambda_A$ and hence we have $x_t\ \overline{\varepsilon_\delta \vee q_\gamma}\ \lambda_A$ and $y_t\ \overline{\varepsilon_\delta \vee q_\gamma}\ \lambda_A$, which is a contradiction.Thus $\lambda_A(x) \wedge \lambda_A(y) \wedge \delta \leq \lambda_A(xy) \vee \gamma$.
Conversely, suppose that $\lambda_A(x) \wedge \lambda_A(y) \wedge \delta \leq \lambda_A(xy) \vee \gamma$
Let $(xy)_t \in_\delta \lambda_A$. Then $\lambda_A(xy) < t$ and so $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(xy) \vee \gamma < t \vee \gamma$
We consider the following two cases.
(i) If $t \in (\gamma, \delta]$, then $\lambda_A(x) < t$ (or) $\lambda_A(y) < t$.(i.e)$x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$.
Thus,$x_t \in_\delta \vee q_\gamma \lambda_A$ (or) $y_t \in_\delta \vee q_\gamma \lambda_A$
(ii) If $t \in (0, \gamma]$, then $\lambda_A(x) \wedge \lambda_A(y) < \gamma$ and so $\lambda_A(x) < \gamma$ (or) $\lambda_A(y) < \gamma$.Hence,$\lambda_A(x) + t \leq 2\gamma$ (or) $\lambda_A(y) + t \leq 2\gamma$. (i.e)$x_t\ q_\gamma\ \lambda_A$ (or) $y_t\ q_\gamma\ \lambda_A$.
Thus,$x_t \in_\delta \vee q_\gamma\ \lambda_A$ (or) $y_t \in_\delta \vee q_\gamma\ \lambda_A$
This proves that $A = (\mu_A, \lambda_A)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$ Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal of R. □

Theorem 3.4. *An Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy ideal $A = (\mu_A, \lambda_A)$ of a near ring R is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal iff A is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal.
Let $x, y \in R$ and $t \in (\gamma, \delta]$
(i)Let $(xy)_t \in_\gamma \mu_A \Rightarrow \mu_A(xy) \geq t$

Now,

$$\begin{aligned}\mu_A(x) \vee \mu_A(y) \vee \gamma &\geq \mu_A(xy) \wedge \delta \\ &\geq t \wedge \delta \\ &= t\end{aligned}$$

Therefore,$\mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$
$\Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
Therefore,$(xy)_t \in_\gamma \mu_A \Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
(ii)Again let $(xy)_t \in_\delta \lambda_A \Rightarrow \lambda_A(xy) < t$
Now,

$$\begin{aligned}\lambda_A(x) \wedge \lambda_A(y) \wedge \delta &\leq \lambda_A(xy) \vee \gamma \\ &< t \vee \gamma \\ &= t\end{aligned}$$

Therefore,$\lambda_A(x) < t$ (or) $\lambda_A(y) < t$
$\Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
Therefore,$(xy)_t \in_\delta \lambda_A \Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
Hence,$A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R.
Conversely, Let $A = (\mu_A, \lambda_A)$ be an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R.Then $\mu_A(xy) \geq t \Rightarrow (xy)_t \in_\gamma \mu_A$
Let $x, y \in R$ and $\mu_A(xy) = t$, $t \in (\gamma, \delta]$.
$\Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
$\Rightarrow \mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$
Now,

$$\begin{aligned}\mu_A(x) \vee \mu_A(y) \vee \gamma &\geq t \vee t \vee \gamma \\ &= t = t \wedge \delta \\ &= \mu_A(xy) \wedge \delta\end{aligned}$$

Therefore,$\mu_A(x) \vee \mu_A(y) \vee \gamma \geq \mu_A(xy) \wedge \delta$
Again let $x, y \in R$ such that $\lambda_A(xy) = t$, $t \in (\gamma, \delta]$.
Then $\lambda_A(xy) < t \Rightarrow (xy)_t \in_\delta \lambda_A$
$\Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
$\Rightarrow \lambda_A(x) < t$ (or) $\lambda_A(y) < t$
Now,

$$\begin{aligned}\lambda_A(x) \wedge \lambda_A(y) \wedge \delta &\leq t \wedge t \wedge \delta \\ &= t = t \vee \gamma \\ &= \lambda_A(xy) \vee \gamma\end{aligned}$$

Therefore,$\lambda_A(x) \wedge \lambda_A(y) \wedge \delta \leq \lambda_A(xy) \vee \gamma$
Therefore,$A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R. □

Theorem 3.5. *An $A = (\mu_A, \lambda_A)$ is a Intuitionistic (q_δ, q_γ) fuzzy prime ideal iff $A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an Intuitionistic (q_δ, q_γ) fuzzy prime ideal of R.
Let $x, y \in R$ such that $(xy)_t \in_\gamma \mu_A \Rightarrow \mu_A(xy) \geq t$
$\Rightarrow \mu_A(xy) + t > 2t > 2\delta \Rightarrow (xy)_t\ q_\delta\ \mu_A$
Since $A = (\mu_A, \lambda_A)$ is a Intuitionistic (q_δ, q_γ) fuzzy prime ideal, we have $x_t\ q_\delta\ \mu_A$ (or) $y_t\ q_\delta\ \mu_A$
$\Rightarrow \mu_A(x) + t > 2\delta$ (or) $\mu_A(y) + t > 2\delta$
$\Rightarrow \mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$
$\Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
Therefore,$(xy)_t \in_\gamma \mu_A \Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
Let $x, y \in R$ such that $(xy)_t \in_\delta \lambda_A \Rightarrow \lambda_A(xy) < t$
$\Rightarrow \lambda_A(xy) + t < 2t \leq 2\gamma \Rightarrow (xy)_t\ q_\gamma\ \lambda_A$
Since $A = (\mu_A, \lambda_A)$ is a Intuitionistic (q_δ, q_γ) fuzzy prime ideal, we have $x_t\ q_\gamma\ \lambda_A$ (or) $y_t\ q_\gamma\ \lambda_A$
$\Rightarrow \lambda_A(x) + t \leq 2\gamma$ (or) $\lambda_A(y) + t \leq 2\gamma$
$\Rightarrow \lambda_A(x) < t$ (or) $\lambda_A(y) < t$
$\Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
Therefore,$(xy)_t \in_\delta \lambda_A \Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
Hence,$A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R.
Conversely, assume that $A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R.
Let $(xy)_t\ q_\delta\ \mu_A \Rightarrow \mu_A(xy) + t > 2\delta \Rightarrow \mu_A(xy) > 2\delta - t \geq 2t - t = t$
$\Rightarrow \mu_A(xy) \geq t$
$\Rightarrow (xy)_t \in_\gamma \mu_A$
Since $A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R, we have $x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
$\Rightarrow \mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$
$\Rightarrow \mu_A(x) + t > 2t \geq 2\delta$ (or) $\mu_A(y) + t > 2t \geq 2\delta$
$\Rightarrow \mu_A(x) + t > 2\delta$ (or) $\mu_A(y) + t > 2\delta$
$\Rightarrow x_t\ q_\delta\ \mu_A$ (or) $y_t\ q_\delta\ \mu_A$
Therefore,$(xy)_t\ q_\delta\ \mu_A \Rightarrow x_t\ q_\delta\ \mu_A$ (or) $y_t\ q_\delta\ \mu_A$
Let $(xy)_t\ q_\gamma\ \lambda_A \Rightarrow \lambda_A(xy) + t < 2\gamma \Rightarrow \lambda_A(xy) < 2\gamma - t < 2t - t = t$
$\Rightarrow \lambda_A(xy) < t$
$\Rightarrow (xy)_t \in_\delta \lambda_A$
Since $A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R, we have $x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
$\Rightarrow \lambda_A(x) < t$ (or) $\lambda_A(y) < t$
$\Rightarrow \lambda_A(x) < 2t - t = 2\gamma - t$ (or) $\lambda_A(y) < 2t - t = 2\gamma - t$
$\Rightarrow \lambda_A(x) + t \leq 2\gamma$ (or) $\lambda_A(y) + t \leq 2\gamma$
$\Rightarrow x_t\ q_\gamma\ \lambda_A$ (or) $y_t\ q_\gamma\ \lambda_A$
Therefore,$A = (\mu_A, \lambda_A)$ is a Intuitionistic (q_δ, q_γ) fuzzy prime ideal of R. □

Remark 3.6. *The notion of Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal, Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal and Intuitionistic (q_δ, q_γ) fuzzy prime ideal are equivalent.*

Theorem 3.7. *If an IFS $A = (\mu_A, \lambda_A)$ of a near ring R is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R and $\mu_A(x) < \delta, \lambda_A(x) > \gamma\ \forall x, y \in R$ then $A = (\mu_A, \lambda_A)$ is also an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R and $\mu_A(x) < \delta$,$\lambda_A(x) > \gamma\ \forall x, y \in R$
Let $(xy)_t \in_\gamma \mu_A \Rightarrow \mu_A(xy) \geq t$
Therefore,$t \leq \mu_A(xy)$
We know that,
$\mu_A(xy) \wedge \delta \leq \mu_A(x) \vee \mu_A(y) \vee \gamma$

$t \wedge \delta < \delta \vee \delta \vee \gamma$
$t \leq \mu_A(xy) < \delta$
and also $\mu_A(x) < \delta, \mu_A(y) < \delta, t < \delta$
Therefore,$\mu_A(x) + t < \delta + \delta = 2\delta, \mu_A(y) + t < \delta + \delta = 2\delta$
$\Rightarrow (xy)_t \in_\gamma \mu_A \Rightarrow x_t \in_\gamma \mu_A$ (or) $y_t \in_\gamma \mu_A$
Again let $(xy)_t \in_\delta \lambda_A \Rightarrow \lambda_A(xy) < t$
$\lambda_A(x) \wedge \lambda_A(y) \wedge \delta \leq \lambda_A(xy) \vee \gamma$
$t > \lambda_A(xy) > \gamma$
$\lambda_A(x) > \gamma, \lambda_A(y) > \gamma, t > \gamma$
Therefore,$\lambda_A(x) + t > \gamma + \gamma = 2\gamma, \lambda_A(y) + t > \gamma + \gamma = 2\gamma$
$\Rightarrow x_t q\ \gamma\ \lambda_A$ (or) $y_t\ q_\gamma\ \lambda_A$
Hence $(xy)_t \in_\delta \lambda_A \Rightarrow x_t \in_\delta \lambda_A$ (or) $y_t \in_\delta \lambda_A$
Therefore,$A = (\mu_A, \lambda_A)$ is an Intuitionistic $(\in_\delta, \in_\gamma)$ fuzzy prime ideal of R. □

Theorem 3.8. *An IFS $A = (\mu_A, \lambda_A)$ of a near ring R is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R iff the set $(\mu_A)_t = \{x \in X / \mu_A(x) \geq t, t \in (0, 0.5]\}$ and $(\lambda_A)_s = \{x \in X / \lambda_A(x) < s, s \in (0.5, 1]\}$ are prime ideals of R.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R. Let $t \in (0, 0.5]$ and $(xy) \in_\gamma (\mu_A)_t$
Therefore,$\mu_A(xy) \geq t$
It follows that

$$\begin{aligned}\mu_A(x) \vee \mu_A(y) \vee \gamma &\geq \mu_A(xy) \wedge \delta \\ &\geq t \wedge 0.5 \\ &= t\end{aligned}$$

Therefore,$\mu_A(x) \geq t$ (or) $\mu_A(y) \geq t$
(i.e)$x \in_\gamma (\mu_A)_t$ (or) $y \in_\gamma (\mu_A)_t$
Therefore,$(\mu_A)_t$ is a prime ideal of R
Let $s \in (0.5, 1]$ and $(xy) \in_\delta (\lambda_A)_s$
Therefore,$\lambda_A(xy) < s$
It follows that

$$\begin{aligned}\lambda_A(x) \wedge \lambda_A(y) \wedge \delta &\leq \lambda_A(xy) \vee \gamma \\ &< s \vee 0.5 \\ &= s\end{aligned}$$

Therefore,$\lambda_A(x) < s$ (or) $\lambda_A(y) < s$
(i.e)$x \in_\delta (\lambda_A)_s$ (or) $y \in_\delta (\lambda_A)_s$
Therefore,$(\lambda_A)_s$ is a prime ideal of R.
Conversely, let $A = (\mu_A, \lambda_A)$ be an IFS and the sets $(\mu_A)_t, (\lambda_A)_s$ are prime ideals of R.
Suppose $A = (\mu_A, \lambda_A)$ is not an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R, then there exists some $p, q \in R$ such that atleast one of conditions
$\mu_A(p) \vee \mu_A(q) \vee \gamma < \mu_A(pq) \wedge \delta$ and
$\lambda_A(p) \wedge \lambda_A(q) \wedge \delta > \lambda_A(pq) \vee \gamma$ hold.
Suppose $\mu_A(p) \vee \mu_A(q) \vee \gamma < \mu_A(pq) \wedge \delta$ hold. Then choose t such that
$\mu_A(p) \vee \mu_A(q) \vee \gamma < t \leq \mu_A(pq) \wedge \delta \quad \rightarrow (1)$
$\Rightarrow t \leq \mu_A(pq)$
(i.e)$pq \in_\gamma (\mu_A)_t$
Since $(\mu_A)_t$ is an ideal, it follows that $p \in_\gamma (\mu_A)_t$ (or) $q \in_\gamma (\mu_A)_t$
(i.e)$\mu_A(p) > t$ (or) $\mu_A(q) > t$
which contradicts (1)
Therefore,$\mu_A(p) \vee \mu_A(q) \vee \gamma \geq \mu_A(pq) \wedge \delta$
Again if $\lambda_A(p) \wedge \lambda_A(q) \wedge \delta > \lambda_A(pq) \vee \gamma$ hold. Then choose s such that
$\lambda_A(p) \wedge \lambda_A(q) \wedge \delta > s > \lambda_A(pq) \vee \gamma \quad \rightarrow (2)$
$\Rightarrow s > \lambda_A(pq)$
(i.e)$pq \in_\delta (\lambda_A)_s$
Since $(\lambda_A)_s$ is a prime ideal,
$\Rightarrow p \in_\delta (\lambda_A)_s$ (or) $q \in_\delta (\lambda_A)_s$
(i.e)$\lambda_A(p) < s$ (or) $\lambda_A(q) < s$
which contradicts (2)
Therefore,$\lambda_A(p) \wedge \lambda_A(q) \wedge \delta \leq \lambda_A(pq) \vee \gamma$
Hence,$A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R. □

Theorem 3.9. *Let A be a nonempty subset of a near ring R. Consider the IFS $A = (\mu_A, \lambda_A)$ in R is defined by*

$$\mu_A(x) = \begin{cases} 1 \ if\ x \in A \\ 0 \ otherwise \end{cases} \qquad \lambda_A(x) = \begin{cases} 0 \ if\ x \in A \\ 1 \ otherwise \end{cases}$$

then A is a prime ideal of R iff $A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R.

Proof. Let A be a prime ideal of R, then $(\mu_A)_t = \{x \in X / \mu_A(x) \geq t\}\ \forall\ t \in (0, 0.5] = A$ and $(\lambda_A)_s = \{x \in X / \lambda_A(x) < s\}\ \forall\ s \in (0.5, 1] = A$ which is a prime ideal. Hence by previous theorem, $A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R
Conversely,assume that $A = (\mu_A, \lambda_A)$ is an Intuitionistic$(\in_\gamma, \in_\gamma \vee q_\delta)$ fuzzy prime ideal of R
Let $xy \in_\gamma A$ then $\mu_A(xy) = 1$
Now,

$$\begin{aligned}\mu_A(x) \vee \mu_A(y) \vee \gamma &\geq \mu_A(xy) \wedge \delta \\ &= 1 \wedge \delta \\ &= \delta\end{aligned}$$

Therefore,$\mu_A(x) \geq \delta$ (or) $\mu_A(y) \geq \delta$
$\Rightarrow \mu_A(x) = 1$ (or) $\mu_A(y) = 1$
$\Rightarrow x \in_\gamma A$ (or) $y \in_\gamma A$
Again if $xy \in_\gamma A$ then $\lambda_A(xy) = 0$
Now,

$$\begin{aligned}\lambda_A(x) \wedge \lambda_A(y) \wedge \delta &\leq \lambda_A(xy) \vee \gamma \\ &= o \vee \gamma \\ &= \gamma\end{aligned}$$

$\Rightarrow \lambda_A(x) \wedge \lambda_A(y) \leq \gamma$
$\Rightarrow \lambda_A(x) \leq \gamma$ (or) $\lambda_A(y) \leq \gamma$
$\Rightarrow \lambda_A(x) = 0$ (or) $\lambda_A(y) = 0$
$\Rightarrow x \in_\gamma A$ (or) $y \in_\gamma A$
Therefore,$xy \in_\gamma A \Rightarrow x \in_\gamma A$ (or) $y \in_\gamma A$
Hence,A is a prime ideal of R □

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