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Fuzzy chaotic centred pre-distinctiveness space

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Abstract

In this paper, the concept of fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, Efremovie property, reverse Kolmogorov property and weak nested neighbourhood property are introduced and studied. Some of their related properties are discussed.

Keywords

Fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, Efremovie property, reverse Kolmogorov property and weak nested neighbourhood property.

AMS Subject Classification

54A40, 03E72.

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Contents

1. Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [12].Thereafter, fuzzy set theory found applications in different areas of mathematics and its applications in other sciences. Fuzzy sets have applications in many fields such as information [7] and control [8]. Chang [4] introduced and developed the concept of fuzzy topological spaces. In 2007, the concept of centred sysytems in fuzzy topological spaces introduced by Uma, Roja and Balasubramanian [10]. The concept of chaotic in general metric space was introduced by R. L. Devaney [5]. The elementary properties of chaos (Devaney definition of chaos) were established in [1] and [2]. Futhermore, the properties of chaos were developed and studied in [11]. In this paper, the concept of fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, Efremovie property, reverse Kolmogorov property and weak nested neighbourhood property are introduced and studied. Some of their interesting properties are discussed.

2. Preliminaries

Definition 2.1. **[12]** A fuzzy set in X is a function with domain X and values in I, that is an element of I^X.

Definition 2.2. **[3]** A ditopology on a texture (S, $\mathfrak{S}$) is a pair (τ, κ) of subsets of $\mathfrak{S}$, where the set of open sets τ satisfies

(1) S, $\phi \in \tau$,

(2) $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$ and

(3) $G_i \in \tau$, i $\in$ I, $\Rightarrow \vee_i G_i \in \tau$ and

the set of closed sets κ satisfies

(1) S, $\phi \in \kappa$,

(2) $K_1, K_2 \in \kappa \Rightarrow K_1 \cup K_2 \in \kappa$ and

(3) $K_i \in \kappa$, i $\in$ I, $\Rightarrow \wedge_i K_i \in \kappa$.

Hence a ditopology essentially a "topology" for which there is no a priori relation between the open and closed sets. But if σ is a complementation on (S, $\mathfrak{S}$) and τ, κ are connected by the relation $\kappa = \sigma(\tau)$, then we call (τ, κ) a complemented ditopology on (S, $\mathfrak{S}$, σ).

For A $\in \tau$ we define the closure [A] or cl(A) and the interior]A[or int(A) under (τ, κ) by the equalities
cl(A) = $\cap$ { K $\in \kappa$ / A $\subset$ K } and
int(A) = $\cup$ { G $\in \tau$ / A $\subseteq$ G }.

Definition 2.3. [6] Let X be a nonempty set and let f : X→ X be any mapping. Let λ be any fuzzy set in X. The fuzzy orbit $O_f(\lambda)$ of λ under the mapping f is defined as $O_f(\lambda) = \{\lambda, f(\lambda), f^2(\lambda), ...\}$.

Definition 2.4. [6] Let X be a nonempty set and let f : X→ X be any mapping. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \{\lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge ...\}$the intersection of all members of $O_f(\lambda)$.

Definition 2.5. [6] Let (X, τ) be a fuzzy topological space. Let f : X → X be any mapping. The fuzzy orbit set under the mapping f which is in fuzzy topology τ is called fuzzy orbit open set under the mapping f. Its complement is called a fuzzy orbit closed set under the mapping f.

Definition 2.6. [6] Let X be a nonempty set and let f : X → X be any mapping. Then a fuzzy set γ of X is called fuzzy periodic set with respect to f if $f^n(\gamma) = \gamma$, for some n ∈ Z_+. Smallest of these n is called fuzzy periodic of X.

Definition 2.7. [6] Let (X, τ) be a fuzzy topological space and let f : X → X be any mapping. The fuzzy periodic set with respect to f which is in fuzzy topology τ is called fuzzy periodic open set with respect to f. Its complement is called a fuzzy periodic closed set with respect to f.

Notation 2.8. P = ∧ { fuzzy periodic open sets with respect to f }

Definition 2.9. [9] Let (X, τ) be a fuzzy topological space and λ ∈ KF(X) (Where KF(X) is a collection of all nonempty fuzzy compact subsets of X).Let f : X → X be any mapping. Then f is fuzzy chaotic with respect to λ if

(i) cl $FO_f(\lambda) = 1$,

(ii) P is fuzzy dense.

Notation 2.10. (i) FC (λ) = { f : X → X / f is fuzzy chaotic with respect to λ where λ is a fuzzy set in X }.

(ii) FCH(X) = { λ ∈ KF(X) / FC(λ) ≠ ϕ }.

Definition 2.11. [9] A fuzzy topological space (X, τ) is called a fuzzy chaos space if FCH (X) ≠ ϕ . If (X, τ) is fuzzy chaos space then the elements of the FCH(X) are called chaotic sets in X.

Definition 2.12. [9] Let (X, τ) be a fuzzy chaos space. Let $\mathfrak{C}$ be the collection of fuzzy chaotic sets in X satisfying the following conditions:

(i) $0, 1 \in \mathfrak{C}$,

(ii) if $\mu_1, \mu_2 \in \mathfrak{C}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{C}$,

(iii) if $\{\mu_j : j \in J\} \subset \mathfrak{C}$, then $\vee_{j \in J} \mu_j \in \mathfrak{C}$.

Then $\mathfrak{C}$ is called the fuzzy chaotic structure in X. The triple (X, τ, $\mathfrak{C}$) is called fuzzy chaotic structure space. The elements of $\mathfrak{C}$ are called fuzzy chaotic open sets. The complement of fuzzy chaotic open set is called fuzzy chaotic closed set.

Definition 2.13. [9] Let (X, τ, $\mathfrak{C}$) be a fuzzy chaotic Hausdorff space and let
p = { A_i } where each A_i is an fuzzy chaotic set. Then p is said to be a fuzzy chaotic centred system if any finite collection of A_i such that A_i q A_j for i ≠ j. The system p is said to be a fuzzy maximal chaotic centred system (or) fuzzy chaotic end if it cannot be included in any larger fuzzy chaotic centred system.

Notation 2.14. Let $\mathfrak{X}$ = { $p_i/i \in J$} be a non empty set where each p_i is a fuzzy chaotic centred system in fuzzy chaotic Hausdorff space (X, τ, $\mathfrak{C}$) and J be an indexed set. Now, $\mathfrak{P}(\mathfrak{X})$ denotes the power set of $\mathfrak{X}$.

3. Fuzzy Chaotic Centred Pre-Distinctiveness Space

Definition 3.1. Let $\mathfrak{X}$ = { $p_i/i \in J$} be a nonempty set with an inequality relation, where each p_i is a fuzzy chaotic centred system and J be an indexed set. Let $\mathfrak{R}$ be a relation between subsets of $\mathfrak{X}$ that satisfies the following conditions:

(i) $p_i \mathfrak{R} q_i$ implies ¬ ($p_i = q_i$)

(ii) $p_i \mathfrak{R} q_i$ implies $q_i \mathfrak{R} p_i$.

The types of complement for a subset A of $\mathfrak{X}$ are as follows:

$\neg A = \{ p_i \in \mathfrak{X}\colon p_i \notin A \}$,

$\sim A = \{ p_i \in \mathfrak{X}\colon \forall q_i \in \tau \text{ s.t } p_i \neq q_i \}$,

$-A = \{ p_i \in \mathfrak{X}\colon \{p_i\} \mathfrak{R} A \}$.

For $p_i \in \mathfrak{X}$, $\mathfrak{R}$ is a fuzzy chaotic centred pre-distinctiveness on $\mathfrak{X}$ if it satisfies the following four axioms:

(D1) $\mathfrak{X} \mathfrak{R} \phi$

(D2) $-A \subset \sim A$

(D3) $((A_1 \cup A_2) \mathfrak{R} (B_1 \cup B_2)) \Leftrightarrow \forall\, i, j \in \{1,2\}, A_i \mathfrak{R} B_j$

(D4) $-A \subset \sim B \Rightarrow -A \subset -B$.

Then the pair ($\mathfrak{X}$, $\mathfrak{R}$) is called a fuzzy chaotic centred pre-distinctiveness space. If in addition, $\mathfrak{R}$ satisfies

(D5) $p_i \in -A \Rightarrow \exists B \subset \mathfrak{X}$ such that $p_i \in -B$ and $\mathfrak{X} = -A \cup B$

then it is called a fuzzy chaotic centred distinctiveness space.

Definition 3.2. Let ($\mathfrak{X}$, $\mathfrak{R}$) be a fuzzy chaotic centred pre-distinctiveness space. Then the relation $\mathfrak{R}$ is said to be symmetric if for all $A, B \subset \mathfrak{X}$, $A \mathfrak{R} B \Leftrightarrow B \mathfrak{R} A$.

Definition 3.3. Let ($\mathfrak{X}$, $\mathfrak{R}$) be a fuzzy chaotic centred pre-distinctiveness space and let A, B be subsets of $\mathfrak{X}$. If $\mathfrak{R}$ is a symmetric relation and $A \mathfrak{R} B$,then A, B are said to be distinctive (from each other).

Notation 3.4. The fuzzy chaotic centred point set pre distinctiveness associated with the given set is obtained by defining $p_i \,\mathfrak{R}\, A \Leftrightarrow \{\ p_i\ \} \,\mathfrak{R}\, A$.

Proposition 3.5. Let $(\mathfrak{X}, \mathfrak{R})$ be a fuzzy chaotic centred pre-distinctiveness space and let S, T be subsets of $\mathfrak{X}$. If $S \,\mathfrak{R}\, T$, then $A \,\mathfrak{R}\, B$ for all $A \subset S$ and $B \subset T$.

Proof. Let $A \subset S$ and $B \subset T$. Then $S = A \cup S$ and $T = B \cup T$. Therefore
$A \cup S \,\mathfrak{R}\, B \cup T$. Hence by (D3) $A \,\mathfrak{R}\, B$. □

Proposition 3.6. In any fuzzy chaotic centred pre distinctiveness space $\mathfrak{X}$, $\phi \,\mathfrak{R}\, \phi$.

Proof. The proof follows from (D1) and Proposition 3.5 □

Proposition 3.7. Let $(\mathfrak{X}, \mathfrak{R})$ be a fuzzy chaotic centred pre-distinctiveness space and let S, T be subsets of $\mathfrak{X}$. If $A \,\mathfrak{R}\, B$, then $A \subset -B$ and $B \subset \sim A$.

Proof. Let $p_i \in A$. By Proposition 3.5 $\{\ p_i\ \} \,\mathfrak{R}\, B$, that is $p_i \in -B$. Therefore $A \subset -B$. By (D2) $A \subset \sim B$. Hence $B \subset \sim A$. □

Proposition 3.8. Let $(\mathfrak{X}, \mathfrak{R})$ be a fuzzy chaotic centred pre-distinctiveness space and let T be a subset of $\mathfrak{X}$ such that $-T$ is nonempty. Then $\phi \,\mathfrak{R}\, T$.

Proof. Let $p_i \in -T$. Then $\{\ p_i\ \} \,\mathfrak{R}\, T$ and $\phi \subset \{\ p_i\ \}$. By Proposition 3.5 $\phi \,\mathfrak{R}\, T$. □

Proposition 3.9. Let $(\mathfrak{X}, \mathfrak{R})$ be a symmetric fuzzy chaotic centred distinctiveness space, let $p_i \in \mathfrak{X}$ and $A \subset \mathfrak{X}$. If $p_i \in -A$, then $\mathfrak{X} = -\{p_i\ \} \cup -A$.

Proof. By (D5), there exists $S \subset \mathfrak{X}$ such that $p_i \in -S$ and $\mathfrak{X} = -A \cup S$. Since $p_i \,\mathfrak{R}\, S$, by proposition 3.5 if $q_i \in S$, then $p_i \,\mathfrak{R}\, \{\ q_i\ \}$. Since $(\mathfrak{X}, \mathfrak{R})$ is symmetric, $q_i \,\mathfrak{R}\, \{\ p_i\ \}$. Hence $S \subset -\{\ p_i\ \}$ and therefore $\mathfrak{X} = -\{p_i\ \} \cup -A$. □

Note 3.10. The following three axioms hold in fuzzy chaotic centred pre-distinctiveness space.

(E1) $A \,\mathfrak{R}\, B$ and $-B \subset \sim C \Rightarrow A \,\mathfrak{R}\, C$

(E2) $A \,\mathfrak{R}\, B$ and $-B \subset \neg C \Rightarrow A \,\mathfrak{R}\, C$.

(E3) $p_i \,\mathfrak{R}\, A \Rightarrow \forall\, q_i \in \mathfrak{X}$ either $p_i \neq q_i$ or $q_i \,\mathfrak{R}\, A$.

Proposition 3.11. If $\mathfrak{X}$ is a fuzzy chaotic centred pre distinctiveness space satisfying E1, then $A \,\mathfrak{R}\, B \Leftrightarrow A \,\mathfrak{R} \sim\sim B$, for all subsets A and B of $\mathfrak{X}$.

Proof. Assume that $A \,\mathfrak{R}\, B$. Since $-B \subset \sim B = \sim\sim\sim B$, $A \,\mathfrak{R} \sim\sim B$. Conversely, assume that $A \,\mathfrak{R} \sim\sim B$. Since $B \subset \sim\sim B$ and by Proposition 3.5, $A \,\mathfrak{R}\, B$. □

Definition 3.12. A fuzzy chaotic centred pre-distinctiveness space $\mathfrak{X}$ is said to have Efremovic property if $S \,\mathfrak{R}\, T \Rightarrow \exists\, E \subset \mathfrak{X}$ such that $S \,\mathfrak{R} \neg E$ and $E \,\mathfrak{R}\, T$.

Definition 3.13. A fuzzy chaotic centred pre-distinctiveness space $\mathfrak{X}$ is said to have reverse Kolmogorov property if $\forall\, p_i$, $q_i \in \mathfrak{X}$, $\forall\, S \subset \mathfrak{X}$ such that $p_i \in -S$ and $q_i \notin -S \Rightarrow p_i \neq q_i$.

Proposition 3.14. A fuzzy chaotic centred pre-distinctiveness space $\mathfrak{X}$ with Efremovic property has reverse Kolmogorov property.

Proof. Let $(\mathfrak{X}, \mathfrak{R})$ be a fuzzy chaotic centred pre-distinctiveness space. Let U be a subset of $\mathfrak{X}$ and let $p_i, q_i \in \mathfrak{X}$ such that $p_i \in -U$ and $q_i \notin -U$. By Efremovic property, there exists $E \subset \mathfrak{X}$ such that $p_i \,\mathfrak{R} \neg E$ and $E \,\mathfrak{R}\, U$. If $q_i \in E$, then $q_i \in -U$. This is a contradiction. Hence $\{\ q_i\ \} \subset \neg E$. By Proposition 3.5 $p_i \,\mathfrak{R}\, \{\ q_i\ \}$ and by (D2) $p_i \neq q_i$. □

Proposition 3.15. For a fuzzy chaotic centred pre distinctiveness space $\mathfrak{X}$, Efremovic property implies the axiom E2.

Proof. Let A, B and C be subsets of $\mathfrak{X}$. Let $A \,\mathfrak{R}\, B$ and $-B \subset \neg C$. By Efremovic property, there exists $E \subset \mathfrak{X}$ such that $A \,\mathfrak{R} \neg E$ and $E \,\mathfrak{R}\, B$. $E \subset -B \subset \neg C$ and therefore $C \subset \neg\neg C \subset \neg E$. By Proposition 3.5 $A \,\mathfrak{R}\, C$. Hence the proof. □

Definition 3.16. A fuzzy chaotic centred pre-distinctiveness space $\mathfrak{X}$ is said to be T_1 fuzzy chaotic centred pre-distinctiveness space if $\forall\, p_i, q_i \in \mathfrak{X}$ such that $p_i \neq q_i \Rightarrow p_i \,\mathfrak{R}\, \{\ q_i\ \}$.

Definition 3.17. A fuzzy chaotic centred pre-distinctiveness space is fuzzy chaotic pre-distinctiveness Hausdorff space if for every $p_i, q_i \in \mathfrak{X}$ such that $p_i \neq q_i$, there exists $U \subset \mathfrak{X}$, $V \subset \mathfrak{X}$ s.t $U \,\mathfrak{R}\, V$ and $p_i \in U$, $q_i \in V$.

Proposition 3.18. A symmetric T_1 fuzzy chaotic pre-distinctiveness space with Efremovic property is fuzzy chaotic pre-distinctiveness Hausdorff.

Proof. Let $\mathfrak{X}$ be a symmetric T_1 fuzzy chaotic pre-distinctiveness space and let $p_i, q_i \in \mathfrak{X}$ and $p_i \neq q_i$. Since $\mathfrak{X}$ is T_1, $p_i \,\mathfrak{R}\, \{q_i\ \}$. By Efremovic property and symmetry, there exists $V \subset \mathfrak{X}$ such that $p_i \,\mathfrak{R} \neg V$ and $q_i \,\mathfrak{R}\, V$. Let $U \equiv \neg V$, by (D2) $p_i \in -U$, $q_i \in -V$ and $-U \subset \sim \neg V \subset \sim -V$. □

Definition 3.19. A fuzzy chaotic centred pre-distinctiveness space $\mathfrak{X}$ is said to have weak nested neighbourhood property if $p_i \in -S$, then there exists $T \subset \mathfrak{X}$ such that $p_i \in -T$ and $\neg T \subset -S$.

Proposition 3.20. Let $\mathfrak{X}$ be a symmetric fuzzy chaotic centred pre-distinctiveness space and the Efremovic property implies the weak nested neighbourhood property.

Proof. Let $\mathfrak{X}$ be a fuzzy chaotic centred pre-distinctiveness space and A be a subset of $\mathfrak{X}$. Let $p_i \in -A$. Since $\mathfrak{X}$ is symmetric, $A \,\mathfrak{R}\, \{p_i\ \}$. By Efremovic property , there exists $E \subset \mathfrak{X}$ such that $A \,\mathfrak{R} \neg E$ and $E \,\mathfrak{R}\, \{p_i\ \}$. Then by symmetric property and Proposition 3.5, $\neg E \subset -A$ and $p_i \in -E$. □

Definition 3.21. Let $(\mathfrak{X}, \mathfrak{R})$ be a fuzzy chaotic centred pre-distinctiveness space and $\mathfrak{Y}$ be a nonempty subset of $\mathfrak{X}$. Define the relation $\mathfrak{R}_{\mathfrak{Y}}$ between subsets A, B of $\mathfrak{Y}$ by $A\ \mathfrak{R}_{\mathfrak{Y}}\ B \Leftrightarrow A\ \mathfrak{R}\ B$. We say that $\mathfrak{R}_{\mathfrak{Y}}$ is induced on $\mathfrak{Y}$ and it satisfies (D1-D3). If also, $(\mathfrak{Y} - A \subset \mathfrak{Y} \sim B) \Rightarrow (\mathfrak{Y} - A \subset \mathfrak{Y} - B)$, then $\mathfrak{R}_{\mathfrak{Y}}$ is fuzzy chaotic centred pre-distinctiveness on $\mathfrak{Y}$. The space $(\mathfrak{Y}, \mathfrak{R}_{\mathfrak{Y}})$ is called a fuzzy chaotic centred pre-distinctiveness subspace of $\mathfrak{X}$. If $\mathfrak{R}_{\mathfrak{Y}}$ satisfies (D5), then it is called a fuzzy chaotic centred distinctiveness subspace of $\mathfrak{X}$.

Definition 3.22. A fuzzy chaotic centred pre-distinctiveness space $(\mathfrak{X}, \mathfrak{R})$ or the pre-distinctiveness $\mathfrak{R}$ itself is a fuzzy chaotic centred locally decomposable if $\forall\ p_i \in \mathfrak{X}$ and $\forall\ S \subset \mathfrak{X}$ such that $p_i \in -S \Rightarrow \exists\ T \subset \mathfrak{X}$ such that $p_i \in -T$ and $\mathfrak{X} = -S \cup T$.

Proposition 3.23. Every nonempty subset of a fuzzy chaotic centred distinctiveness space is a fuzzy chaotic centred distinctiveness subspace.

Proof. Let $\mathfrak{Y}$ be a nonempty subset of a fuzzy chaotic centred distinctiveness space $\mathfrak{X}$. Let $\mathfrak{X}$ be a fuzzy chaotic centred locally decomposable and let $p_i \in -S$ and choose T such that $p_i \in -T$ and $\mathfrak{X} = -S \cup T$. For each $q_i \in \mathfrak{X}$, either $q_i \in -S$ or $q_i \in T$ hence $p_i \neq q_i$ and satisfies (E3). Therefore $\mathfrak{Y}$ has the reverse Kolmogorov property. To prove $(\mathfrak{Y}, \mathfrak{R}_{\mathfrak{Y}})$ is a fuzzy chaotic centred locally decomposable, consider, $q_i \in \mathfrak{Y}$ and $A \subset \mathfrak{Y}$ such that $q_i\ \mathfrak{R}_{\mathfrak{Y}}\ A$. Then $q_i\ \mathfrak{R}\ A$ in $\mathfrak{X}$, Therefore there exists $S \subset \mathfrak{X}$ such that $q_i\ \mathfrak{R}\ S$ in $\mathfrak{X}$ and $\mathfrak{X} = (\mathfrak{X} - A) \cup S$. Clearly, $Y = (\mathfrak{Y} - A) \cup (\mathfrak{Y} \cap S)$. Since $\mathfrak{Y} \cap S \subset S$, $q_i\ \mathfrak{R}\ (\mathfrak{Y} \cap S)$ in $\mathfrak{X}$ and therefore, $q_i\ \mathfrak{R}_{\mathfrak{Y}}\ (\mathfrak{Y} \cap S)$. Hence the proof. □

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