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Pythagorean triangles and addition of nonagonal, triangular numbers

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Abstract

Oblong numbers as figurate numbers, which were first studied by the Pythagoreans are studied in terms of special Pythagorean Triangles. The two consecutive sides and their perimeters of Pythagorean triangles are investigated. In this study, the perimeter of Pythagorean triangles is obtained as addition of nonagonal and triangular numbers.

Keywords

Nonagonal numbers, Triangle numbers, Pythagorean Triangles, Diophantine equation.

AMS Subject Classification

11D09.

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Contents

1. Introduction

In 2005, Gopalan and Devibala [2] studied Special Pythagorean triangle. In 2008, Gopalan and Janaki [3] investigated Pythago -rean triangles with perimeter as a pentagonal number. In 2010, Gopalan and Vijayalakshmi [1] observed Special Pythagorean triangles generated through the integral solutions of the equation $y2 = (k2+1)x2+1$. After that Mita [4] investigated about oblong numbers and Pythagorean triangles. He found that perimeter of the Pythagorean triangles are as oblong numbers. In 2017, Jayakumar. P and Shankarakalidoss. G [5] and [6] investigated about Hexagonal numbers and Pythagorean triangles. He investigated that perimeter as a double of hexagonal numbers.

2. Method of Analysis

The primitive solutions of the Pythagorean Equation,

$$X^2+Y^2=Z^2 \tag{2.1}$$

is given by [5]

$$X=m^2-n^2, Y=2mn, Z=m^2+n^2 \tag{2.2}$$

for some integers m,n of opposite parity such that $m>n>0$ and $(m,n)=1$.

2.1 Perimeter is an addition of nonagonal and triangular numbers

Definition 2.1. *A natural number P is called addition of nonagonal and triangular numbers if it can be written in the form*

$$\frac{\left(7w^2-5w\right)}{2}+\frac{\left(w^2+w\right)}{2}=2\left(2w^2-w\right), w\in N.$$

If the perimeter of the Pythagorean triangle (X,Y,Z) is addition of nonagonal and triangular numbers W, then

$$X+Y+Z=2\left(2w^2-w\right)=P \tag{2.3}$$

From the equations (2.2) & (2.3) $2m^2+2mn=2\left(2w^2-w\right)$, $w\in N$

$$m(m+n)=w(2w-1) \tag{2.4}$$

2.2 Hypotenuse and one leg are consecutive

In such cases,

$$m = n + 1. \tag{2.5}$$

This gives equation (2.4) as $(n+1)(2n+1) = w(2w-1)$ Take,

$$w = n + 1. \tag{2.6}$$

Equations (2.2),(2.5) & (2.6) give solution of equations (2.1) in correspondence with equations (2.3) and (2.4) i.e., $X = 2n+1$; $Y = 2n(n+1)$; $Z = 2n(n+1)+1$;
First ten such special Pythagorean triangles (X,Y,Z) are given in the Table 1 below:

Table 1. Special Pythagorean Triangles

S. No.	n	w	P	X	Y	Z
1	1	2	12	3	4	5
2	2	3	30	5	12	13
3	3	4	56	7	24	25
4	4	5	90	9	40	41
5	5	6	132	11	60	61
6	6	7	182	13	84	85
7	7	8	240	15	112	113
8	8	9	306	17	144	145
9	9	10	380	19	180	181
10	10	11	462	21	220	221

Table 2. Verification of $X^2+Y^2=Z^2$ and $X+Y+Z = 2w(2w-1)$

S.No	X^2	Y^2	X^2+Y^2	Z^2	$X+Y+Z$ =2 w(2 w-1)
1	9	16	25	25	$12 = 2.2.3$
2	25	144	169	169	$30 = 2.3.5$
3	49	576	625	625	$56 = 2.4.7$
4	81	1600	1681	1681	$90 = 2.5.9$
5	121	3600	3721	3721	$132 = 2.6.11$
6	169	7056	7225	7225	$182 = 2.7.13$
7	225	12544	12769	12769	$240 = 2.8.15$
8	289	20736	21025	21025	$306 = 2.9.17$
9	361	32400	32761	32761	$380 = 2.10.19$
10	441	48400	48841	48841	$462 = 2.11.21$

3. Observations and conclusion

1. $(X+Y-Z)^2 = (Y+Z-2X+1)$
2. $(X+Z-Y)^2 = (Y+Z+2X+1)$
3. $Y+Z = X^2$
4. $(2X-Y+Z)^2 = X^2+2(X+Y+Z)+2(X+Z)$

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