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Fuzzy chaotic centred quasi-uniform space

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Abstract
In this paper, the concept of fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, fuzzy chaotic centred filter base, fuzzy chaotic centred quasi-uniform structure, fuzzy chaotic centred quasi-uniform space, fuzzy chaotic centred tight and fuzzy chaotic centred quasi-uniform subspace are introduced and their interesting properties are discussed.

Keywords
Fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, fuzzy chaotic centred filter base, fuzzy chaotic centred quasi-uniform structure, fuzzy chaotic centred quasi-uniform space, fuzzy chaotic centred tight and fuzzy chaotic centred quasi-uniform subspace.

AMS Subject Classification
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Contents

1. Introduction

Fuzzy set theory, which was established by Zadeh[12] (1965), has emerged as a incredible method of representing quantitatively and controlling the imprecision in decision-making problems. Fuzzy sets have applications in many fields such as information [7] and control [8]. Chang [4] introduced and developed the concept of fuzzy topological spaces. In 2007, the concept centred sysytems in fuzzy topological spaces introduced by Uma, Roja and Balasubramanian [10]. The concept of chaotic in general metric space was introduced by R. L. Devaney [5]. The elementary properties of chaos (Devaney definition of chaos) were established in [1] and [2]. Futhermore, the properties of chaos were developed and studied in [11]. In this paper, the concept of fuzzy chaotic centred pre-distinctiveness space, fuzzy chaotic centred distinctiveness space, fuzzy chaotic centred filter base, fuzzy chaotic centred quasi-uniform structure, fuzzy chaotic centred quasi-uniform space, fuzzy chaotic centred tight and fuzzy chaotic centred quasi-uniform subspace are introduced and their interesting properties are discussed.

2. Preliminaries

Definition 2.1. **[12]** A fuzzy set in X is a function with domain X and values in I, that is an element of I^X.

Definition 2.2. **[3]** Let S be a set. Then $\mathfrak{S} \subseteq \mathfrak{P}(s)$ is called a texturing of S and (S, $\mathfrak{S}$) is called a texture space or simply a texture if

(1) ($\mathfrak{S}$, $\subseteq$) is complete lattice containing S and ϕ which has the property that arbitrary meets coincide with intersections, and finite joins coincide with unions. ie.,
$\wedge_{i\in I} A_i = \cap_{i\in I} A_i$, $A_i \in T$, i $\in$ I, for all index sets I,
$\vee_{i\in I} A_i = \cup_{i\in I} A_i$, $A_i \in T$, i $\in$ I, for all index sets I.

(2) $\mathfrak{S}$ is completely distributive.

(3) $\mathfrak{S}$ separates the points of S. That is, given $s_1 \neq s_2$ in S, we have A $\in \mathfrak{S}$ with $s_2 \in$ A, $s_1 \notin$ A.

A surjection $\sigma : \mathfrak{S} \to \mathfrak{S}$ is called a complementation if σ^2(P) = P for all P $\in \mathfrak{S}$ and P $\subseteq$ Q in $\mathfrak{S}$ implies $\sigma(Q) \subseteq \sigma$(P). A texture with a complementation is said to be complemented.

Example 2.3. If X is a set and P(X) the powerset of X, then (X, P(X)) is the discrete texture on X. For x $\in$ X, P_x = x and Q_x = X / x.

Definition 2.4. **[6]** Let (X, τ) be a fuzzy topological space. Let f : X $\to$ X be any mapping. The fuzzy orbit set under the mapping f which is in fuzzy topology τ is called fuzzy orbit open set under the mapping f. Its complement is called a fuzzy orbit closed set under the mapping f.

Definition 2.5. **[9]** Let (X, τ) be a fuzzy topological space and $\lambda \in$ KF(X) (Where KF(X) is a collection of all nonempty fuzzy compact subsets of X).Let $f : X \to X$ be any mapping. Then f is fuzzy chaotic with respect to λ if

(i) cl $FO_f(\lambda) = 1$,

(ii) P is fuzzy dense.

Notation 2.6. (i) FC (λ) = { f : X $\to$ X / f is fuzzy chaotic with respect to λ where λ is a fuzzy set in X }.

(ii) FCH(X) = { $\lambda \in$ KF(X) / FC(λ) $\neq \phi$ }.

Definition 2.7. **[9]** A fuzzy topological space (X, τ) is called a fuzzy chaos space if FCH (X) $\neq \phi$. If (X, τ) is fuzzy chaos space then the elements of the FCH(X) are called chaotic sets in X.

Definition 2.8. **[9]** Let (X, τ) be a fuzzy chaos space. Let $\mathfrak{C}$ be the collection of fuzzy chaotic sets in X satisfying the following conditions:

(i) $0, 1 \in \mathfrak{C}$,

(ii) if $\mu_1, \mu_2 \in \mathfrak{C}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{C}$,

(iii) if $\{\mu_j : j \in J\} \subset \mathfrak{C}$, then $\vee_{j\in J}\mu_j \in \mathfrak{C}$.

Then $\mathfrak{C}$ is called the fuzzy chaotic structure in X. The triple (X, τ, $\mathfrak{C}$) is called fuzzy chaotic structure space. The elements of $\mathfrak{C}$ are called fuzzy chaotic open sets. The complement of fuzzy chaotic open set is called fuzzy chaotic closed set.

Definition 2.9. **[9]** A fuzzy chaotic structure space (X, τ, $\mathfrak{C}$) is called a fuzzy chaotic Hausdorff space if for each pair of non zero fuzzy chaotic sets λ and μ such that $\lambda \neq \mu$, then there exist fuzzy chaotic open sets γ and δ such that $\lambda \leq \gamma$ and $\mu \leq \delta$ and γ q δ.

3. Fuzzy Chaotic Centred Quasi-Uniform Space

Definition 3.1. Let $\mathfrak{X}$ = { $p_i/i \in J$} be a nonempty set of fuzzy chaotic centred systems and let U, V be subsets of the cartesian product of $\mathfrak{X} \times \mathfrak{X}$. Define certain subsets as follows:

$U \circ V \equiv \{ (p_i, q_i) : \exists\, r_i \in \mathfrak{X}$ such that $((p_i, r_i) \in U$ and $(r_i, q_i) \in V) \}$,

$U^1 \equiv U, U^{n+1} \equiv U \circ U^n$ (n = 1, 2,...),

$U^{-1} \equiv \{ (p_i, q_i) : (q_i, p_i) \in U \}$ and, for $p_i \in \mathfrak{X}$,

$U[\,p_i\,] \equiv \{ q_i \in \mathfrak{X} : (p_i, q_i) \in U \}$.

U is called symmetric if $U = U^{-1}$.

Definition 3.2. Let S be any set and $\mathscr{B}$ be a nonempty subset of S. Then $\mathscr{B}$ is called a fuzzy chaotic centred filter base on S if the intersection of two sets in $\mathscr{B}$ contains a set in $\mathscr{B}$.

Definition 3.3. Let S be any set and $\mathscr{B}$ is a fuzzy chaotic centred filter base on S. If $\mathscr{B}$ satisfies the following conditions:

(F1) The intersection of two sets in $\mathscr{B}$ belongs to $\mathscr{B}$

(F2) All supersets of sets in $\mathscr{B}$ belong to $\mathscr{B}$.

. Then $\mathscr{B}$ is called fuzzy chaotic centred filter on S.

Definition 3.4. Let $\mathfrak{X}$ be a nonempty set of fuzzy chaotic centred systems and let $\mathscr{U}$ be a family of subsets of $\mathfrak{X} \times \mathfrak{X}$. If $\mathscr{U}$ satisfies the following conditions:

(U1) $\mathscr{U}$ is a fuzzy chaotic centred filter on $\mathfrak{X} \times \mathfrak{X}$.

(U2) For all $(p_i, q_i) \in \mathfrak{X}$, $p_i = q_i$ if and only if $(p_i, q_i) \in U$ for each $U \in \mathscr{U}$.

(U3) For each $U \in \mathscr{U}$ there exists $V \in \mathscr{U}$ such that $V^2 \subset U$.

(U4) For each $U \in \mathscr{U}$ there exists $V \in \mathscr{U}$ such that $\mathfrak{X} \times \mathfrak{X} = U \cup \neg V$.

Then $\mathscr{U}$ is called a fuzzy chaotic centred quasi-uniform structure on $\mathfrak{X}$. The elements of $\mathscr{U}$ are called the fuzzy chaotic centred entourages of (fuzzy chaotic centred quasi-uniform structure on) $\mathfrak{X}$, and the pair $(\mathfrak{X}, \mathscr{U})$ itself is called a fuzzy chaotic centred quasi-uniform space.

Note 3.5. The intersection of all the fuzzy chaotic centred entourages in $\mathscr{U}$ is the diagonal. ie, $\Delta \equiv \{ (p_i, p_i) : p_i \in \mathfrak{X} \}$ of $\mathfrak{X} \times \mathfrak{X}$ by (U2).

Note 3.6. The inequality relation is symmetric, (U2) implies that if $p_i = q_i$, then $(p_i, q_i) \in U$ for all $U \in \mathscr{U}$.

Definition 3.7. The standard (fuzzy chaotic centred uniform) inequality on a fuzzy chaotic centred quasi-uniform space $(\mathfrak{X}, \mathscr{U})$ is defined by
$p_i \neq q_i \Leftrightarrow$ there exists $U \in \mathscr{U}$ such that $(p_i, q_i) \notin U$ or there exists $U \in \mathscr{U}$ such that $(q_i, p_i) \notin U$.

Definition 3.8. The standard inequality on a fuzzy chaotic centred quasi-uniform space $(\mathfrak{X}, \mathscr{U})$ is said to be fuzzy chaotic centred tight if $\neg\,(p_i \neq q_i) \Rightarrow p_i = q_i$

Definition 3.9. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic centred quasi-uniform space and let $\mathfrak{Y}$ be a nonempty subset of $\mathfrak{X}$. A fuzzy chaotic centred quasi-uniform structure $\mathscr{U}$ on $\mathfrak{X}$ induces a fuzzy chaotic centred quasi-uniform structure $\mathscr{U}_{\mathfrak{Y}}$ on a nonempty subset $\mathfrak{Y}$ of $\mathfrak{X}$. The fuzzy chaotic centred entourages of $\mathscr{U}_{\mathfrak{Y}}$ are the sets $U \cap (\mathfrak{Y} \times \mathfrak{Y})$ with $U \in \mathscr{U}$. Then $\mathfrak{Y}$ together with $\mathscr{U}_{\mathfrak{Y}}$ is called fuzzy chaotic centred quasi-uniform subspace of $\mathfrak{X}$. The fuzzy chaotic centred quasi-uniform structure $\mathscr{U}_{\mathfrak{Y}}$ is called the fuzzy chaotic centred subspace of quasi-uniform structure on $\mathfrak{Y}$.

Proposition 3.10. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic centred quasi-unform space and let U be a fuzzy chaotic centred entourage of $(\mathfrak{X}, \mathscr{U})$. Then either $p_i \neq q_i$ or $(p_i, q_i) \in U$, for every $p_i, q_i \in \mathfrak{X}$.

Proof. By (U4), there exists $V \in \mathscr{U}$ such that $\mathfrak{X} \times \mathfrak{X} = U \cup \neg V$. If $(p_i, q_i) \in \neg V$, then $p_i \neq q_i$ by the definition of the standard inequality on $\mathfrak{X}$. □

Proposition 3.11. The standard inequality on a fuzzy chaotic centred quasi-uniform space is fuzzy chaotic centred tight.

Proof. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic cnetred quasi-uniform space and let p_i and q_i be fuzzy chaotic centred systems of $\mathfrak{X}$ such that $\neg(p_i \neq q_i)$. Then by Proposition 3.10 $(p_i, q_i) \in U$ for each $U \in \mathscr{U}$. Therefore by (U2) $p_i = q_i$. □

Proposition 3.12. Let$(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic centred quasi-uniform space and let V be a fuzzy chaotic centred entourage of $(\mathfrak{X}, \mathscr{U})$. Let n be an integer ≥ 2. Then V^n is a fuzzy chaotic centred entourage, and $V^{n-1} \subset V^n$.

Proof. Let $(p_i, q_i) \in V^{n-1}$, by (U2) $(p_i, p_i) \in V$. Hence $(p_i, q_i) \in V \circ V^{n-1} = V^n$. Therefore by (U1) and (F2) V^n is fuzzy chaotic centred entourage. □

Proposition 3.13. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic cnetred quasi-uniform space and let $U \in \mathscr{U}$. Then there exists $V \in \mathscr{U}$ such that $V^3 \subset U$.

Proof. Let us construct $W, V \in \mathscr{U}$ such that $W^2 \subset U$ and $V^2 \subset W$ by using (U3) twice. By Proposition 3.12 $V \subset V^2 \subset W$. Therefore $V^3 = V \circ V^2 \subset W \circ V^2 \subset W \circ W \subset U$. Hence $V^3 \subset U$. □

Proposition 3.14. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic centred quasi-unform space and let U be a fuzzy chaotic centred entourage of $(\mathfrak{X}, \mathscr{U})$.Then there exists a fuzzy chaotic centred entourage V such that $V^2 \subset U$ and $\neg U \subset \sim V$.

Proof. By Proposition 3.13, choose a fuzzy chaotic centred entourage V such that $V^3 \subset U$. By Proposition 3.12, $V^2 \subset U$ and there exists a fuzzy chaotic centred entourage W such that $\mathfrak{X} \times \mathfrak{X} = V \cup \neg W$ by (U4). Consider (p_i, q_i) in $\neg U$ and (r_i, s_i) in V. If $(p_i, r_i) \in V$ and $(si, q_i) \in V$, then $(p_i, q_i) \in V^3 \subset U$, which is a constradiction. Hence either $(p_i, r_i) \in \neg W$ and so $p_i \neq r_i$, or else $(s_i, q_i) \in \neg W$ and so $s_i \neq q_i$. Thus $(p_i, q_i) \neq (r_i, s_i)$. It follows that $\neg U \subset \sim V$. □

Proposition 3.15. Let $(\mathfrak{X}, \mathscr{U})$ be a fuzzy chaotic centred quasi-unform space.If U is a fuzzy chaotic centred entourage of $(\mathfrak{X}, \mathscr{U})$, then for all integers $n \geq 2$, there exists a fuzzy chaotic centred entourage V such that $V^n \subset U$ and $\mathfrak{X} \times \mathfrak{X} = U \cup \sim V$.

Proof. If U is a fuzzy chaotic centred entourage of $(\mathfrak{X}, \mathscr{U})$, then there exists a fuzzy chaotic centred entourage $W \in \mathscr{U}$ such that $\mathfrak{X} \times \mathfrak{X} = U \cup \neg W$. By Proposition 3.14, there exists a fuzzy chaotic centred entourage $V \in \mathscr{U}$ such that $\neg W \subset \sim V$. Also by Proposition 3.14, there exists a fuzzy chaotic centred entourage V such that $V^2 \subset U$, which gives the proof for $n = 2$. Since the proof is true for $n = 2$, by the induction hypothesis it is true for power of 2, say 2^k where k is a positive integer. Let $2^k > n$. Hence there exists a fuzzy chaotic centred entourage $V \in \mathscr{U}$ such that $V^{2k} \subset U$ and $\mathfrak{X} \times \mathfrak{X} = U \cup \sim V$. By Proposition 3.12, $V^n \subset V^{2k}$.Thus $V^n \subset V^{2k} \subset U$, which completes the proof. □

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