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EPQ model under imperfect production process with customer return and partial backlogging

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Abstract
In the proposed paper, an imperfect production model is developed by considering the management of inventory at two different stages. In the first stage, we consider raw material inventory management while in the second stage inventory management for the finished goods is considered. The total cost for both systems is optimized together. For the raw material inventory, demand depends on the discount rate offered by the suppliers while the demand for finished goods depends on the selling price of goods. Different deterioration rates are considered for the inventories in different stages. The production rate of finished goods depends on demand. Partial backlogging and shortages are also considered. Customer return rate is considered as the function of selling price and demand. The whole of the study is carried out under the effect of inflation. A numerical example and sensitivity analysis have been done in the paper to show the practical utility of the model. A convexity graph of the total cost function is also included in the paper to show the behavior of the total cost function.

Keywords
Imperfect production, deterioration, shortage, partial backlogging, inflation.

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Contents

1. Introduction

In competitive market structure manufacturer face lot of challenges. These challenges are related with many practical conditions which are arise in production process or market, like fluctuation in demand, production, deterioration of goods, shortages of goods in the market etc. Different researchers worked with different assumptions to resolve these issues. Teng et al. (2007) proposed a comparative study between pricing and lot-sizing stock models for deteriorated items and partial backlogging. An inventory model was proposed by Huo et al. (2011) for perishable items with stock-dependent selling price and partial backlogging. Yang (2012) examined an inventory model for deteriorating goods with partial backlogging. Fluctuation in price and purchasing cost was also studied in that model. To study the effect of discount Taleizadeh and Pentico (2014) presented an inventory model with partial backlogging. Relation between quality improvement and backorder was discussed by Sarkar et al. (2015). Relation between price discount and other factors like partial backlogging, supplier retailer relationship etc. was revealed by Wu and Wu (2015), Li et al. (2016), Goyal et al. (2017), Kumar et al. (2018), and Yadav et al. (2021).

Imperfect production system is a production system in which all produced goods are not perfect. Imperfect production is always a serious issue for the manufacturer as well as for the researcher. Imperfect production affects production process as well as the profit of the manufacturer. In this field, Sarkar and Moon (2011) presented an imperfect production system with stochastic demand under the effect of inflation. Yadav et al. (2013) designed a three stage integrated model by considering random demand and imperfect product system. Pal et al. (2014) proposed an ordering policy for joint pricing and two echelon imperfect production model. Paul et al. (2015) observed management of disruption in an imperfect production model. Nourelfath et al. (2016) examined an integrated strategy for production and prevented maintence decision for imperfect process. Shaikh et al. (2018) examined a fuzzy production model for perishable goods with variable demand, shortage and partial backlogging. Lai et al. (2019) exposed an economic production quantity model for imperfect manufacturing goods with hybrid maintenance policy, shortages and partial backlogging. Recently, Tahami and Fakhravar (2020) observed a fuzzy inventory model in which imperfect quality items are considered with reparative batch and order overlapping.

To fill the gap in literature, in this paper we propose a production inventory model in which management of raw and finished material is considered. Here, production system is imperfect where demand of the finished products depends on selling price of the product. Customer return is considered which depends on selling price and customer's demand. Whole of the study is performed under the effect of inflation.

This study is organized as follows: Problem definition, assumptions and notations are illustrated in section 2. Mathematical formulation is illustrated in section 3. Section 4 contains the solution methodology of the problem. Section 5 contains the numerical analysis with sensitivity analysis. Section 6 contains the concluding remarks with future extension of the present work.

2. Problem Definition, Notations and Assumptions

In this section, problem definition, notations and assumptions are presented on the basis of which mathematical formulation of model is derived in section 3.

2.1 Problem Definition

In the proposed model, an imperfect production system is considered in which raw material as well as finished goods inventory management is considered. Before starting the production, manufacturer ordered raw material to start the production process. Therefore, production cycle starts with shortages. In time period 0 to t_1 manufacturer get the ordered raw material. At $t = t_1$ manufacturer starts production process to fulfill the backlogged demand. Produced units during $[t_1, t_2]$ is utilized to satisfy backlogged and the demand of the customer. In time t_2 to t_3, production process continues and during that period demand of the consumers is also fulfilled. At time t3 production stops and demand is fulfilled by the stored goods. Inventory level becomes zero at time t_4. Graphical representation of production inventory system is represented in Fig.1.

2.2 Notations

P Production rate

D_r $(= b_r)$ Demand rate for raw material where r is discount rate.

D_F $(= ap^{-\gamma}, \gamma > 0)$ Demand rate for finished goods where p is the selling price.

r Rate of discount offered by the supplier

θ_F Deterioration rate for raw material

θ_R Deterioration rate for finished goods

μ Backlogging rate

I_R (t) Inventory level for raw material (case I) in time interval 0 to t $_1$

I_1 (t) Inventory level for finished goods (case II) in time interval 0 to t_1

I_2 (t) Inventory level for finished goods (case II) in time interval t_1 to t_2

I_3 (t) Inventory level for finished goods (case II) in time interval t_2 to t_3

I_4 (t) Inventory level of for finished goods (case II) in time interval t_3 to t_4

Q_0 Maximum inventory level of raw material

h_R Fixed holding cost for raw material ($ per unit item)

d_R Fixed deterioration cost for raw material ($ per unit item)

p_R Fixed purchasing cost for raw material ($ per unit item)

A_R Fixed ordering cost for raw material ($ per order)

HC_R Total holding cost for raw material

DC_R Total deterioration cost for raw material

PC_R Total purchasing cost for raw material

d_F Fixed deterioration cost for finished goods ($ per unit item)

h_F Fixed holding cost for finished goods ($ per unit item)

l Fixed lost sale cost for finished goods ($ per unit item)

SV Per unit salvage cost

p_f Fixed production cost for finished goods ($ per unit item)

c_s Fixed shortage cost for finished goods ($ per unit item)

δ	Proportion of imperfect production rate
TC	Total inventory cost of the production system
TC_1	Total cost for raw material
TC_2	Total cost for finished goods

2.3 Assumptions

(a) Deterioration rate for raw material inventory and finished goods are different.

(b) Production rate for finished goods depends on demand. Production rate for finished goods is $P = KD$

(c) Partial backlogging and shortages are allowed. Backlogging rate μ is for the shortage period.

(d) Demand for finished goods is as follow:

$$D = \begin{cases} a, & \text{when inventory level is negative} \\ ap^{-\gamma}, & \text{when inventory level is positive} \end{cases}$$

where p is the selling price and " a ' is constant.

(e) Demand for raw material depends on discount rate i.e. $D_r = b^r$.

(f) Customer retums rate is the function of demand and selling price of the product. Following form of the customer's retum rate is considered: (Anderson et al. (2008)) $R(D, p) = \alpha D + \beta p, (\beta \geq 0 \text{ and } 0 \leq \alpha < 1)$

3. Mathematical Formulation of Production Inventory Model

In the proposed model different costs for both raw material and finished goods are calculated. To calculate total cost for raw material and finished goods two different cases are considered here. Case one is considered for raw material inventory and case two is considered for finished goods inventory.

Case I: Raw material inventory: For the smooth production process, manufacturer purchase raw material in bulk. Purchasing of raw material depends on several factors and discount offered by the supplier is one of them. Hence in this case, demand of raw material depends on rate of discount offered by the supplier to the manufacturer.

Inventory level of raw material during the time interval $[0, t_1]$ is governed by the following differential equation:

$$\frac{dI_R(t)}{dt} + \theta_R I_R(t) = -b^r \tag{3.1}$$

With the condition $I_R(t_1) = 0$ Solution of above equation under the boundary condition is

$$I_R(t) = \frac{b^r}{\theta_R}\left(e^{\theta_R(t_1-t)} - 1\right) \tag{3.2}$$

The maximum inventory level of raw material is

$$Q_0 = I_R(0) = \frac{b^r}{\theta_R}\left(e^{\theta_R t_1} - 1\right) \tag{3.3}$$

Now, we evaluate different costs associated with raw material inventory one by one under the effect of inflation.

Holding cost for the raw material:

$$\begin{aligned} HC_R =& h_R \int_0^{t_1} I_R(t) e^{-Rt} dt \\ =& \frac{h_R b^r}{\theta_R}\left[\frac{e^{\theta_R t_1}}{(\theta_R + R)} - \frac{e^{-Rt_1}}{(\theta_R + R)} + \frac{e^{-Rt_1}}{R} - \frac{1}{R}\right] \end{aligned} \tag{3.4}$$

Deterioration cost for raw material:

$$\begin{aligned} DC_r =& d_R \int_0^{t_1} \theta_R I_R(t) e^{-Rt} dt \\ =& d_R b^r \left[\frac{e^{\theta_R t_1}}{(\theta_R + R)} - \frac{e^{-Rt_1}}{(\theta_R + R)} + \frac{e^{-Rt_1}}{R} - \frac{1}{R}\right] \end{aligned} \tag{3.5}$$

Purchasing cost for raw material:

$$PC_R = p_R[Q_0] = p_R\left[\frac{b^r}{\theta_R}\left(e^{\theta_R t_1} - 1\right)\right] \tag{3.6}$$

Total cost for raw material:
Average inventory cost for the raw material under the effect of inflation is the sum of purchasing cost, deterioration cost, holding cost, and ordering cost. Thus,

$$\begin{aligned} TC_1 =& \frac{PC_R + DC_R + HC_R + A_R}{T} \\ TC_1 =& \frac{1}{T}\Bigg[p_r\left[\frac{b^r}{\theta_R}\left(e^{\theta_R t_1} - 1\right)\right] \\ &+ d_R b^r \left[\frac{e^{\theta_R t_1}}{(\theta_R + R)} - \frac{e^{-Rt_1}}{(\theta_1 + R)} + \frac{e^{-Rt_1}}{R} - \frac{1}{R}\right] \\ &+ \frac{h_R b^r}{\theta_R}\left[\frac{e^{\theta_R t_1}}{(\theta_R + R)} - \frac{e^{-Rt_1}}{(\theta_R + R)}\right. \\ &+ \left.\frac{e^{-Rt_1}}{R} - \frac{1}{R}\right] + A_R\Bigg] \end{aligned} \tag{3.7}$$

Case II: Production of finished inventory system:
Now, the manufacturer starts production after receiving the raw material. First, the produced items utilized for backlogged as well as to satisfy the demand. Production process of finished goods stops after reaching the certain level. After that the inventory level starts decreasing due to demand and deterioration.

First, we assume that during $[0, t_1]$ inventory level drops and below zero as the production process not started at the manufacturer end. Fraction of demand during this period is fulfilled when the production process starts. Thus, the inventory level of manufacturer is governed with the help of following differential equation:

$$\frac{dI_1(t)}{dt} = -\mu a, \quad 0 \le t \le t_1 \tag{3.8}$$

with the condition $I_1(t_1) = 0$.
Secondly, we consider the inventory level of manufacturer during the interval $[t_1 t_2]$. This period is the recovery period of the shortages that occur during $[0, t_1]$. During $[t_1 t_2]$ produced quantity is utilized to satisfy the demand and the backlogged quantity. Thus, the governing equation during this period is

$$\frac{dI_2(t)}{dt} = \delta K a - a, \quad t_1 \le t \le t_2 \tag{3.9}$$

with the condition $I_2(t_2) = 0$.

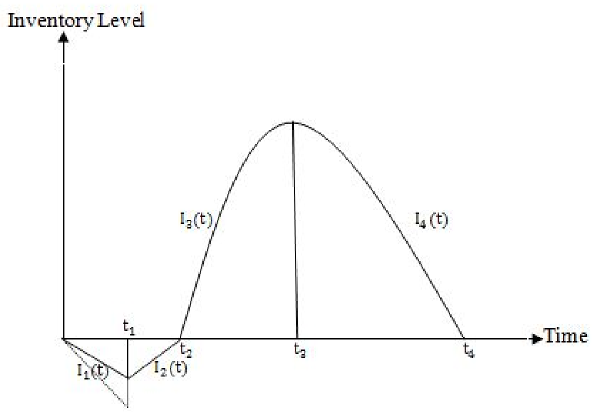

Figure 1. Graphical Representation of Inventory Level of Manufacturer.

Thirdly, during the interval $[t_2 t_3]$ inventory accumulated due to the product and decline due to the demand and deterioration. Thus, inventory level of manufacture can be represented with the help of following equation:

$$\frac{dI_g(t)}{dt} = \delta K a p^{-\gamma} - ap - \theta_F I_3(t), \quad t_2 \le t \le t_3 \tag{3.10}$$

with the condition $I_3(t_2) = 0$.
Finally, during the interval $[t_3 t_4]$ production process stop and inventory level decline due to demand and deterioration. Thus, governing equation of inventory level of manufacturer is

$$\frac{dI_4(t)}{dt} = -ap^{-\gamma} - \theta_F I_4(t), \quad t_3 \le t \le t_4 \tag{3.11}$$

with the condition $I_4(t_4) = 0$.
Now, we solve the above system of equations one by one. Solution of equation (3.8) under the given condition is

$$I_1(t) = \mu a t \tag{3.12}$$

Solution of equation (3.9) under the given condition is

$$I_2(t) = a(\delta K - 1)(t - t_2) \tag{3.13}$$

Solution of equation (3.10) under the given condition is

$$I_3(t) = \frac{(K\delta - 1)(ap^{-\gamma})}{\theta_F}\left(1 - e^{(\theta_F)(t_2 - t)}\right) \tag{3.14}$$

Solution of equation (3.11) under the given condition is

$$I_4(t) = \frac{ap^{-\gamma}}{(\theta_F)}\left(e^{(\theta_P)(t_4 - t)} - 1\right) \tag{3.15}$$

On using the following equation of continuity condition, we get

$$I_1(t_1) = I_2(t_1) \Rightarrow t_2 = t_1\left(1 - \frac{\mu}{\delta K - 1}\right) \tag{3.16}$$

On applying the following equation of continuity, we get

$$I_3(t_3) = I_4(t_3) \Rightarrow t_4 = t_3 - \frac{\log[(K-1)] + (\theta_F)(t_2 - t_3)}{(\theta_F)\log[(K-2)]} \tag{3.17}$$

Finally now, we evaluate the different cost associated finished product inventory.
Deterioration cost:

$$\begin{aligned} DC_F =& \theta_F d_F\left[\int_{t_2}^{t_3} I_3(t)e^{-Rt}dt + \int_{t_3}^{t_4} I_4(t)e^{-Rt}dt\right] \\ =& \theta_F d_F\left[\frac{(\delta K - 1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R} + \frac{e^{-Rt_2}}{R}\right.\right. \\ & \left. + \frac{e^{(\theta_F)(t_2 - t_3) - Rt_3}}{(\theta_F + R)}\right] \\ & \left. + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4 - t_3 + \frac{e^{(\theta_F)(t_4 - t_3)}}{(\theta_F)}\right)\right] \end{aligned} \tag{3.18}$$

Holding cost:

$$\begin{aligned} HC_F =& h_F\left[\int_{t_2}^{t_3} I_3(t)e^{-Rt}dt + \int_{t_3}^{t_4} I_4(t)e^{-Rt}dt\right] \\ =& h_F\left[\frac{(\delta K - 1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R} + \frac{e^{-Rt_2}}{R}\right.\right. \\ & \left. + \frac{e^{(\theta_F)(t_2 - t_3) - Rt_3}}{(\theta_F + R)}\right] \\ & \left. + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4 - t_3 + \frac{e^{(\theta_F)(t_4 - t_3)}}{(\theta_F)}\right)\right] \end{aligned} \tag{3.19}$$

Lost sale cost:

$$\begin{aligned} LC_F =& l\int_0^{t_1}(1 - \mu)ae^{-Rt}dt \\ =& l(\mu - 1)\left[(a)\frac{e^{-Rt_1}}{R}\right] \end{aligned} \tag{3.20}$$

Production cost:

$$PC_F = P_f\left[\int_{t_1}^{t_2} Kae^{-Rt}dt + \int_{t_2}^{t_3} Kap^{-\gamma}e^{-Rt}dt + \int_{t_3}^{t_4} Kap^{-\gamma}e^{-Rt}dt\right] = P_f\left[\frac{Ka}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right)+\frac{Kap^{-\gamma}}{R}\left[e^{-Rt_4}-e^{-Rt_3}\right]\right] \tag{3.21}$$

Shortage cost:

$$SC_F = C_s\left[-\int_0^{t_1} I_1(t)e^{-Rt}dt - \int_{t_1}^{t_2} I_2(t)e^{-Rt}dt\right] = C_s\left[(K\delta-1)(a)\left(\frac{e^{-Rt_1}}{R^2}-\frac{e^{-Rt_1}}{R}\left(t_1-t_2+\frac{1}{R}\right)\right) + \mu(a)\left(\frac{e^{-Rt_1}}{R}\left(t_1+\frac{1}{R}\right)\right)\right] \tag{3.22}$$

Return Cost:

$$RC = (p-SV)\left[\int_{t_1}^{t_2}(\alpha a+\beta p)e^{-Rt}dt + \int_{t_2}^{t_4}\left(\alpha ap^{-\gamma}+\beta p\right)e^{-Rt}dt\right] = \frac{(\alpha a+\beta p)}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right) + \frac{(\alpha ap^{-\gamma}+\beta p)}{R}\left(e^{-Rt_2}-e^{-Rt_4}\right) \tag{3.23}$$

Total Inventory cost for finished goods: Total inventory cost of the finished products is the sum deterioration cost, holding cost, lost sale cost, production cost, shortage cost, and retum cost. Thus, average total inventory cost is

$$\begin{aligned}TC_2 =& \frac{1}{T}\left\{\theta_F d_F\left[\frac{(K-1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R}+\frac{e^{-Rt_2}}{R} + \frac{e^{(\theta_F)(t_2-t_3)-Rt_3}}{(\theta_F+R)}\right] + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4-t_3+\frac{e^{(\theta_F)(t_3-t_4)}}{(\theta_F)}\right)\right]\right.\\ &+P_f\left[\frac{Ka}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right)+\frac{Kap^{-\gamma}}{R}\left[e^{-Rt_4}-e^{-Rt_3}\right]\right]\\ &+C_s\left[(K\delta)(ap)\left(\frac{e^{-Rt_1}}{R^2}-\frac{e^{-Rt_1}}{R}\left(t_1-t_2+\frac{1}{R}\right)\right) + \mu(ap)\left(\frac{e^{-Rt_1}}{R}\left(t_1+\frac{1}{R}\right)\right)\right]\\ &+h_F\left[\frac{(K-1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R}+\frac{e^{-Rt_2}}{R} + \frac{e^{(\theta_F)(t_2-t_3)-Rt_3}}{(\theta_F+R)}\right] + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4-t_3+\frac{e^{(\theta_F)(t_3-t_4)}}{(\theta_F)}\right)\right]\\ &+l(\mu-1)\left[(ap)\frac{e^{-Rt_1}}{R}\right]+\frac{(\alpha a+\beta p)}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right)\\ &\left.+\frac{(\alpha ap^{-\gamma}+\beta p)}{R}\left(e^{-Rt_2}-e^{-Rt_4}\right)\right\}\end{aligned} \tag{3.24}$$

Now, the total cost for production process is sum of total inventory cost associated with raw material and finished products. Thus, total average inventory cost of the production system is

$$TC = TC_1 + TC_2$$

$$\begin{aligned}TC =& \frac{1}{T}\left\{\theta_F d_F\left[\frac{(K-1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R}+\frac{e^{-Rt_2}}{R} + \frac{e^{(\theta_F)(t_2-t_3)-Rt_3}}{(\theta_F+R)}\right] + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4-t_3+\frac{e^{(\theta_F)(t_3-t_4)}}{(\theta_F)}\right)\right]\right.\\ &+P_f\left[\frac{Ka}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right)+\frac{Kap^{-\gamma}}{R}\left[e^{-Rt_4}-e^{-Rt_3}\right]\right]\\ &+C_s\left[(K\delta)(ap)\left(\frac{e^{-Rt_1}}{R^2}-\frac{e^{-Rt_1}}{R}\left(t_1-t_2+\frac{1}{R}\right)\right) + \mu(ap)\left(\frac{e^{-Rt_1}}{R}\left(t_1+\frac{1}{R}\right)\right)\right]\\ &+h_F\left[\frac{(K-1)(ap^{-\gamma})}{\theta_F}\left[\frac{-2e^{-Rt_3}}{R}+\frac{e^{-Rt_2}}{R} + \frac{e^{(\theta_F)(t_2-t_3)-Rt_3}}{(\theta_F+R)}\right] + \frac{ap^{-\gamma}}{(\theta_F)}\left(t_4-t_3+\frac{e^{(\theta_F)(t_3-t_4)}}{(\theta_F)}\right)\right]\\ &+l(\mu-1)\left[(ap)\frac{e^{-Rt_1}}{R}\right]+\left[p_r\left[\frac{b^r}{\theta_R}\left(e^{\theta_R t_1}-1\right)\right]\right.\\ &+d_R b^r\left[\frac{e^{\theta_R t_1}}{(\theta_R+R)}-\frac{e^{-Rt_1}}{(\theta_1+R)}+\frac{e^{-Rt_1}}{R}-\frac{1}{R}\right]\\ &\left.+\frac{h_R b^r}{\theta_R}\left[\frac{e^{\theta_R t_1}}{(\theta_R+R)}-\frac{e^{-Rt_1}}{(\theta_R+R)}+\frac{e^{-Rt_1}}{R}-\frac{1}{R}\right]+A_R\right]\\ &+\frac{(\alpha a+\beta p)}{R}\left(e^{-Rt_1}-e^{-Rt_2}\right)\\ &\left.+\frac{(\alpha ap^{-\gamma}+\beta p)}{R}\left(e^{-Rt_2}-e^{-Rt_4}\right)\right\}\end{aligned} \tag{3.25}$$

4. Solution Methodology

Model formulated in section 3, have t_1 and t_3 as decision variable. Here, expression of objective function is non-linear in t_1 and t_3.*So*, it is not an easy task to obtain the closed form solution. Therefore, calculus based search algorithmis adopted to optimize the objective function. First, assume that the objective function satisfies the following necessary conditions:

$$\frac{\partial TC}{\partial t_1} = 0 \tag{4.1}$$

$$\frac{\partial TC}{\partial t_3} = 0 \tag{4.2}$$

Step 1. $t_3 = 0$ and find t_1^* from equation (4.1)
Step 2. Find t_3^* from equation (4.2) , using the revised values of Step 1.
Step 3. Repeat the above process until the values of t_1^*, and t_3^*

remain unchanged.

Step 4. Evaluate the different principal minors of Hessian matix of TC i.e.

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial t_3} \\ \frac{\partial^2 TC}{\partial t_3 \partial t_1} & \frac{\partial^2 TC}{\partial t_3^2} \end{bmatrix}$$

at the point (t_1^*, t_3^*) and if it is found to be positive definite. Then, $TC(t_1^*, t_3^*)$ is the optimal value of the objective function. Whole of the process is carried out with the help of software MATHEMATICA 5.2 .

5. Numerical Example and Sensitivity Analysis

Proposed model is analyzed with the help of numerical example in this section and further sensitivity analyses is also carried out in this section with respect to important parameters. In order to analyze the model numerically we consider the following data in appropriate units.

$a = 70; p = 20$ \$per unit; $K = 0.5; r = 0.5; \theta_F = 0.4; \theta_R = 0.2; \mu = 0.75; \alpha = 0.1$ $b = 6000; h_R = 0.3$ \$per unit; $d_R = 0.6$\$ per unit; $p_r = 5; A_r = 50$ \$per order; $d_F = 0.4$ \$per unit; $h_F = 0.5$ per unit; $1 = 0.7$ per unit; $pF = 11$\$ per unit; $c_s = 0.2$ \$per unit ; $SV = 10$ \$per unit : $\delta = 0.90; \alpha = 0.2; \beta = 0.3$

With the help of above data and the solution methodology presented in section 4, optimal solution of decision variables and total cost is as follows:

$$TC = 23293.3; t_1 = 0.278017; t_3 = 1.40861$$

Fig. 2 shows the convexity of the total cost function. It shows that obtained result is global optimal solution.

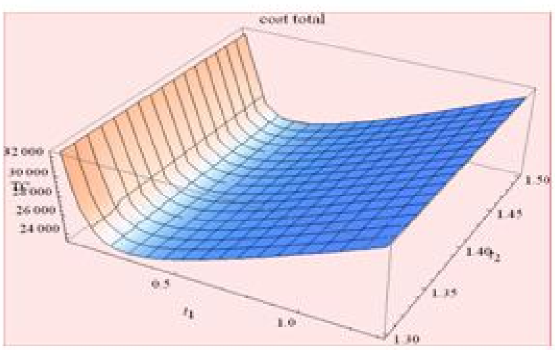

Figure 2. Convexity of Total Cost Function

5.1 Sensitivity Analyses

In this section sensitivity analyses is performed with respect to salvage value of the product, partial backlogging rate, shortage cost, and deterioration rate.

5.2 Observation

1. Sensitivity analysis of salvage value of the commodity has been done in Table 1 . According to this analysis, on increasing the salvage value of any commodity total cost will decrease. On increasing the value of SV, values of TC, t_1 and t_3 will decrease.

2. In Table 2 sensitivity analysis of μ (backlogging rate) has been done. On increasing the backlogging rate of demand, it is quite obvious that it is beneficial for the manufacturer. Increase in backlogging rate will increase the production. On increasing the value of μ, values of TC, t_1 and t_3 will decrease.

3. In Table 3 sensitivity analysis of c_s (shortage cost) has been done. Increase in shortage cost will increase the total cost. On increasing the value of c_s, values of TC and t_3 will increase but the values of t_1 decreases.

4. In Table 4 sensitivity analysis of θ_F (deterioration cost) has been done. Increase in shortage cost will increase the total cost. On increasing the value of θ_F, values of TC and t_3 will increase but the values of t_1 decreases.

Table 1. Sensitivity analysis for parameter SV (salvage value of product)

SV	TC	t_1	t_3
10	23293.3	0.278017	1.40861
12	23228.6	0.274894	1.40158
14	23096.3	0.268542	1.38730
16	22961.8	0.262039	1.37271

Table 2. Sensitivity analysis for parameter μ (backlogging rate)

μ	TC	t_1	t_3
0.1	23293.3	0.278017	1.40861
0.2	23111.0	0.195549	1.36795
0.3	22584.7	0.159441	1.34237
0.4	22185.7	0.138136	1.31570

Table 3. Sensitivity analysis for parameter c_s (shortage cost for finished goods)

c_s	TC	t_1	t_3
0.2	23293.3	0.278017	1.40861
0.3	24544.3	0.225173	1.42827
0.4	25581.2	0.193668	1.45805
0.5	26480.9	0.172551	1.49085

Table 4. Sensitivity analysis for parameter θ_F (deterioration rate for finished goods)

θ_F	TC	t_1	t_3
0.4	23293.3	0.278017	1.40861
0.5	24130.1	0.318154	1.38465
0.6	24567.8	0.339342	1.33527
0.7	24833.4	0.352164	1.28178

6. Concluding Remark and Future Extension

An optimal policy for a production model is derived in the present study. The production model is considered with the assumptions like shortages, partial backlogging, imperfect production, inflation etc. In most of the production model total cost for the finished goods is calculated, but in proposed model total cost for the raw material is also calculated. Two different deterioration rates for the finished goods and raw material are considered. Selling price dependent demand is considered for the finished goods and demand of raw material depends on rate of discount. According to the sensitivity analysis of the optimal policy when deterioration rate for finished goods and shortage cost for finished goods are increased total cost is increased. On increasing the backlogging rate and salvage value of the product, total cost decreased. Convexity of the total cost function is also presented graphically. Volume flexibility, learning, and screening in the production process are the few promising areas of future extension. Present model can be made environmentally sustainable in future by considering different carbon emission policy.

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