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On fuzzy γ^* continuity with compare other forms of continuity in fuzzy topological space

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Abstract
The aim the paper is investigated the relationships between fuzzy γ^* continuity and other forms of continuity of fuzzy functions.

Keywords
Fuzzy γ open set, γ closed set, γ continuity, γ^* open set, γ^* closed set, fuzzy strong and normal space.

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Contents

1. Introduction

Joseph and Kwack [7] introduced (θ, s) - continuous functions in order to investigate S-closed due to Thompson [13]. A function f is called (θ, s) - continuous if the inverse image of each regular open set is closed.

Chang in [1] introduced fuzzy S-closed spaces in 1968. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. One of them is fuzzy γ-continuity. In 1999, Hanafy in [7] introduced the concept of fuzzy γ - continuity.

2. Preliminaries

A fuzzy set X is a function with domain X and values in I. That is an element of I^X. Let $A \in I^X$. The subset of X in which A assumes non-zero values is known as the support of A for every $x \in X$, $A(x)$ is called the grade of membership of x in A.

Definition 2.1. *A family $\tau \subseteq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms.*

1. $\bar{0}, \bar{1} \in \tau$
2. $\forall A, B \in \tau \Rightarrow A \wedge B \in \tau$
3. $\forall (A_j)_{j \in J} \in \tau \Rightarrow \vee_{j \in J} A_j \in \tau$

The pair (X, τ) is called a fuzzy topological space (or) fts for short. The elements of τ are called fuzzy open sets.

Definition 2.2. *A fuzzy set A in a space X is called*

1. *Fuzzy γ-open if $A \leq (int(cl(A))) \vee (cl(intA))$ & fuzzy γ-closed if $(cl(intA)) \wedge (int(cl(A))) \leq A$.*
2. *Let (X, τ) be a fuzzy topological space then a fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy γ^* open set if $A \leq cl(\gamma intA)$.*
3. *Fuzzy γ^*- semi open if $int(A) \leq cl(\gamma - int(A))$ and fuzzy γ^*- semi closed if $cl(A) \geq int(\gamma - cl(A))$.*

3. Relationships between the fuzzy γ^* continuous functions and other continuous functions

Definition 3.1. *A function $f : X \to Y$ is called fuzzy γ^* continuous if for each $x \in X$ and each fuzzy irregular closed set η of γ containing $f(x_\varepsilon)$. There exists a fuzzy γ^* open set μ in X containing x_ε such that $int(f(\mu)) \leq \eta$.*

Definition 3.2. *A function $\mathscr{F} : X \to Y$ is called fuzzy (γ^*, S) open if the image of each fuzzy γ^* open set is fuzzy semi-open.*

Theorem 3.3. *If a function $f : X \to Y$ is fuzzy weakly γ^* continuous and fuzzy (γ^*, S) open then f is fuzzy almost γ^* continuous.*

Proof. Let $x \in X$ and η be a fuzzy regular closed set containing $f(x_\varepsilon)$. Since f is fuzzy weakly γ^* continuous. There exist a fuzzy γ^* open set μ in X containing x_ε such that $int(f(\mu)) \leq \eta$. Since f is fuzzy (γ^*, S) open $f(\mu)$ is a semi open set in Y and $f(\mu) \leq cl\,(int\,(f(\mu))) \leq \eta$. This shows that f is fuzzy almost γ^* continuous. □

Definition 3.4. *Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be*

1. *Fuzzy almost precontinuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy pre closed in X.*
2. *Fuzzy almost contra semi continuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy semi-closed in X.*
3. *Fuzzy almost continuous [5] if the inverse image of each regular open set in Y is fuzzy closed in X.*
4. *Fuzzy almost α-continuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy α-closed in X.*
5. *Fuzzy β-continuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy β-closed in X.*

Example 3.5. *Fuzzy almost α-continuity not be a fuzzy almost continuity.*

Proof. Let X be non empty set and $C_\alpha : X \to [0,1]$ be defined as $C_\alpha(x) = a, \forall x \in X$, and $a \in [0,1]$. Then $\tau_1 = \{C_0, C_{6/10}, C_1\}$, $\tau_2 = \{C_0, C_{3/10}, C_1\}$ are fuzzy topologies and (X, τ_1), (X, τ_2) are fuzzy topological space. The identity function $f : (X, \tau_1) \to (X, \tau_2)$ is fuzzy almost α-continuous but not fuzzy almost continuous. □

Definition 3.6. *A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ on X and each $x_\varepsilon \in \mu$ there exists fuzzy regular closed set τ containing x_ε such that $x_\varepsilon \in \tau \leq \mu$.*

Definition 3.7. *A fuzzy function $f : X \to Y$ is said to be fuzzy γ^* continuous $f^{-1}(\alpha)$ is fuzzy γ^* - open in X for every fuzzy open set (α) in Y.*

Theorem 3.8. *Let $f : X \to Y$ be a fuzzy function. Then if f is fuzzy almost γ^* continuous and Y is fuzzy P_Σ, then f is fuzzy γ^* - continuous.*

Proof. Let α be any fuzzy open set in Y. Since Y is fuzzy P_Σ, there exists a family ψ whose members are fuzzy irregular closed sets of Y such that $\alpha = V\{\tau : \tau \in \psi\}$. Since f is fuzzy almost γ^* continuous. $f^{-1}(\tau)$ is fuzzy γ^* - open in X, for each $\tau \in \psi$ and $f^{-1}(\alpha)$ is fuzzy open in X. Therefore, f^{-1} is almost γ^* continuous. □

Definition 3.9. *A Space is said to be fuzzy weakly P_Σ if any fuzzy regular open set α and each $x_\varepsilon \in \alpha$. There exists a fuzzy regular closed set τ containing x_ε such that $x_\varepsilon \in \tau \leq \mu$*

Definition 3.10. *A fuzzy function $f : X \to Y$ is said to be fuzzy almost γ^* continuous at $x_\varepsilon \in X$ if for each fuzzy open set η containing $f\,(x_\varepsilon)$, there exists a fuzzy γ^* open set α containing x_ε such that $f(\alpha) \leq int\,(cl\,(\eta))$.*

Theorem 3.11. *Let $f : X \to Y$ be a fuzzy almost γ^* continuous function. If Y is fuzzy weakly P_Σ then f is fuzzy almost γ^* continuous.*

Proof. Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_Σ, there exists a family ψ whose are fuzzy irregular closed sets of Y such that $\mu \forall \{\tau : \tau \in \psi\}$. Since f is fuzzy almost γ^* continuous, $f^{-1}(\tau)$ is fuzzy γ^* - open in X, for each $\tau \in \psi\}$ and $f^{-1}(\mu)$ is fuzzy γ^* open in X. Here, f is fuzzy almost γ^* continuous. □

Theorem 3.12. *Let X, Y, Z be fuzzy topological spaces. Let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If f is fuzzy γ^* - irreducible and g is fuzzy almost γ^* continuous,$g \circ f : X \to Z$ is a fuzzy almost contra γ^* continuous function.*

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set. Since g is fuzzy almost γ^* continuous, $g^{-1}(\mu)$ is fuzzy γ^* open in Y. But f is fuzzy γ^* -irresolute. $\Rightarrow f^{-1}\left(g^{-1}(\mu)\right)$ is fuzzy γ^* open in X. Thus $(g \circ f)^{-1}(mu) = f^{-1}\left(g^{-1}(\mu)\right)$ is fuzzy γ^* open in X and this proves that $g \circ f$ is a fuzzy almost γ^* continuous function. □

Definition 3.13. *A function $f : X \to Y$ is called always fuzzy γ^* open if the image of each fuzzy γ^* open set is fuzzy γ^* open.*

Theorem 3.14. *If $f : X \to Y$ is a surjective always fuzzy γ^* open function and $g : Y \to Z$ is a fuzzy function such that $g \circ f : X \to Z$ is fuzzy almost γ^* continuous, then g is fuzzy γ^* continuous.*

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set. Since $g \circ f$ is fuzzy γ^* continuous, $(g \circ f)^{-1}(\mu)$ is fuzzy γ^* open in X. Therefore $f^{-1}\left(g^{-1}(\mu)\right) = (g \circ f)^{-1}(mu)$ is fuzzy γ^* open in X. f is always fuzzy γ^* open surjection implies $f\left(f^{-1}\left(g^{-1}(\mu)\right)\right) = g^{-1}(\mu)$ is fuzzy γ^* open in Y. Thus g is fuzzy γ^* continuous. □

Definition 3.15. *A space X is said to be fuzzy γ^* compact if every γ^* open cover of X has a finite subcover.*

Theorem 3.16. *The fuzzy γ^* continuous image of a fuzzy γ^* compact space is fuzzy S -closed.*

Proof. Suppose $f : X \to Y$ is a fuzzy γ^* continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of Y. Since f is fuzzy γ^* continuous $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy γ^* open cover of X and X being fuzzy γ^* compact. There exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$ Since f is surjection, we have $Y = \vee\{(\eta_i) : i \in I_0\}$ and thus Y is fuzzy S-closed. □

4. Conclusion

In general fuzzy topology Fuzzy - closed and - open sets are major role. Since its inception several weak forms of fuzzy γ^* - closed sets and γ^*- open sets have been introduced in general fuzzy topology. The present paper investigated in Relationships between the fuzzy γ^* continuous functions and other continuous functions. Some basic properties and examples are given.

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