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Logarithmic coefficients for starlike and convex functions of complex order defined by subordination

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Abstract
The aim of this paper is to find the bounds for the logarithmic coefficients $\gamma_n$of the general classes of starlike and convex functions of complex order,$S_d^*(\Psi)$and $K_d(\Psi)$respectively. Our results would generalize some of the previous paper like [1] E. A. Adegani et al., [3] Ali et al.,etc.

Keywords
Starlike function and convex function of Complex order; subordination; logarithmic coefficients.

AMS Subject Classification
30C45.

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Contents

1. Introduction

Suppose $\mathscr{A}$ be the class containing functions which are of the form:

$$f(z)=z+\sum_{n=2}^{\infty} t_n z^n \tag{1.1}$$

and are also analytic in the unit disk $f(z)=\{z:|z|<1\}$.
Furthermore we assumes to be the subclass of $\mathscr{A}$which consists of all univalent functions in Δ,then the logarithmic coefficients $\gamma_n$of fS, satisfies:

$$\log\left(\frac{f(z)}{z}\right)=2\sum_{n=1}^{\infty}\gamma_n(f)z^n, z\in\Delta \tag{1.2}$$

γ_n (f)can be written as γ_n. In the history of univalent function, these logarithmic coefficients play a significant role in various estimates. [2] Kayumov solved Brennan's conjecture for conformal mappings using these logarithmic coefficients.
Equation (1.2) can be written as

$$2\sum_{n=1}^{\infty}\gamma_n z^n=\left[t_2z+t_3z^2+t_4z^3+\dots\right]-\frac{1}{2}\left[t_2z+t_3z^2+t_4z^3\dots\right]^2+\frac{1}{3}\left[t_2z+t_3z^2+t_4z^3+\dots\right]^3+\dots$$

Equating the coefficients of z^n for $n=1,2,3$,on both sides of the above equation ,we get:

$$\begin{cases} 2\gamma_1=t_2 \\ 2\gamma_2=t_3-\frac{1}{2}t_2{}^2 \\ 2\gamma_3=t_4-t_2t_3+\frac{1}{3}t_2{}^3 \end{cases} \tag{1.3}$$

Definition 1.1 Starlike function of complex order d: For the function $f(z)\in\mathscr{A}$ to be starlike of complex order d $(d\in C\setminus\{0\})$,it must follow the condition: $\frac{f(z)}{z}\neq 0\,(z\in\triangle)$and

$$Re\left\{1+\frac{1}{d}\left(\frac{zf'(z)}{f(z)}-1\right)\right\}>0$$

we denote this class by $S_o^*(d)$.
Definition 1.2 Convex function of complex order d: For the function $f(z)\,\varepsilon\mathscr{A}$ to be convex of complex order d $(d\in C\setminus\{0\})$, it must follow the conditions given below:
$f'(z)\neq 0$ and
$\text{Re}\left\{1+\frac{1}{d}\left(\frac{zf''(z)}{f'(z)}\right)\right\}>0,\ (z\triangle)$
We denote this class by $K_o(d)$.
A function $f(z)\,\varepsilon\mathscr{A}$ is close-to-convex of complex order dd $(d\in C\setminus\{0\})$ if there exists a function $g(z)\in K_o(d)(d\in C\setminus\{0\})$which satisfy the following condition :-
$\text{Re}\left\{1+\frac{1}{d}\left(\frac{f'(z)}{g'(z)}-1\right)\right\}>0,(z\in\Delta)$
We denote this class by C_o(d).
Definition 1.3 Subordination: If f and g are two functions analytic in Δ, then the function f is subordinate to g in

Δ,$ie.f(z) \prec g(z)$, if there exists a Schwarz function ω, analytic in Δ with $\omega(0) = 0$ and $|\omega(z)| < 1$ such tha $f(z) = g(\omega(z))(z\mathscr{E}\triangle)$. articularly, if the function $g$is univalent inΔ, then $f \prec g$ if the following conditions hold $f(0) = g(0)$and $f(\triangle) \subseteq g(\triangle)$

Nasr and Aouf [4] introduced and studied the classes$S_0^*(b)$ *and* $K_0(b)$. Ma and Minda [5] introduced and studied the class $S^*(\phi)$ which consists of functions$f \in S$satisfying the following conditions

$$\frac{zf'(z)}{f(z)} \prec \phi(z), (z\Delta).$$

In this paper we define a more general class of starlike function and convex function of complex order following Ma and Minda and find bounds for logarithmic coefficients for this class.

Definitions 1.4 : Let $S_d^*(\Psi)$be a class consisting of all analytic function $f\mathscr{A}$ where $d(C/\{0\})$ and Ψ(z)is any analytic function with positive real part on Δ satisfying $\Psi(0) = 1$, $\Psi'(0) > 0$ and maps Δ ontoa region starlikewith respect to 1 and symmetric with respect to the real axis. Then $S_d^*(\Psi)$ consists of all analytic functions $f\mathscr{A}$satisfying

$$1 + \frac{1}{d}\left(\frac{zf'(z)}{f(z)} - 1\right) \prec \Psi(z) \tag{1.4}$$

The class $K_d(\Psi)$ consists of the functions $f\mathscr{A}$which satisfies the following condition:

$$1 + \frac{1}{d}\left(\frac{zf''(z)}{f'(z)}\right) \prec \Psi(z) \tag{1.5}$$

Furthermore, we let $S^*(M,N,d)$and $K\ (M,N,d)\ (d \neq 0$,complex) denote the class $S_d^*(\Psi)$ and $K_d(\Psi)$respectively, where

$$\Psi(\mathrm{z}) = \frac{1+Mz}{1+Nz}, (-1 \leq N < M \leq 1).$$

The Class $S^*(M,N,d)$, and therefore the class $S_d^*(\Psi)$, specialize to many well known classes of univalent functions for suitable choice of M,N and d.

Recently many researchers have worked on the similar problems of logarithmic coefficients, such as the function$k(z) = z(1-e^{i\theta})^{-2}$ has logarithmic coefficients$\gamma_n = \frac{e^{i\theta n}}{n}, n \geq 1$ for every θ. In [6] (Theorem 4), it has been proved that the logarithmic coefficients $\gamma_n$of every function $f\varepsilon S$ satisfy:

$$\sum_{n=1}^{\infty} |\gamma_n|^2 \leq \frac{\pi^2}{6},$$

and the equality is attained for the Koebefunction. Ali et al. [3] and P. Kumar et al. [7] in 2018 found the bounds for logarithmic coefficient $\gamma_n$for definite classes of close-to-convex functions. In 2019, E.A. Adegani, NakEun Cho and Mostafa Jafari [1] obtained bounds for logarithmic coefficients for certain subclasses of starlike and convex functions defined by subordination. But the problem for n ≥3, for the logarithmic coefficients of univalent function is still a matter of concern.

On the basis of the results obtained in the previous paper, we have tried to obtain the bounds for the logarithmic coefficients $\gamma_n$of the general classes $S_d^*(\Psi)$ and $K_d(\Psi)$ in this paper.

The lemmas will be using in our proofs are as follows:

Lemma 1. [8]. Let w be a Schwarz function such that $w(z) = \sum_{n=1}^{\infty} w_n z^n$, then

$$|w_1| \leq 1, |w_n| \leq 1 - |w_1|^2, n = 2, 3, \ldots$$

Lemma 2. [9] Suppose $\psi, \varphi \in \mathscr{A}$ be convex in Δ, such that $f(z) \prec \psi(z)$ and $g(z) \prec (z)$, then $f(z) * g(z) \prec \psi(z) * \varphi(z)$, where $f, g\ \mathscr{A}$ and "$*$" represents convolution.

Lemma 3. [6,10]Suppose $l(z) = \sum_{n=1}^{\infty} l_n z^n$and $k(z) = \sum_{n=1}^{\infty} k_n z^n$be analytic in Δ, and assume $l \prec k$where k is univalent in$\triangle$. Then $\sum_{m=1}^{n} |l_m|^2 \leq \sum_{m=1}^{n} |k_m|^2, n = 1, 2, \ldots$

Lemma 4. [6,10] (Theorom 6.4(i)). Suppose $j(z) = \sum_{n=1}^{\infty} j_n z^n$and $h(z) = \sum_{n=1}^{\infty} h_n z^n$be analytic in $\triangle$and assuming $j \prec h$where$h$is univalent in $\triangle$, then

[1] On condition that$h$is convex; $|j_n| \leq |h'(0)| = h_1, n = 1, 2, \ldots.$

[2] On condition that h is starlike (starlike with respect to 1);$|j_n| \leq n|h$

Lemma 5. [11] If v (z) = $\sum_{n=1}^{\infty} v_n z^n \in \Omega$,where Ωdenotes the class of schwarz functions in Δ. Then for any real number p_1 and p_2, the following sharp estimate holds:

$$\left|v_3 + p_1 v_1 v_2 + p_2 v_1^3\right| \leq H(p_1; p_2),$$

where,
$H(p_1, p_2)$

$$H(p_1,p_2) = \begin{cases} 1, \quad if\ (p_1,p_2) \in D_1 \cup D_2 \cup \{(2,1)\}, \\ |p_2|, \quad if\ (p_1,p_2) \in \cup_{K=3}^{7} D_k \\ \frac{2}{3}(|p_1|+1)\left(\frac{|p_1|+1}{3|p_1|+1+p_2}\right)^{\frac{1}{2}}, if\ (p_1,p_2) \in D_8 \cup D_9 \\ \frac{p_2}{3}\left(\frac{p_1^2-4}{p_1^2-4p_2}\right)\left(\frac{p_1^2-4}{3(p_2-1)}\right)^{\frac{1}{2}}, if\ (p_1,p_2) \in D_{10} \cup D_{11} \setminus \{(2,1)\} \\ \frac{2}{3}(|p_1|-1)\left(\frac{|p_1|-1}{3|p_1|-1-p_2}\right)^{\frac{1}{2}}, if\ (p_1,p_2) \in D_{12} \end{cases}$$

where the sets $D_k, k = 1, 2, \ldots, 12$are given by

$D_1 = \left\{(p_1, p_2) : |p_1| \leq \frac{1}{2}, |p_2| \leq 1\right\}$,

$$D_2 = \left\{(p_1,p_2) : \frac{1}{2} \leq |p_1| \leq 2, \frac{4}{27}\left((|p_1|+1)^3\right) - (|p_1|+1) \leq |p_2| \leq 1\right\}$$

$D_3 = \left\{(p_1, p_2) : |p_1| \leq \frac{1}{2}, |p_2| \leq -1\right\}$,
$D_4 = \left\{(p_1, p_2) : |p_1| \geq \frac{1}{2}, |p_2| \leq -\frac{2}{3}(|p_1|+1)\right\}$,
$D_5 = \{(p_1, p_2) : |p_1| \leq 2, |p_2| \geq 1\}$,

$D_6=\left\{(p_1,p_2):2\leq|p_1|\leq 4,|p_2|\geq\frac{1}{12}\left(p_1^2+8\right)\right\}$,
$D_7=\left\{(p_1,p_2):|p_1|\geq 4,|p_2|\geq\frac{2}{3}\left(|p_1|-1\right)\right\}$,
$$D_8=\left\{\begin{array}{l}(p_1,p_2):\frac{1}{2}\leq|p_1|\leq 2,\\ -\frac{2}{3}\left(|p_1|+1\right)\leq p_2\leq\frac{4}{27}\left(|p_1|+1\right)^3-\left(|p_1|+1\right)\end{array}\right\}$$
$D_9=\left\{(p_1,p_2):|p_1|\geq 2,-\frac{2}{3}\left(|p_1|+1\right)\leq p_2\leq\frac{2|p_1|(|p_1+1|)}{p_1^2+2|p_1|+4}\right\}$
$D_{10}=\left\{(p_1,p_2):2\leq|p_1|\leq 4,\frac{2|p_1|(|p_1+1|)}{p_1^2+2|p_1|+4}\leq p_2\leq\frac{1}{12}\left(p_1^2+8\right)\right\}$
$D_{11}=\left\{(p_1,p_2):|p_1|\geq 4,\frac{2|p_1|(|p_1+1|)}{p_1^2+2|p_1|+4}\leq p_2\leq\frac{2|p_1|(|p_1-1|)}{p_1^2-2|p_1|+4}\right\}$,
$D_{12}=\left\{(p_1,p_2):|p_1|\geq 4,\frac{2|p_1|(|p_1-1|)}{p_1^2-2|p_1|+4}\leq p_2\leq\frac{2}{3}\left(|p_1|-1\right)\right\}$.

2. Main Results

Here we are assuming $\Psi(z)$ to be an analytic univalent function in $\triangle$ which follows the condition; $\Psi(0)=1$ and is given by

$$\Psi\ (z)=1+\sum_{n=1}^{\infty}D_nz^n\ ,\qquad D_1\neq 0 \tag{2.1}$$

Theorem 1. Suppose the function $f\in S_d^*(\psi)$. Then the logarithmic coefficients follows the conditions

[3] In case that Ψ is convex;

$$|\gamma_n|\leq\frac{d\,|D_1|}{2n},n\in\mathrm{N}, \tag{2.2}$$

$$\sum_{n=1}^{k}|\gamma_n|^2\leq\frac{d^2}{4}\sum_{n=1}^{k}\frac{|D_n|^2}{n^2},k\in\mathrm{N} \tag{2.3}$$

and

$$\sum_{n=1}^{\infty}|\gamma_n|^2\leq\frac{d^2}{4}\sum_{n=1}^{\infty}\frac{|D_n|^2}{n^2}. \tag{2.4}$$

(ii) In case that $\Psi(z)$is starlike with respect to 1;

$$|\gamma_n|\leq\frac{d}{2}|D_1|,n\in\mathrm{N} \tag{2.5}$$

The above inequalities in the cases (i) and (ii)are sharp such that for any n $\in$ N, there exists function f_n satisfying: $1+\frac{1}{d}\left(\frac{zf_n'(z)}{f_n(z)}-1\right)=\psi(z^n)$ and the function f satisfying: $1+\frac{1}{d}\left(\frac{zf'(z)}{f(z)}-1\right)=\psi(z)$, respectively.

Proof : Assume that $f\in S_d^*(\Psi)$.Then by the definition of $S_d^*(\Psi)$ and using (1.2), we deduce that

$$1+\frac{1}{d}\left[z\frac{d}{dz}\left(\log\frac{f(z)}{z}\right)\right]=1+\frac{1}{d}\left[\frac{zf'(z)}{f(z)}-1\right]\prec\Psi(z),z\in\triangle$$

and thus we obtain

$$\frac{1}{d}\left[\sum_{n=1}^{\infty}2n\gamma_nz^n\right]\prec\Psi(z)-1:=\phi(z),z\in\Delta$$

Now, firstly to prove inequality (2.2), let us suppose that $\Psi(z)$is convex in Δ. Then $\phi(z)$is also convex with $\phi'(0)=D_1$, so by applying Lemma 4 (i), we obtain

$$\frac{2n}{d}|\gamma_n|\leq|\phi'(0)|=|D_1|$$

which gives the result:

$$|\gamma_n|\leq\frac{d}{2n}|D_1|,n\in N$$

Again, to prove inequality (2.3), we define the analytic function $h(z)=\left(\frac{f(z)}{z}\right)^{\frac{1}{d}}$,which satisfy the following :

$$\frac{zh'(z)}{h(z)}=\frac{1}{d}\left(\frac{zf'(z)}{f(z)}-1\right)\prec\phi(z),z\in\triangle \tag{2.6}$$

Alsowe know that the function (see [12])

$E_o(z)=\log\left(\frac{1}{1-z}\right)=\sum_{n=1}^{\infty}\frac{z^n}{n}$

belongs to the class K, and forf $\in\mathscr{A}$,

$$f(z)_*E_0(z)=\int_0^z\frac{f(x)}{x}dx \tag{2.7}$$

Then, by Lemma 2 and equation (2.6), we get

$$\frac{zh'(z)}{h(z)}*E_o(z)\prec\phi(z)*E_o(z)$$

Using (2.7), the exceeding equation reduces to

$$\frac{1}{d}\log\left(\frac{f(z)}{z}\right)\prec\int_0^z\frac{\varphi(x)}{x}dx$$

Also we know that (see [13]), the function $\int_o^z\frac{\phi(x)}{x}dx$, is convex univalent. Using (1.2),the above relation becomes

$$\frac{1}{d}\sum_{n=1}^{\infty}2\gamma_nz^n\prec\sum_{n=1}^{\infty}\frac{D_nz^n}{n}$$

Now by using Lemma 3, the above subordination yields

$$\frac{4}{d^2}\sum_{n=1}^{k}|\gamma_n|^2\leq\sum_{n=1}^{k}\frac{|D_n|^2}{n^2}$$

This concludes inequality (2.3).
Assuming $k\to\infty$,

$$\sum_{n=1}^{\infty}|\gamma_n|^2\leq\frac{d^2}{4}\sum_{n=1}^{\infty}\frac{|D_n|^2}{n^2}$$

this gives equation (2.4).
Lastly assume that $\Psi(z)$ is starlike with respect to 1 in Δ, this implies $\phi(z)$ is starlike, therefore using lemma 4(ii), we deduce

$$\frac{2n}{d}|\gamma_n|\leq n|\phi'(o)|=n|D_1|,n\in N$$

This gives equation (2.5),
To get the sharp bounds, it is sufficient to consider the following :

$$\frac{1}{d}\left[z\frac{d}{dz}\left(\log\left(\frac{f(z)}{z}\right)\right)\right]=\frac{1}{d}\left[\frac{zf'(z)}{f(z)}-1\right]$$

and so these results are sharp in cases (i) and (ii), such that for any n∈N, there exists the function f_n given by $1+\frac{1}{d}\left[\frac{zf_n'(z)}{f_n(z)}-1\right]=\Psi(z^n)$, and the function f given by $1+\frac{1}{d}\left[\frac{zf'(z)}{f(z)}-1\right]=\Psi(z)$, respectively, hence proved.

Corollary 1. For $0\le a<1$, if $f\in S_d^*(\alpha+(1-\alpha)e^z)$,Then the logarithmic coefficients of f, follows the conditions given below $|\gamma_n|\le\frac{d}{2n}(1-\alpha)$, n∈ N and

$$\sum_{n=1}^{\infty}|\gamma_n|^2\le\frac{d^2}{4}\sum_{n=1}^{\infty}\frac{(1-\alpha)^2}{(n!)^2n^2}$$

The above conditions are sharp for function f_n satisfying :

$$1+\frac{1}{d}\left(\frac{zf_n'(z)}{f_n(z)}-1\right)=\alpha+(1-\alpha)e^{z^n},n\in N$$

and the function f given by:

$$1+\frac{1}{d}\left(\frac{zf'(z)}{f(z)}-1\right)=\alpha+(1-\alpha)e^{z}.$$

Corollary 2. Assuming $d=1$, Class $S_d^*(\Psi(z))$ reduces to $S^*(\Psi(z))$ defined by Ma and Minda [5]. For the function $f\in S^*(\Psi(z))$,the results of Theorem 1 reduces to the logarithmic coefficients γ_n given by E.A. Adegani et al. [3],(see Theorem 1).

Corollary 3. Suppose the function $f\in S_d^*\left(1+\frac{z}{1-\alpha z^2}\right)$ and $0\le\alpha<1$. Then the logarithmic coefficients of f assures:

$$|\gamma_n|\le\frac{d}{2},\in N$$

The result is sharp for any n∈ N, there exists function f_n satisfying:

$$1+\frac{1}{d}\left(\frac{zf_n'(z)}{f_n(z)}-1\right)=1+\frac{z^n}{1-\alpha z^{2n}}.$$

Corollary 4. Suppose the function $f\in S_d^*(z+\sqrt{(1+z^2)})$ then the logarithmic coefficients of f satisfies:

$$|\gamma_n|\le\frac{d}{2},\in N.$$

This result is sharp such that for any $n\in N$, there exists function f_n satisfying:

$$1+\frac{1}{d}\left(\frac{zf_n{}'(z)}{f_n(z)}-1\right)=(z^n+\sqrt{(1+z^{2n})}).$$

Theorem 2. Suppose the function $f\in K_d(\Psi)$. Then the logarithmic coefficients of f satisfies the following conditions:

$$|\gamma_1|\le\frac{d|D_1|}{4} \tag{2.8}$$

$$|\gamma_2|\le\begin{cases}\frac{d|D_1|}{12}, & if\ \left|4D_2+dD_1^2\right|\le4|D_1|\\ \frac{d\left|4D_2+dD_1^2\right|}{48}, & if\ \left|4D_2+dD_1^2\right|>4|D_1|\end{cases} \tag{2.9}$$

and if D_1,D_2 and D_3 are real values,

$$|\gamma_3|\le\frac{d|D_1|}{24}H(p_1;p_2) \tag{2.10}$$

where $H(p_1;p_2)$ is stated in Lemma 5, $p_1=\frac{dD_1+\frac{4D_2}{D_1}}{2}$ and $p_2=\frac{\left[(3-2d)D_2+\frac{2D_3}{D_1}\right]}{2}$
The bounds of equations(2.8) and (2.9) are sharp.
Proof: Assume $f\in K_d(\Psi)$. By considering the definition of subordination, there exists $w\in\ \Omega$ with $w(z)=\sum_{n=1}^{\infty}b_nz^n$, so that

$$1+\frac{1}{d}\left[\frac{zf''(z)}{f'(z)}\right]=\Psi(w(z))$$

$$\begin{aligned}&1+\tfrac{1}{d}\left[\tfrac{zf''(z)}{f'(z)}\right]=1+D_1b_1z+(D_1b_2+D_2b_1^2)z^2\\&+(D_1b_3+2b_1b_2D_2+D_3b_1^3)z^3+......\end{aligned} \tag{2.11}$$

Equating the coefficients of z^n(n=1,2,3), we obtain

$$\begin{cases}\frac{2t_2}{d}=D_1b_1\\ \frac{6t_3-4t_2^2}{d}=D_1b_2+D_2b_1{}^2\\ \frac{12t_4-18t_2t_3+8t_2^3}{d}=D_1b_3+2b_1b_2D_2+b_1{}^3D_3\end{cases} \tag{2.12}$$

Now by putting the values of t_n (n=1,2,3) from (2.11) in (1.3);

$$\begin{cases}2\gamma_1=\frac{dD_1b_1}{2}\\ 2\gamma_2=\frac{8dD_1b_2+db_1^2\left(2dD_1^2+8D_2\right)}{48}\\ 2\gamma_3=\frac{dD_1}{12}\left[b_3+\left(\frac{dD_1+\frac{4D_2}{D_1}}{2}\right)b_1b_2+\left(\frac{(3-2d)D_2+\frac{2D_3}{D_1}}{2}\right)b_1^3\right]\end{cases} \tag{2.13}$$

Now, for γ_1 we apply Lemma 1 and get

$$|\gamma_1|\le\frac{d|D_1|}{4}$$

and this bound is sharp for $|b_1|=1$

Again, for γ_2, we apply Lemma 1and obtain

$$|\gamma_2| \leq d\left[\frac{4|D_1|\left(1-|b_1|^2\right)+\left|4D_2+dD_1{}^2\right||b_1|^2}{48}\right]$$

$$= \frac{d}{48}\left[4|D_1|+\left(\left|4D_2+dD_1^2\right|-4|D_1|\right)|b_1|^2\right]$$

$$\leq \begin{cases} \frac{4d|D_1|}{48}, & if\ \left|4D_2+dD_1^2\right| \leq 4|D_1| \\ \frac{d\left|4D_2+dD_1^2\right|}{48}, & if\ \left|4D_2+dD_1^2\right| \leq 4|D_1| \end{cases}$$

These bounds are sharp for $b_1 = 0$and $|b_1| = 1$ respectively.
At the last, for γ_3, using Lemma 5, we get

$$2|\gamma_3| \leq \frac{d|D_1|}{12}\left|b_3+\frac{\left(dD_1+\frac{4D_2}{D_1}\right)}{2}.b_1b_2+\frac{(3-2d)D_2+\frac{2D_3}{D_1}}{2}.b_1^3\right|$$

$$\leq H(p_1;p_2).\frac{d|D_1|}{12}$$

Where
$p_1 = \frac{dD_1+\frac{4D_2}{D_1}}{2}$,and$p_2 = \frac{(3-2d)D_2+\frac{2D_3}{D_1}}{2}$
Thus we get the result.
Remark 1. Assuming

$$\Psi(z) = 1+\frac{cz}{1-z} \quad (C(0,3])$$

and d =1, we obtain the result given by ponnusamy et al. [14]
Remark 2. Let d=1, in theorem 2, then we get the result obtained by E.A. Adeganiet al. [2].

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