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Onto minus domination number of paths and cycles

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Abstract

Let $G=(V,E)$ be a graph with n vertices. An onto minus dominating function of a graph G is a minus dominating function of G which is onto. The onto minus domination number of a graph G is a minimum weight of a set of onto minus dominating functions of G. In this paper we discuss the onto minus domination number of a path P_n, cycle C_n.

Keywords

Onto Minus Dominating Function, Onto Minus Domination Number, Paths, Cycles.

AMS Subject Classification

05C69.

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Contents

1. Introduction

Domination theory is one of the most interesting and application oriented branch in Graph theory. Oystein Ore [9] introduces the terms dominating set and domination number. Later Cockayne and Hedetniemi used the notation $\gamma(G)$ for the domination number of a graph G. The minus dominating function was introduced by Dunbar et al.[1] Zelinka [3] gave a lower bound of a minus domination number for a cubic graph and Dunbar et al.[4] did the same work for regular graphs[5].

The closed neighborhood of a vertex $v \in V(G)$ denoted by $N[v]$ consists of the vertex v and all the vertices which are adjacent to v in G. For a real valued function f defined on V we denote the weight of the function to be $f(V)$ and is defined by $f(V)=\sum_{v\in V} f(v)$. Also for $S \subseteq V$, we define $f(S)=\sum_{v\in S} f(v)$. A minus dominating function of a graph G [4] is a function $f: V(G) \to \{-1,0,1\}$ such that $f(N[v]) \geq 1$ for all $v \in V(G)$. The minus domination number[4] of a graph G denoted by $\gamma^-(G)$ is the minimum weight of a set of minus dominating functions of G. A function $f: A \to B$ is said to be onto if every element in B has a pre-image in A. That is a function $f: A \to B$ is said to be onto if the range of f is equal to B. A vertex of degree zero is called an isolated vertex. A vertex of degree one is called an end vertex. A vertex which is adjacent to an end vertex is called a remote vertex [6].

2. Main Results

An onto dominating function is a dominating function which is onto. The onto domination number denoted by $\gamma_o(G)$ is the minimum weight of a set of onto dominating functions of G. The function $f: V(G) \to \{0,1\}$ defined by $f(v)=1$ for all $v \in V(G)$ is a dominating function of G, which is not onto. We call the function defined above as the trivial dominating function. Every dominating function other than the trivial dominating function is onto. Since the trivial dominating function is the only dominating function of $\bar{K}_n$, we conclude that onto dominating function do not exists only for the graph $\bar{K}_n$ Thus except the graph $\bar{K}_n$ we have $\gamma_0(G)=\gamma(G)$. An onto minus dominating function is a minus dominating function which is onto. An onto minus domination number denoted by $\gamma_o^-(G)$ is the minimum weight of a set of onto minus dominating functions of G. By the definition, every onto minus dominating function of G is a minus dominating function of G. Hence $\gamma^-(G) \leq \gamma_0^-(G)$. Thus if there exists an onto minus dominating function of weight $\gamma^-(G)$ then $\gamma_o^-(G)=\gamma^-(G)$.

In this paper we discuss the onto minus domination number of a path P_n and cycle C_n. For the function $f : V \to \{-1,0,1\}$, the co-domain contains three elements and so an onto minus dominating function is defined only for the graphs with number of vertices $n \geq 3$. Also every graph with order $n = 3$ has no onto minus dominating function and hence an onto minus dominating function exists only for $n \geq 4$. But we cannot say that onto minus dominating function exists for all graphs with $n \geq 4$. For example the graph C_4 given below has no onto minus dominating function.

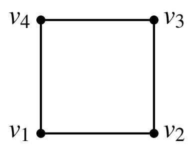

Fig 2.1. $G = C_4$

Since if we assign -1 to the vertex v_1 under the onto minus dominating function f, then in order to obtain $f(N[v_1]) \geq 1$ we must assign 1 to the vertices v_2, v_4. Now if we assign 0 or -1 to the vertex v_3 then $f(N[v_2]) < 1$ & $f(N[v_4]) < 1$. Hence we must assign 1 to the vertex v_3. Thus we cannot assign 0 to any vertex. Hence the graph C_4 given above has no onto minus dominating function. Onto minus dominating function does not exists for all graphs G with every vertex is either an end vertex or isolated vertex or remote vertex. Minus domination number is not always equal to onto minus domination number. For example, for a wheel graph W_n minus domination number is equal to 1 while onto minus domination number is equal to 2.

3. Onto Minus Domination Number of P_n.

For $n \leq 5$ the graph P_n has no onto minus dominating function, because for $n \leq 4$, every vertex in P_n is either an end vertex or remote vertex and hence it has no onto minus dominating function and for $n = 5$, the only possibility of assigning -1 is to the mid vertex and if we assign -1 to the mid vertex then in order to obtain an onto minus dominating function we must assign 1 to the remaining vertices and hence 0 is not assigned to any vertex and so the function is not onto and hence P_5 also has no onto minus dominating function. Thus for $n \leq 5$ the graph P_n has no onto minus dominating function.

Theorem 3.1. *For $n > 5, \gamma_o^-(P_n) = \gamma^-(P_n)$ or $\gamma^-(P_n) + 1$.*

Proof. We prove this result by considering three cases. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n. We know that $\gamma^-(P_n) = \lceil n/3 \rceil$.

Case (i) $n \equiv 0(mod 3)$.

In this case $n = 3m$, where $m > 1$ is a positive integer. Define a function $f : V \to \{-1,0,1\}$ by

$$f(v_i) = \begin{cases} 1 \; if \; i \equiv 1(mod3) \text{ or } i \equiv 2(mod3) \\ 0 \; if \; i = n \\ -1 \; otherwise \end{cases}$$

Then $f[v_1] = 2$ and $f[v_{n-1}] = 2$ and $f[v_i] = 1$ for all other vertices. Thus f is an onto minus dominating function of P_n if $n \equiv 0(mod3)$. Also the weight of the function f is $f(V) = (m-1)(-1) + 2(m-1) + 2 = -m+1+2m-2+2 = m+1 = \lceil 3m/3 \rceil + 1 = \lceil n/3 \rceil + 1 = \gamma^-(P_n) + 1$.

Case (ii) $n \equiv 1(mod3)$. In this case $n = 3m+1$, where $m > 1$ is a positive integer. Define a function $f : V \to \{-1,0,1\}$ by

$$f(v_i) = \begin{cases} 1 \; if \; i \equiv 1(mod3) \text{ or } i \equiv 2(mod3) \\ 0 \; if \; i = n-1 = 3m \\ -1 \; otherwise \end{cases}$$

Then $f[v_1] = 2$ and $f[v_{n-2}] = 2$ and $f[v_i] = 1$ for all other vertices. Hence f is an onto minus dominating function of P_n if $n \equiv 1(mod3)$. Also the weight of the function f is $f(V) = (m-1)(-1) + 2(m-1) + 3 = -m+1+2m-2+3 = m+2 = m+1+1 = \lceil (3m+1)/3 \rceil + 1 = \lceil n/3 \rceil + 1 = \gamma^-(P_n) + 1$.

Case (iii) $n \equiv 2(mod3)$. In this case $n = 3m+2$, where $m > 1$ is a positive integer. Define a function $f : V \to \{-1,0,1\}$ by

$$f(v_i) = \begin{cases} 1 \; if \; i \equiv 1(mod3) \text{ or } i \equiv 2(mod3) and i \neq 3m+2 \\ 0 \; if \; i = 3m and 3m+2 \\ -1 \; otherwise \end{cases}$$

Then $f[v_1] = 2$ and $f[v_{n-3}] = 2$ and $f[v_i] = 1$ for all other vertices. Hence f is an onto minus dominating function of $P_n if n \equiv 2(mod3)$. Also the weight of the function f is $f(V) = (m-1)(-1) + 2(m-1) + 3 = -m+1+2m-2+3 = m+2 = m+1+1 = \lceil (3m+2)/3 \rceil + 1 = \lceil n/3 \rceil + 1 = \gamma^-(P_n) + 1$. From the above three cases we conclude that for $n > 5$, the graph P_n has an onto minus dominating function of weight $\gamma^-(P_n) + 1$. Hence $\gamma_o^-(P_n) \leq \gamma^-(P_n) + 1$. Also we know that $\gamma^-(P_n) \leq \gamma_o^-(P_n)$. Hence $\gamma^-(P_n) \leq \gamma_o^-(P_n) \leq \gamma^-(P_n) + 1$. Thus $\gamma_o^-(P_n) = \gamma^-(P_n)$ or $\gamma^-(P_n) + 1$.

□

Theorem 3.2. *For $n > 6$ $\gamma_o^-(P_n) = \gamma^-(P_n) = \lceil n/3 \rceil if n \equiv 1(mod3)$.*

Proof. Assume that $n \equiv 1(mod3)$ & $n > 6$. Then there exists an integer $m > 1$ such that $n = 3m+1$. Define a function $f : V \to \{-1,0,1\}$ by

$$f(v_i) = \begin{cases} 0 \; if \; i \equiv 1(mod3) \text{ except } i = 4 \text{ and } i \equiv 2(mod3) \text{ except } i = 2,5 \\ 1 \; if \; i = 0(mod3) \text{ and } i = 2,5 \\ -1 \; if \; i = 4 \end{cases}$$

Then $f(N[v_i]) = 1$ for all i except $i = 2$ and for $i = 2$, $f(N[v_2]) = 2$. Thus f is an onto minus dominating function of P_n. Also the weight of the function is $f(V) = \sum_{i=1}^{n} f(v_i) = 0(m+1-1+m-2) + 1(m+2) + (-1) = 0+m+2-1 = m+1 = \lceil (3m+1)/3 \rceil = \lceil n/3 \rceil = \gamma^-(P_n)$. Hence f is an onto minus dominating function of weight $\gamma^-(P_n)$. Therefore

$\gamma_o^-(P_n) \leq \gamma^-(P_n)$. Also we know that $\gamma_o^-(P_n) \geq \gamma^-(P_n)$. Hence $\gamma_o^-(P_n) = \gamma^-(P_n)$. Thus $\gamma_o^-(P_n) = \gamma^-(P_n) = \lceil n/3 \rceil if n \equiv 1(mod3)$.

If $n \equiv 0(mod3)$ then $\gamma_o^-(P_n) \neq \gamma^-(P_n)$. For example the possible onto minus dominating functions of P_6 are as follows. Let $v_1, v_2, \ldots, v_6$ be the vertices of P_6. P_6 has only two onto minus dominating functions $f, g : V(P_6) \to \{-1, 0, 1\}$ and are defined as follows. $f(v_1) = 1, f(v_2) = 1, f(v_3) = -1$, $f(v_4) = 1, f(v_5) = 1, f(v_6) = 0$ & $g(v_1) = 0, g(v_2) = 1, g(v_3) = 1, g(v_4) = -1, g(v_5) = 1, g(v_6) = 1$. Hence $f(V) = 3$ & $g(V) = 3$. Thus $\gamma_o^-(P_6) = 3$. But $\gamma^-(P_6) = \lceil 6/3 \rceil = 2$. Therefore $\gamma_o^-(P_6) \neq \gamma^-(P_6)$ and $\gamma_o^-(P_6) = \gamma^-(P_6) + 1$. If $n \equiv 2(mod3)$ then $\gamma_o^-(P_n) \neq \gamma^-(P_n)$. For example the possible onto minus dominating functions of P_8 with minimum weight are as follows. Let $v_1, v_2, \ldots, v_8$ be the vertices of P_8.P_8 has only four onto minus dominating functions f, g, h, i : $V(P_8) \to -1, 0, 1$ of minimum weight 4 and are defined as follows. $f(v_1) = 1, f(v_2) = 1, f(v_3) = -1, f(v_4) = 1, f(v_5) = 1, f(v_6) = 0, f(v_7) = 0, f(v_8) = 1$ & $g(v_1) = 0, g(v_2) = 1, g(v_3) = 1, g(v_4) = -1, g(v_5) = 1, g(v_6) = 1, g(v_7) = 0, g(v_8) = 1, h(v_1) = 1, h(v_2) = 0, h(v_3) = 1, h(v_4) = 1, h(v_5) = -1, h(v_6) = 1, h(v_7) = 1, h(v_8) = 0, i(v_1) = 1, i(v_2) = 0, i(v_3) = 0, i(v_4) = 1, i(v_5) = 1, i(v_6) = -1, i(v_7) = 1, i(v_8) = 1$. Thus $\gamma_o^-(P_8) = 4$. But $\gamma^-(P_8) = \lceil 8/3 \rceil = 3$. Therefore $\gamma_o^-(P_8) \neq \gamma^-(P_8) and \gamma_o^-(P_8) = \gamma^-(P_8) + 1$. In general if $n \equiv 2(mod3)$ then $\gamma_o^-(P_n) = \gamma^-(P_n) + 1$. □

4. Onto Minus Domination Number of C_n.

We already discuss the graph C_4 has no onto minus dominating function. Also the graph C_5 has no onto minus dominating function. Suppose C_5 has an onto minus dominating function. Let f be an onto minus dominating function of C_5. Let $v_1, v_2, \ldots, v_5$ be the vertices of C_5. If we assign -1 to one of the vertex then we must assign 1 to the remaining four vertices and so 0 is not assigned to any of the vertices of C_5 under f. Hence the function f is not onto. Which is a contradiction. Hence the graph C_5 has no onto minus dominating function.

Theorem 4.1. *For* $n > 5, \gamma_o^-(C_n) = \gamma^-(C_n)$ *or* $\gamma^-(C_n) + 1$.

Proof. Since every vertex adjacent in P_n is also adjacent in C_n and hence every minus dominating function of P_n is also a minus dominating function of C_n. Furthermore every onto minus dominating function of P_n is also an onto minus dominating function of C_n. Thus $\gamma_o^-(C_n) \leq \gamma_o^-(P_n)$. We know that $\gamma^-(C_n) \leq \gamma_o^-(C_n)$. Therefore $\lceil n/3 \rceil = \gamma^-(C_n) \leq \gamma_o^-(C_n) \leq \gamma_o^-(P_n) = \lceil n/3 \rceil + 1$. Thus $\gamma_o^-(C_n) = \lceil n/3 \rceil$ or $\lceil n/3 \rceil + 1$. □

Theorem 4.2. *For* $n > 6$, $\gamma_o^-(C_n) = \lceil n/3 \rceil$ *if* $n \equiv 1(mod3)$.

Proof. We have $\gamma^-(C_n) \leq \gamma_o^-(C_n) \leq \gamma_o^-(P_n)$. Also we know that for $n > 6, \leq \gamma^-(C_n) = \lceil n/3 \rceil$ and $\gamma_o^-(P_n) = \lceil n/3 \rceil$ if $n \equiv 1(mod3)$. Thus $\lceil n/3 \rceil \leq \gamma_o^-(C_n) \leq \lceil n/3 \rceil if n \equiv 1(mod3)$. Hence for $n > 6, \leq \gamma_o^-(C_n) = \lceil n/3 \rceil$ if $n \equiv 1(mod3)$. □

5. Conclusion

Onto minus dominating functions in graphs is an interesting and application oriented area in domination theory. In this paper we discussed the onto minus domination number of path P_n and cycle C_n. This work increases the scope for an extensive study of minus dominating functions in graph theory.

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