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Unbalanced transportation problem with pentagonal intuitionsitic fuzzy number solved using ambiguity index

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Abstract

This paper provides a solution to an unbalanced Pentagonal Intuitionistic fuzzy transportation problem in which the total supply is more than the total demand. The cost of the transportation problem is Pentagonal Intuitionistic Fuzzy Number with degree of membership and degree of non-membership function. A ranking method based on value and ambiguity index of Pentagonal Intuitionistic Fuzzy Number is used to solve the unbalanced transportation problem.

Keywords

Pentagonal Intuitionistic Fuzzy Number (PIFN), value index, ambiguity index, alpha cut, beta cut .

AMS Subject Classification

03E72, 03F55, 90B06, 90C08.

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Contents

1. Introduction

Fuzzy set theory was introduced by Zadeh [18] in the year 1965. The concept on Intuitionistic Fuzzy Number was introduced by Atanassov [6]. An Intuitionistic Fuzzy set is a powerful tool which deals with vagueness. There are many models in transportation problem which play an important role in reducing cost and improving service. Pentagonal Intuitionistic Fuzzy Numbers are used in neural networks and also in many other applications of research. In this paper an illustrate example for unbalanced transportation problem with pentagonal intuitionistic fuzzy cost along with degree of acceptance and degree of rejection is solved. Where the supply and demand are real numbers. Further initial basic feasible solution and optimal solution for Intuitionistic fuzzy Vogel's Approximation method is evaluated by converting the pentagonal intutionistic fuzzy number to crisp values by using value and ambiguity index based ranking method.

2. Preliminaries

Definition 2.1. *[3]* **Intuitionistic fuzzy set:** *Let X be a universal set. An Intuitionistic fuzzy set A in X is* $A^I = \{x, \mu_A^I(x), \vartheta_A^I(x)) : x \in X\}$ *where the function* $\mu_{\tilde{A}} : x \to [0,1]$, $\vartheta_{\tilde{A}} : x \to [0,1]$ *define the degree of membership and the degree of non-membership of the element* $x \in X$ *to the set* A^I *respectively and for every* $x \in X$ *in* A^I, $0 \le \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}} \le 1$ *holds.*

Definition 2.2. *[9]* **Pentagonal intuitionistic fuzzy number:** *A PIFN* $\tilde{a} = \langle (a_1,a_2,a_3,a_4,a_5)(b_1,b_2,b_3,b_4,b_5); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ *is a Intuitionistic Fuzzy set on a set of real number R, whose membership and non-membership functions are defined as:*

MEMBERSHIP FUNCTION

$$\mu_{\tilde{a}}(x) = \begin{cases} w_1 \frac{(x-a_1)}{a_2-a_1} & for \quad a_1 \le x \le a_2 \\ w_1 + \frac{(w_{\tilde{a}}-w_1)(x-a_2)}{a_3-a_2} & for \quad a_2 \le x \le a_3 \\ w_{\tilde{a}} & for \quad x = a_3 \\ w_1 + \frac{(w_{\tilde{a}}-w_1)(a_4-x)}{a_4-a_3} & for \quad a_3 \le x \le a_4 \\ w_1 \frac{(a_5-x)}{a_5-a_4} & for \quad a_4 \le x \le a_5 \\ 0 & for \quad a_5 < x \ \& \ a_1 > x \end{cases}$$

NON-MEMBERSHIP FUNCTION

$$\vartheta_{\tilde{a}}(x) = \begin{cases} 1 - \frac{(1-w_1)(x-b_1)}{b_2-b_1} & for \quad b_1 \le x \le b_2 \\ \frac{w_1-(w_1-u_{\tilde{a}})(x-b_2)}{b_3-b_2} & for \quad b_2 \le x \le b_3 \\ u_{\tilde{a}} & for \quad x = b_3 \\ \frac{w_1-(w_1-u_{\tilde{a}})(b_4-x)}{b_4-b_3} & for \quad b_3 \le x \le b_4 \\ 1 - \frac{(1-w_1)(b_5-x)}{b_5-b_4} & for \quad b_4 \le x \le b_5 \\ 1 & for \quad b_5 < x \ \& \ b_1 > x \end{cases}$$

The maximum degree of membership $w_{\tilde{a}}$ and minimum degree of non-membership $u_{\tilde{a}}$ satisfy the conditions $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ and $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$. The parameters $w_{\tilde{a}}$ and $u_{\tilde{a}}$ reflects the confidence level and non-confidence level of the Pentagonal Intuitionistic Fuzzy Number $\langle (a_1,a_2,a_3,a_4,a_5)(b_1,b_2,b_3,b_4,b_5); w_{\tilde{a}} u_{\tilde{a}} \rangle$.

Let $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \vartheta_{\tilde{a}}(x)$, which is an indeterminacy factor index of an element x in $\tilde{a}$. It is the degree of the indeterminacy membership of an element x in $\tilde{a}$.

Definition 2.3. *[9] Let* $\tilde{a} = \langle (a_1,a_2,a_3,a_4,a_5)(c_1,c_2,c_3,c_4,c_5); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ *and* $\tilde{b} = \langle (b_1,b_2,b_3,b_4,b_5)(d_1,d_2,d_3,d_4,d_5); w_{\tilde{b}}, u_{\tilde{b}} \rangle$ *be two PIFNs and* λ *be a real number. Then the arithmetical operations are*

$$\tilde{a} + \tilde{b} = \langle a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5; c_1+d_1, c_2+d_2, c_3+d_3, c_4+d_4, c_5+d_5); min\{w_{\tilde{a}}, w_{\tilde{b}}\}, max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle$$

$$\tilde{a} - \tilde{b} = \langle (a_1-b_5, a_2-b_4, a_3-b_3, a_4-b_2, a_5-b_1; c_1-d_5, c_2-d_4, c_3-d_3, c_4-d_2, c_5-d_1); min\{w_{\tilde{a}}, w_{\tilde{b}}\}, max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle$$

$\tilde{a} * \tilde{b} = \langle a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5; c_1d_1, c_2d_2, c_3d_3, c_4d_4, c_5d_5);$ $min\{w_{\tilde{a}}, w_{\tilde{b}}\}, max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle$ *where* $\tilde{a}$ *and* $\tilde{b}$ *are non-negative pentagonal intuitionistic fuzzy numbers.*

$\lambda\tilde{a} = \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5; \lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5); w_{\tilde{a}}, u_{\tilde{a}} \rangle;$ $\lambda \ge 0$

$\lambda\tilde{a} = \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5; \lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5); w_{\tilde{a}}, u_{\tilde{a}} \rangle;$ $\lambda < 0$

$\tilde{a}^{-1} = \langle (\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}, \frac{1}{a_5}; \frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3}, \frac{1}{b_4}, \frac{1}{b_5}; w_{\tilde{a}}, u_{\tilde{a}} \rangle$

3. $\alpha-$cut sets and $\beta-$cuts sets of Pentagonal Intuitionistic Fuzzy Number (PIFN)

[9] $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ are both closed sets and are denoted by $\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)]$ and $\tilde{a}_\beta = [L_{\tilde{a}}(\beta), R_{\tilde{a}}(\beta)]$ repectively. The respective values of $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ are calculated as follows:

$$[L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)] = [a_1 + \frac{\alpha(a_2-a_1)}{w_1}, a_2 + \frac{(\alpha-w_1)(a_3-a_2)}{(w_{\tilde{a}}-w_1)}, a_4 + \frac{(\alpha-w_1)(a_4-a_3)}{(w_{\tilde{a}}-w_1)}, a_5 + \frac{\alpha(a_5-a_4)}{w_1}]$$

$$[L_{\tilde{a}}(\beta), R_{\tilde{a}}(\beta)] = [b_1 + \frac{(1-\beta)(b_2-b_1)}{(1-w_1)}, b_2 + \frac{(w_1-\beta)(b_3-b_2)}{(w_1-w_{\tilde{a}})}, b_4 + \frac{(w_1-\beta)(b_4-b_3)}{(w_1-w_{\tilde{a}})}, b_5 + \frac{(1-\beta)(b_5-b_4)}{(1-w_1)}]$$

4. Ranking of PIFNs based on Value and Ambiguity

The value and ambiguity of a PIFN can be defined similar to those of a TIFNs introduced by D.F.Li [11].

Definition 4.1. *[9] Let* $\tilde{a}_\alpha$ *and* $\tilde{a}_\beta$ *be an* $\alpha-$*cut set and* $\beta-$*set of a Pentagonal Intuitionistic Fuzzy Number* $\tilde{a} = \langle (a_1,a_2,a_3,a_4,a_5)(b_1,b_2,b_3,b_4,b_5); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ *respectively. Then the values of the membership function* $\mu_{\tilde{a}}(x)$ *and the values of the non-membership function* $\vartheta_{\tilde{a}}(x)$ *for the PIFN* $\tilde{a}$ *are defined as follows*

$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} \frac{L_{\tilde{a}}(\alpha) + R_{\tilde{a}}(\alpha)}{2} f(\alpha) d\alpha$$

$$V_\vartheta(\tilde{a}) = \int_0^{w_{\tilde{a}}} \frac{L_{\tilde{a}}(\beta) + R_{\tilde{a}}(\beta)}{2} g(\beta) d\beta$$

Respectively, where the function $f(\alpha)$ *is a non-negative and non-decreasing function on the interval* $[0, w_{\tilde{a}}]$ *with* $f(0) = 0$ *and* $\int_0^{w_{\tilde{a}}} f(\alpha) d\alpha = w_{\tilde{a}}$. *The function* $g(\beta)$ *is a non-negative and non-increasing function on the interval* $[u_{\tilde{a}}, 1]$ *with* $g(1) = 0$ *and* $\int_{u_{\tilde{a}}}^1 g(\beta) d\beta = 1 - u_{\tilde{a}}$. *Throughout the paper we shall choose* $f(\alpha) = \frac{2\alpha}{w_{\tilde{a}}}$, $\alpha \in [0, w_{\tilde{a}}]$ *and* $g(\beta) = \frac{2(1-\beta)}{1-u_{\tilde{a}}}$ *where* $\beta \in [u_{\tilde{a}}, 1]$.

The value of the membership function of a PIFN $\tilde{a}$ is calculated as follows:

$$V_\mu(\tilde{a}) = \frac{w_1^2(a_1+2a_2+2a-4+a_5)}{6w_{\tilde{a}}} + \frac{(w_{\tilde{a}}+w_1)[w_{\tilde{a}}(a_2+a_4)-2w_1a_3]}{2w_{\tilde{a}}} + \frac{w_{\tilde{a}}^2+w_{\tilde{a}}w_1+w_1^2)(2a_3-a_2-a_4)}{3w_{\tilde{a}}}.$$

The value of the non- membership function of a PIFN $\tilde{a}^I$ is calculated as follows:

$$V_\vartheta(\tilde{a}) = \frac{1}{(1-u_{\tilde{a}})(w_1-u_{\tilde{a}})}\left[\frac{2w_1-w_1^2-2u_{\tilde{a}}+u_{\tilde{a}}^2)(2w_1b_3-u_{\tilde{a}}(b_2+b_4))}{2} + \frac{(3w_1^2-2w_1^3-3u_{\tilde{a}}^2+2u_{\tilde{a}}^3)(b_4-2b_3+b_2)}{6}\right] + \left[\frac{(1-w_1)\big((b_2+b_4)-w_1(b_1+b_5)\big)}{2(1-u_{\tilde{a}})}\right] + \left[\frac{(1-3w_1^2+2w_1^3)(b_5-b_4-b_2+b_1)}{6(1-u_{\tilde{a}})(1-w_1)}\right]$$

With the condition that $0 \le w_{\tilde{a}}+u_{\tilde{a}} \le 1$, it follows that $V_\mu(\tilde{a}) \le V_\vartheta(\tilde{a})$ thus the values of the membership and non-membership function of a PIFN $\tilde{a}$ can be expressed as an interval $[V_\mu(\tilde{a}), V_\vartheta(\tilde{a})]$.

Definition 4.2. *[9] Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an $\alpha-$cut set and $\beta-$set of a PIFN $\tilde{a} = \langle(a_1,a_2,a_3,a_4,a_5)(b_1,b_2,b_3,b_4,b_5); w_{\tilde{a}}, u_{\tilde{a}}\rangle$ respectively. Then the ambiguities of the membership function $\mu_{\tilde{a}}(x)$ and the ambiguities of the non-membership function $\vartheta_{\tilde{a}}(x)$ for the PIFN $\tilde{a}$ are defined as follows*

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha) f(\alpha) d\alpha$$

$$A_\vartheta(\tilde{a}) = \int_0^{w_{\tilde{a}}} R_{\tilde{a}}(\beta) - L_{\tilde{a}}(\beta) g(\beta) d\beta \text{ respectively.}$$

It can be followed from definition of $A_\mu(\tilde{a})$ and $A_\vartheta(\tilde{a})$ that $A_\mu(\tilde{a}) \ge 0$, $A_\vartheta(\tilde{a}) \ge 0$

The ambiguity of the membership function of a PIFN $\tilde{a}$ is evaluated as follows.

$$A_\mu(\tilde{a}) = \frac{w_1^2(a_5+2a_4-2a_2-a_1)}{3w_{\tilde{a}}} + \frac{(a_4-a_2)(w_{\tilde{a}}^2+w_{\tilde{a}}w_1-2w_1^2)}{3w_{\tilde{a}}}$$

Similarly, the ambiguity of the non-membership function of a PIFN $\tilde{a}$ is evaluated as follows.

$$A_\vartheta(\tilde{a}) = \frac{(b_4-b_2)[(3w_1^2-2w_1^3-3u_{\tilde{a}}^2+2u_{\tilde{a}}^3)-3u_{\tilde{a}}(2w_1-w_1^2-2u_{\tilde{a}}+u_{\tilde{a}}^2)]}{3(1-u_{\tilde{a}})(w_1-u_{\tilde{a}})} + \frac{(1-w_1)}{} + \frac{(1-3w_1^2+2w_1^3)(b_5-b_4+b_2-b_1)}{3(1-u_{\tilde{a}})(1-w_1)}]$$

With the condition that $0 \le w_{\tilde{a}}+u_{\tilde{a}} \le 1$, it follows that $A_\mu(\tilde{a}) \le A_\vartheta(\tilde{a})$ thus the values of the membership and non-membership function of a PIFN $\tilde{a}$ can be expressed as an interval $[A_\mu(\tilde{a}), A_\vartheta(\tilde{a})]$.

5. The Ranking Technique

[7] Ranking is evaluated by taking the sum of value index and ambiguity index

$$R(\tilde{n}) = V(\tilde{n}, \frac{1}{2}) + A(\tilde{n}, \frac{1}{2})$$

Where $V(\tilde{n}, \frac{1}{2}) = \frac{V_\mu(\tilde{n}) + V_\vartheta(\tilde{n})}{2}$

$$A(\tilde{n}, \frac{1}{2}) = \frac{A_\mu(\tilde{n}) + A_\vartheta(\tilde{n})}{2}$$

6. Initial Basic Feasible Solution by Intuitionistic fuzzy Vogel's Approximation method for pentagonal intuitionistic fuzzy unbalanced transportation problem

1. An unbalanced transportation problem is converted into balanced transportation problem by introducing dummy cost with respect to demand as the total demand is less than the total supply.
2. In Intuitionistic fuzzy transportation problem, the pentagonal intuitionistic fuzzy transportation cost are reduced to crisp numbers using value and ambiguity based ranking.
3. In the reduced PIFTP, identify the row and column difference considering the least two numbers of the respective row and column.
4. Select the maximum among the difference and allocate the respective demand or supply to the minimum value of the corresponding row or column.
5. We take the difference of the corresponding supply and demand of the allocated cell which leads either of the one to zero, eliminating the corresponding row or column (eliminates both demand and supply if both are zero).
6. Repeat step 2, 3 and 4 until all the demands and supplies are satisfied.
7. To find the total minimum cost sum of the product of the cost and the allocated values are calculated.

7. Modified Distribution Optimal Solution by Intuitionistic fuzzy Vogel's Approximation method for pentagonal intuitionistic fuzzy balanced transportation problem

1. The number of allotted cells must be equal to $m+n-1$, if not degeneracy exists for which a very small positive assignment ε is allotted in independent suitable cost cell so that the number of occupied cells is exactly equal to $m+n-1$.

2. For each allotted cell we solve system of equations $u_i + v_j = C_{ij}$ starting with either some u_i or some v_j equating to zero where the number of allocations are maximum and hence finding the values of u_i and v_j respectively.
3. Evaluate $C_{ij} - (u_i + v_j)$ for all unoccupied cells.
4. If $d_{ij} = C_{ij} - (u_i + v_j) \geq 0$, then the basic feasible solution is the optimal solution.

8. AN ILLUSTRATIVE EXAMPLE

Consider a 5×3 Pentagonal Intuitionistic Fuzzy Number with value and Ambiguity index

TABLE 1:

	A	B	C	Supply
$S1$	$\langle(9,10,11,13,15)$ $(7,9,11,14,17);$ $0.6,0.2)\rangle$	$\langle(16,18,20,22,24)$ $(14,17,20,22,24);$ $0.6,0.2)\rangle$	$\langle(3,5,7,9,11)$ $(3,5,7,10,12);$ $0.6,0.2)\rangle$	50
$S2$	$\langle(15,20,21,24,27)$ $(14,19,21,25,28);$ $0.6,0.2)\rangle$	$\langle(12,14,16,18,20)$ $(10,13,16,19,21);$ $0.6,0.2)\rangle$	$\langle(16,18,20,22,24)$ $(14,17,20,23,26);$ $0.6,0.2)\rangle$	40
$S3$	$\langle(4,6,8,10,12)$ $(2,5,8,10,12);$ $0.6,0.2)\rangle$	$\langle(8,10,12,13,15)$ $(7,9,12,15,18);$ $0.6,0.2)\rangle$	$\langle(14,16,18,20,22)$ $(12,15,18,21,23);$ $0.6,0.2)\rangle$	70
Demand	30	25	35	

	D	E	Supply
$S1$	$\langle(4,6,8,10,12)$ $(2,5,8,11,13);$ $0.6,0.2)\rangle$	$\langle(0,0,0,0,0)$ $(0,0,0,0,0);$ $0.6,0.2)\rangle$	50
$S2$	$\langle(8,10,12,14,16)$ $(9,11,12,15,18);$ $0.6,0.2)\rangle$	$\langle(0,0,0,0,0)$ $(0,0,0,0,0);$ $0.6,0.2)\rangle$	40
$S3$	$\langle(5,7,9,11,13)$ $(3,6,9,12,13);$ $0.6,0.2)\rangle$	$\langle(0,0,0,0,0)$ $(0,0,0,0,0);$ $0.6,0.2)\rangle$	70
Demand	40	30	

Since Total Demand $\neq$ Total Supply, this problem is an unbalanced transportation problem. We has introduced a dummy cost column E with demand value 30 which balances the transportation problem. Now for the balanced transportation problem we apply value and ambiguity based ranking on pentagonal intuitionistic fuzzy number.

$\langle(9,10,11,13,15)(7,9,11,14,17);0.6,0.2\rangle$ we have

$V_\mu(\widetilde{a}) = 6.9192$

$V_\vartheta(\widetilde{a}) = 9.1208$

$A_\mu(\widetilde{a}) = 1.9335$

$A_\vartheta(\widetilde{a}) = 2.196$

$$V(\widetilde{a}) = \frac{V_\mu(\widetilde{a}) + V_\vartheta(\widetilde{a})}{2} = 8.02$$

$$A(\widetilde{a}) = \frac{A_\mu(\widetilde{a}) + A_\vartheta(\widetilde{a})}{2} = 2.06$$

$R(\widetilde{a}) = V(\widetilde{a}) + A(\widetilde{a}) = 10.08$

Similarly applying for all the values, we have the following table

TABLE 2:

	A	B	C	D	E	Supply
$S1$	10.08	16.73	7.87	8.74	0	50
$S2$	18.69	14.33	17.21	11.18	0	40
$S3$	8.33	11.21	15.73	9.36	0	70
Demand	30	25	35	40	30	160

Intuitionistic fuzzy Vogel's Approximation method for pentagonal intuitionistic fuzzy balanced transportation problem

TABLE 3: Basic Feasible Solution

	A	B	C	D	E
$S1$	10.08	16.73	35 7.87	15 8.74	0
$S2$	18.69	14.33	17.21	10 11.18	30 0
$S3$	30 8.33	25 11.21	15.73	15 9.36	0
Demand	30	25	35	40	30
Column Difference	1.75	3.12	7.86	0.62	0

	Supply	Row Difference
$S1$	50	7.87
$S2$	40	11.18
$S3$	70	8.33
Demand	160	
Column Difference		

Proceeding the same manner. We get the solution as follows

$$\begin{aligned}\text{Total Cost} &= (35 \times 7.87) + (15 \times 8.74) + (10 \times 11.18) \\ &\quad + (30 \times 0) + (30 \times 8.33) + (25 \times 11.21) \\ &\quad + (15 \times 9.36) \\ &= 1188.9/-\end{aligned}$$

Applying Modified Distribution method for optimal solution of Pentagonal Intuitionistic Fuzzy transportation problem.

TABLE 4: Cost of the allotted cells

	A	B	C	D	E	U_i's
$S1$	-	-	7.87	8.74	-	-0.62
$S2$	-	-	-	11.18	0	1.82
$S3$	8.33	11.21	-	9.36	-	0
V_j's	8.33	11.21	8.49	9.36	-1.82	

TABLE 5: $d_{ij} = C_{ij} - (u_i + v_j)$

	A	B	C	D	E
$S1$	2.37	6.14	-	-	1.2
$S2$	8.54	1.3	6.9	-	-
$S3$	-	-	7.24	-	1.82

Since $d_{ij} \geq 0$, the optimality is obtained.
The fuzzy optimal solution is given by $\widetilde{x}_{13} = 35$, $\widetilde{x}_{14} = 15$, $\widetilde{x}_{24} = 10$, $\widetilde{x}_{25} = 30$, $\widetilde{x}_{31} = 30$, $\widetilde{x}_{32} = 25$, $\widetilde{x}_{34} = 15$
The fuzzy corresponding optimal cost Rs

$$\begin{aligned}\text{Total Cost} &= (35\times 7.87)+(15\times 8.74)+(10\times 11.18)\\ &\quad +(30\times 0)+(30\times 8.33)+(25\times 11.21)\\ &\quad +(15\times 9.36)\\ &= 1188.9/-\end{aligned}$$

The optimal solution of the above problem by fuzzy VAM is Rs.1188.9.

9. Conclusion

A method for finding optimal solution in an intuitionistic fuzzy environment has been proposed using value and ambiguity ranking method for pentagonal intuitionistic fuzzy unbalanced transportation problem. Value and ambiguity ranking method is used to solve intuitionistic Vogel's Approximation method to find the initial basic feasible solution and optimal solution of intuitionistic fuzzy unbalanced transportation problem.

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