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On some fuzzy hyponormal operators

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Abstract

In this work,we focus our study on Fuzzy hyponormal operators acting on a fuzzy Hilbert space(FH space).we have given some properties of Fuzzy hyponormal operators on a FH space.And also we introduced the definition of Fuzzy class (N) operator acting on a Fuzzy Banach space (FB-space) and some definitions, theorems are discussed in detail.

Keywords

Fuzzy Banach Space, Fuzzy Hilbert Space, Fuzzy Normal Operator, Fuzzy Hyponormal Operator, Fuzzy class (N).

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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Contents

1. Introduction

In 1984, Katsaras[4] is first introduced the notion of Fuzzy Norm on a linear space. Then any other mathematicians have studied fuzzy normed space from several points of view [6, 13, 16]. The definition of fuzzy inner product space(FIP-space) headmost started by Biswas[11] and after that according the chronological in [5, 7, 8, 10, 12]. Modulate the definition of fuzzy inner product space (FIP-space) has been inserted by Goudarzi and Vaezpour in [9, 15]. In 2009, Goudarzi and Vaezpour[9] has been introduced the definition of a fuzzy Hilbert space. Noori F.AI- Mayahi and Abbas M.Abbas[19] gave some properties of spectral theory of linear operator defined on a fuzzy normed spaces which is considered as an expansion for the spectral theory of linear operator defined on normed spaces in 2016. The concept of adjoint fuzzy linear operators, self - adjoint fuzzy linear operators was introduced by Sudad.M.Rasheed [3]. In 2017, A.Radharamani et all [2] defined concept of fuzzy normal operator and their properties.The definition of fuzzy unitary operators and their properties has been given by Radharamani et all [21]in 2018.Whereas, the definition of fuzzy hyponormal operator has been given by Radhamani et all [1] in 2019.

In this paper, we consider fuzzy hyponormal operator in FH-space and introduce the definition of fuzzy class (N) operators and we establish some theorems from fuzzy hyponormal operator in FH-space and also from Fuzzy class (N) in FB-space. The classification of this paper is as follows:

In section 2 we provide some preliminary definitions, theorems and results, which are used in this paper. In section 3 we establish some theorems on fuzzy hyponormal operators and we introduce the concept of fuzzy class (N) operator and some properties of fuzzy class operators have been studied.

2. Preliminaries

Definition 2.1. ***[FIP-Space]**[9] A fuzzy inner product space (FIP-Space) is a triplet $(X,F,*)$,where X is a real vector space, $*$ is a continuous t-norm, $F$is a fuzzy set on $X^2\times R$ satisfying the following conditions for every $x,y,z\in X$ and $s,r,t\in R$.*

FI-1: $F(x,x,0)=0$ *and* $F(x,x,t)>0$*, for each* $t>0$

FI-2: $F(x,x,t)\neq H(t)$ *for some* $t\in R$ *if and only if* $x\neq 0$*,where*

$$H(t)=\begin{cases}1 & if\ t>0\\ 0 & if\ t\geq 0\end{cases}$$

FI-3: $F(x,y,t) = F(y,x,t)$

FI-4: *For all* $\alpha \in R$,

$$F(\alpha x,y,t) = \begin{cases} F(x,y,\frac{t}{\alpha}) & : \alpha > 0 \\ H(t) & : \alpha = 0 \\ 1 - F(x,y,\frac{t}{\alpha}) & : \alpha < 0 \end{cases}$$

FI-5: $F(x,x,t) * F(y,y,s) < F(x+y,x+y,t+s)$

FI-6: $Sup_{s+r=t}[F(x,z,s) * F(y,z,r)] = F(x+y,z,t)$

FI-7: $F(x,y,.) : R \to [0,1]$ *is continuous on* $R \setminus \{0\}$

FI-8: $\lim_{t\to\infty} F(x,y,t) = 1$

Definition 2.2. *[15] Let* $(X,F,*)$ *be probabilistic inner product space.*

1. *A sequence* $\{x_n\} \in X$ *is called* τ*- converges to* $x \in X$*, if for any* $\in > 0$ *and* $\lambda > 0, \exists\, N \in Z^+, N = N(\varepsilon,\lambda)$ *such that* $F_{x_n - x, x_n - x}(\varepsilon) > 1 - \lambda$ *, whenever* $n > N$.
2. *A linear functional* $f(x)$ *defined on E is called* τ_F *- continuous, if* $x_n \xrightarrow{\tau_F} x$ *implies* $f(x_n) \xrightarrow{\tau_F} f(x)$ *for any* $\{x_n\}, x \in X$.

Definition 2.3. *[9] Let* $(X,F,*)$ *be a FIP-space, where* $*$ *is strong t- norm, and for each* $x,y \in X$, $\sup\{t \in R : F(x,y,t) < 1\} < \infty$. *Define* $\langle .,. \rangle : X \times X \to R$ *by* $\langle x,y \rangle = \sup\{t \in R : F(x,y,t) < 1\}$*. Then* $(X, \langle .,. \rangle)$ *is an IP space(inner product space), so that* $(X, \|.\|)$ *is N-space (Normed space), where* $\|.\| = \langle x,x \rangle^{1/2}, \forall x \in X$

Definition 2.4. ***[Fuzzy Hilbert Space]****[9] Let*$(X,F,*)$*be a FIP-space with IP:* $\langle x,y \rangle = \sup\{t \in R : F(x,y,t) < 1\}, \forall x,y \in X$*. If X is complete in the* $\|.\|$ *then X is called Fuzzy Hilbert space(FH -space).*

Theorem 2.5. *[9] Let*$(X,F,*)$ *be a FH-space with IP:* $\langle x,y \rangle = \sup\{t \in R : F(x,y,t) < 1\}, \forall x,y \in X$ *for* $x_n \in X$ *and* $x_n \xrightarrow{\|.\|} x$ *then* $x_n \xrightarrow{\tau_F} x$.

Theorem 2.6. ***[Riesz Theorem]*** *[9, 15] Let*$(X,F,*)$ *be a FH-space, for any* τ_F *continuous functional* $\exists$ *unique* $y \in X$ *such that for all* $x \in X$*, then we have*

$$g(x) = \sup\{t \in R : F(x,y,t) < 1\}$$

Theorem 2.7. *[3] Let*$(X,F,*)$ *be a FIP-space, where* $*$ *is strong t- norm , and* $\sup\{x \in R : F(u,v,t) < 1\} < \infty$ *for all* $u,v \in X$*, then* $\sup\{t \in R : F(u+v,w,t) < 1\} = \sup\{t \in R : F(u,w,t) < 1\} + \sup\{t \in R : F(v,w,t) < 1\}$ $\forall u,v,w \in X$.

Remark 2.8. *[2] Let* $FB(X)$ *the set of all fuzzy linear operators on X.*

Theorem 2.9. ***[Adjoint fuzzy operator in FH-space]****[3] Let* $(X,F,*)$ *be a FH-space. let* $S \in FB(X)$ *be* τ_F *- continuous linear functional, then* $\exists$ *unique* $S^* \in FB(X)$ *such that* $\langle Su,v \rangle = \langle u,S^*v \rangle\, \forall u,v \in X$.

Definition 2.10. ***[Self -adjoint fuzzy operator]****[3] Let*$(X,F,*)$ *be a FH-space, with IP:*$\langle u,v \rangle = \sup\{t \in R : F(u,v,t) < 1\}$, $\forall u,v \in X$*. Let* $S \in FB(X)$*, then S is self -adjoint Fuzzy operator. If* $S = S^*$ *where* S^* *is adjoint Fuzzy operator of S.*

Theorem 2.11. *[3] Let*$(X,F,*)$ *be a FH-space, with IP:*$\langle u,v \rangle = \sup\{t \in R : F(u,v,t) < 1\}$*. let* $S \in FB(X)$ *then S self -adjoint Fuzzy operator.*

Theorem 2.12. *[3] Let*$(X,F,*)$ *be a FH-space, with IP:*$\langle u,v \rangle = \sup\{t \in R : F(u,v,t) < 1\}, \forall u,v \in X$*. Let* $S \in FB(X)$*, then,* $\|Su\| = \|S^*u\|$ *for all* $u \in X$.

Remark 2.13. *Let*$(H,F,*)$ *be a FH-space with IP:*$\langle x,y \rangle = \sup\{t \in R : F(x,y,t) < 1\}, \forall x,y \in H$ *and if* $U \in FB(H)$ *it is easy to see that* $U = 0 \Leftrightarrow \langle Ux,y \rangle = 0, \forall x,y \in H$.

Definition 2.14. ***[Fuzzy Normal operator]****[2] Let*$(X,F,*)$ *be a FH-space, with IP:*$\langle u,v \rangle = \sup\{x \in R : F(u,v,x) < 1\}, \forall u,v \in X$*. Let* $S \in FB(X)$*, Then S is a fuzzy normal operator if it commutes with its (fuzzy) adjoint i.e.*$SS^* = S^*S$.

Definition 2.15. ***[Fuzzy unitary operator]****[21] Let* $(H,F,*)$ *be a FH-space with IP:*$\langle u,v \rangle = \sup\{x \in R : F(u,v,x) < 1\}$, $\forall u,v \in H$ *and let* $U \in FB(H)$*, Then U is a fuzzy unitary operator if it satisfies* $UU^* = I = U^*U$.

Definition 2.16. ***[Fuzzy Invariant]****[19] Let* $(X,F,*)$ *be a fuzzy normed linear space, let* $T \in FB(X)$*, A subspace M of a fuzzy normed linear space X is said to be fuzzy invariant under T, if* $TM \subset M$.

Theorem 2.17. *[19] Let T be a normal operator on a finite dimensional fuzzy Hilbert space H over R then,*

1. $T - \lambda I$ *is fuzzy normal.*
2. *Every eigen vector of T is also eigen vector for* T^*

Theorem 2.18. *[19] Let* $(H,F,*)$ *be a FH-space and if* $T \in FB(H)$ *then* $T = 0$ *iff* $\sup\{x \in R : F(Tu,Tu,x) < 1\} = 0\}$, $\forall u \in H$.

Theorem 2.19. *[19] A Closed subspace M of a fuzzy Hilbert space H reduces an operator T iff M is invariant under T and* T^*.

Definition 2.20. *[19] Let M be a closed subspace of a fuzzy Hilbert space H, and let*$T \in FB(H)$*, we say that T is reduced by M if both M and* $M^\perp$ *are invariant under T if T is reduced by M, then we also say that M reduces T.*

Theorem 2.21. *[19] Let M be a closed subspace of a fuzzy Hilbert space X over F ,and let* $T \in FB(H)$*, then M is invariant under T iff* $M^\perp$ *is invariant under* T^*.

Definition 2.22. ***[Fuzzy hyponormal operator]****[1] Let* $(H,F,*)$ *be a FH - space with IP:*$\langle u,v \rangle = \sup\{t \in R : F(u,v,t) < 1\}$, $\forall u,v \in H$*. and let* $T \in FB(H)$*. Then T is a fuzzy hyponormal operator if* $\|T^*u\| \le \|Tu\|, \forall u \in H$ *or equivalently* $T^*T - TT^* \ge 0$.

Theorem 2.23. *[1] Let $(H,F,*)$ be a FH - space with IP:$\langle u,v\rangle = \sup\{t\in R: F(u,v,t)<1\}$, $\forall u,v\in H$ and let $T\in FB(H)$ be a fuzzy hyponormal operator on H. Then $\|(T-zI)u\| \geq \|(T^*-\bar{z}I)u\|$,$\forall u\in H$, i.e. $T-zI$ is FHN.*

Theorem 2.24. *[1] Let T be a FHN - operator on the Hilbert space H. Then $Tu=\lambda u \Rightarrow T^*u=\overline{\lambda}u$.*

Theorem 2.25. *[1] Let T be fuzzy hyponormal iff $\|T^*u\|\leq \|Tu\|$,for all $u\in H$.*

Theorem 2.26. *[1] If T is a fuzzy hyponormal, completely continuous operator then T is fuzzy normal.*

3. Main Results

In this section, we discussed on some elementary properties of fuzzy hyponormal operator in FH-space and we introduce the definition of Fuzzy class(N) operator in FH-space as well as some theorems are also presented.

Theorem 3.1. *Let $(H,F,*)$ be a FH - space with IP:$\langle u,v\rangle = \sup\{t\in R: F(u,v,t)<1\}$,$\forall u,v\in H$ and let $T\in FB(H)$ be a fuzzy hyponormal operator on H. If $T^{*p}T^q$ is completely continuous where p and q are positive integers then T is fuzzy normal.*

Proof. To prove the theorem we need the following lemma. □

Lemma 3.2. *Let T be a fuzzy hyponormal operator and let $\|T\|=1$.Then in the FH-space there exists a sequence $u_n, \|u_n\|=1$ such that*

1. $\|T^*u_n\|\to 1$
2. $\|T^m u_n\|\to 1 \qquad m=1,2,\ldots$
3. $\|T^*Tu_n-u_n\|\to 0$
4. $\|TT^*u_n-u_n\|\to 0$
5. $\left\|T^*T^m u_n - T^{m-1}u_n\right\|\to 0 \qquad m=1,2,\ldots$

Proof. Proof for (1):
By definition there exists a sequence $u_n, \|u_n\|=1$ such that $\|T^*u_n\|\to\|T^*\|=\|T\|=1$
Proof for (2):
By known Theorem, we have that for u, $\|u\|=1, \|Tu\|^2\leq\left\|T^2u\right\|$.
Since,$\|T^*u_n\|^2\leq\|Tu_n\|^2\leq\left\|T^2u_n\right\|\leq 1$.
We have $lim\left\|T^2u_n\right\|=1$. By induction hypothesis, we shall prove that

$$if\left\|T^{k-1}u_n\right\|\to 1, \left\|T^k u_n\right\|\to 1, then\left\|T^{k+1}u_n\right\|\to 1.$$

Now, Since,

$$\left\|T^2u_n\right\|\leq 1$$

$$\Rightarrow\left\|T^2(T^{k-1}u_n)/(T^{k-1}u_n)\right\|\leq\left\|(T^{k-1}u_n)/(T^{k-1}u_n)\right\|$$

$$\Rightarrow\left\|T^{k+1}u_n\right\|\leq\left\|T^{k-1}u_n\leq\right\|1$$

$$\Rightarrow\left\|T^{k+1}u_n\right\|\leq 1$$

By induction we have the relation (2).

$$\|T^m u_n\|\to 1 \qquad m=1,2,\ldots$$

Proof for (3):
From (2),$\|T^m u_n\|\to 1$
Let $m=1$, then $\|Tu_n\|\to 1$
$\Rightarrow \|T^*Tu_n\|\to\|T^*\|$
$\Rightarrow \|T^*Tu_n\|\to\|T\|$
$\Rightarrow \|T^*Tu_n\|-\|u_n\|\to 1-\|u_n\|$
$\Rightarrow \|T^*Tu_n-u_n\|\to 0$
Proof for (4):
From (1),$\|T^*u_n\|\to 1$
$\Rightarrow \|TT^*u_n\|\to\|T\|$
$\Rightarrow \|TT^*u_n\|-\|u_n\|\to 1-\|u_n\|$
$\Rightarrow \|TT^*u_n-u_n\|\to 0$
Proof for (5):
Let $v_n(m)=T^*T^m u_n - T^{m-1}u_n$ and $\delta_n(m)=\|v_n(m)\|^2$.
We have,

$$\begin{aligned}&\delta_n(m)\\ &=\|T^*T^m u_n\|^2-2\|T^*T^m u_n\|\left\|T^{m-1}u_n\right\|+\left\|T^{m-1}u_n\right\|^2\\ &\leq\|T^m u_n\|^2-2\|T^m u_n\|^2+\left\|T^{m-1}u_n\right\|^2\\ &=\left\|T^{m-1}u_n\right\|^2-\|T^m u_n\|^2.\end{aligned}$$

From (2),we obtain that $\delta_n(m)\to 0$ for every m.
This proves the lemma. □

Proof. Proof for Theorem :
Let p and q the positive integers such that $T^{*p}T^q$ is a completely continuous operator. By the lemma$T^*T^q u_n - T^{q-1}u_n\to 0$ where $\{u_n\}$ is the sequence of lemma.

It is clear that $\left\{T^{*p-1}T^{q-1}u_n\right\}$ admits a subsequence which is convergent. Also by the lemma and this result we obtain a sequence of$\left\{T^{*p-2}T^{q-2}u_n\right\}$ which is convergent.

This process can be repeated and we obtain a subsequence $\left\{u_{n_k}\right\} of\left\{u_n\right\}$ which is convergent.
Let $u_0=\lim u_{n_k}$.
Thus $T^*Tu_0=u_0$
$TT^*u_0=0$.
The closed subspace $M_T=\{u, TT_u^*=u\}$ is a non zero subspace. By the known lemma we have that T has an approximate proper value $Tv_n-\lambda v_n\to 0$.
The above arguments show that every sequence of approximate eigen vectors $\{v_n\}$ of T belonging to $\overline{\lambda}$ with $\overline{\lambda}=1$ contains a convergent subsequence so that $\overline{\lambda}$ is an eigen value of T^*, hence λ is of T.

Let M be the smallest closed linear subspace which contains every proper subspace of T and $N=M^{\perp}$. It is known that N is invariant for T^* and thus $T^{*p}T^q$ is a completely continuous on N. It is known that T_N is fuzzy hyponormal. This shows that $N=\{0\}$ and $M=H$. The theorem is proved. □

Definition 3.3. *Let $(H,F,*)$be a FH - space with IP:$\langle u,v\rangle = Sup\{t\in R : F(u,v,t)<1\}\forall u,v\in H$, then the operator $T\in FB(H)$ is said to be of Fuzzy class (N) if u is in the fuzzy Banach space B,$\|u\|=1$ and $\|Tu\|^2\leq \left\|T^2u\right\|$.*

Lemma 3.4. *Every fuzzy hyponormal operator is of Fuzzy class (N).*

Proof. For $u\in H$, $\|u\|=1$,
$\|Tu\|^2=\langle Tu,Tu\rangle$
$= sup\{t\in R : F(Tu,Tu,t)<1\}, u\in H$
$= sup\{t\in R : F(T^*Tu,u,t)<1\}, u\in H$
$=\langle T^*Tu,u\rangle$
$\leq \|T^*Tu\|$
therefore $\|Tu\|^2\leq \left\|T^2u\right\|$ □

Note 3.5. *It is clear by this lemma that these operators are the extension of a fuzzy class(N) of fuzzy hyponormal*

Lemma 3.6. *If the fuzzy hyponormal operator T is of Fuzzy class (N) and 1)$\|T\|=1$, 2)$\|u_n\|\to 1$, 3)$\|Tu_n\|\to 1$, then $\|T^m u_n\|\to 1 \quad (m=1,2,3,\dots)$*

Proof. The Proof is already given in Part (2) of lemma in the above theorem. □

Theorem 3.7. *If the fuzzy hyponormal operator T is of Fuzzy class (N) on a FH-space then $\|T\|=(\lim\|T^n\|^{1/n})=\delta_r$.*

Proof. For every n, the above lemma leads the relation
$Now\,\|T^n\|\,u^2=\langle T^nu,T^nu\rangle$
$=\left\langle T^nu,TT^{n-1}u\right\rangle$
$=\left\langle T^*T^nu,T^{n-1}u\right\rangle$
$\leq \|T^*T^nu\|\left\|T^{n-1}u\right\|$
$\leq \left\|T^*TT^{n-1}u\right\|\left\|T^{n-1}u\right\|$
$\leq \left\|T^{n-1}u\right\|\left\|T^{n-1}u\right\|$
Then $\|T^n\|^2\leq\left\|T^{n-1}\right\|\left\|T^{n-1}\right\|$
And combining this with the equality, a simple induction argument yield,
$\|T^n\|=\|T^n\|$ for $\quad n=1,2,3,\dots$
Therefore, we get $\|T\|=(\lim\|T^n\|^{1/n})=\delta_r$. □

Lemma 3.8. *If the fuzzy hyponormal operator T is of fuzzy class (N) on a FH-space and $\|T\|=1$, then $M_{T^*}=\{x,TT^*=u\}$is invariant under T.*

Proof. Let $u\in M_{T^*}$, $\|u\|=1$.
Then $\|T^*Tu-u\|^2=\|T^*Tu\|^2-2\|T^*Tu\|\,\|u\|+\|u\|^2$
$=\|T^*Tu\|^2-2\|Tu\|^2+1$
$=\|T^*Tu\|^2-2\|TT^*Tu\|^2+1$
$\leq\|T^*Tu\|^2-2\|TTT^*u\|^2+1$
$=\|T^*Tu\|^2-2\left\|T^2\frac{T^*u}{\|T^*u\|}\right\|^2\|T^*u\|^2+1$
$\leq\|T^*Tu\|^2-2\|TT^*u\|^4\frac{1}{\|T^*u\|^2}+1$
$\leq\|T^*Tu\|^2-\frac{2}{\|T^*u\|^2}+1$
≤ 0 thus$\|T^*Tu-u\|^2\leq 0\Rightarrow\|T^*Tu-u\|=0$

It is clear that $Tu=TT^*(Tu)=T(T^*Tu)$ which shows that $Tu\in M_{T^*}$. □

Theorem 3.9. *If the fuzzy hyponormal operator T is of Fuzzy class (N)on a FH-space and $T^*$is a completely continuous for some $k\geq 1$ then T is fuzzy normal.*

Proof. For $\|T\|=1$, from completely continuous property of T^k, it is clear that the subspace $M_{T^*}=\{x,TT^*=u\}$ is not 0. Also $M_{T^*}$is finite dimensional , because it is invariant under T^k which is isometric and completely continuous and M_{T^*} reduces T. We consider the subspace$M_{T^*}^{\perp}$ and continuing in this way and obtain that T is fuzzy normal. □

4. Conclusion

Using the fuzzy hyponormal operator in FH-space is classic form of theorems play the role a prototype in our discussion of this paper. Some concepts and properties have been investigated about fuzzy hyponormal operator in fuzzy Hilbert space. As the definition of Fuzzy class (N) operator on a Fuzzy Banach space (FB-space) is also given and the results of this paper will be helpful for researchers to develop fuzzy functional analysis.

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