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Geodesic convexity in labeled graphs

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Abstract

This paper is an attempt to study geodesic convexity in a graph G with respect to a labeling function $\mathscr{L}$ defined on the vertex set of G. Let $G(V,E)$ be an undirected, connected graph without loops and multiple edges. A bijective function $\mathscr{L}: V(G) \to \{1,2,3,..,|V(G)|\}$ be a vertex labeling of G and it induces a function $\mathscr{L}^*: E(G) \to \{1,2,3,..,|V(G)|\}$ defined by $\mathscr{L}^*(uv) = |\mathscr{L}(u) - \mathscr{L}(v)|$. Let $\Gamma_{\mathscr{L}} = (G,\mathscr{L})$ be a labeled graph. An $\mathscr{L}_g$ convexity space is an ordered pair $(\Gamma_{\mathscr{L}}, \mathscr{C}_{\mathscr{L}})$ where , $\Gamma_{\mathscr{L}}$ is a labeled graph and $\mathscr{C}_{\mathscr{L}}$ is the convexity induced by the label $\mathscr{L}$. The function $\mathscr{L}$ is called a geodesic convex label or simply g- convex label if the convexity $\mathscr{C}_{\mathscr{L}}$ induced by the label $\mathscr{L}$ coincides with the geodesic convexity $\mathscr{C}$ on V. A graph G is defined to be a *geodesically elegant graph* if there exist a g-convex label for G.

Keywords

Graph labeling, Geodesic Convexity, g-convex sets, Weighted graphs

AMS Subject Classification

05C35, 05C78.

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Contents

1. Introduction

Graph Convexities and graph labeling are the two major research areas of Graph theory. Different kinds of convexities and different types of graph labeling can be seen in literature.We begin with simple, finite, connected and undirected graph $G = G(V,E)$. A vertex labeling of a graph G is an assignment $\mathscr{L}$ of labels to the vertices of G that induces for each edge $e = uv$ a label depending on the vertex labels given by $\mathscr{L}(uv) = \mathscr{L}(u) * \mathscr{L}(v)$, where $*$ is a binary operation. In literature one can find that $*$ is any one of the operations addition, subtraction, multiplication, symmetric difference,absolute difference or modulo addition. Clearly in the absence of additional constraints every graph can be labeled in infinetly many ways. Labeled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications[16]. For a dynamic survey of various graph labeling problems along with an extensive bibliography we refer to Gallian [7].

Convexity problems in graphs is an emerging line of research in metric graph theory. An elegant survey on convex structures has been done by Vande Vel.

A convexity $\mathscr{C}$ [13] on a nonempty set V is a collection of subsets of V, convex sets, which contains both V and empty set and satisfying the following conditions

- arbitrary intersection of convex sets are convex
- every nested union of convex sets is convex.

A convexity space is an ordered pair $(V,\mathscr{C})$, where V is a nonempty set and $\mathscr{C}$ is a convexity on V. A graph convexity space is a an ordered pair $(G,\mathscr{C})$ formed by a connected graph $G = (V,E)$ and a convexity $\mathscr{C}$ on V such that$(V,\mathscr{C})$ is a convexity space and satisfying the condition that every member of $\mathscr{C}$ induces a connected subgraph of G [13]. Thus classical convexity can be extended to graphs in a natural way. In the study of convexity in graphs, two types of convexity have played a prominent role - geodesic convexity (also called metric convexity) which arises when we consider shortest paths and the monophonic convexity (also called the minimal path convexity) when we consider chordless paths [1]. Our interest is to focus on geodesic convexity.

For unweighted and weighted graphs different types of convexities and other related parameters are introduced and studied by many authors including Chepoi, Dutchet, Bandelt, Jamison, Changat, Vijayakumar and Parvathy and details are available in the literature. Studies connecting these two major topics in graph theory is not yet found in literature. This motivated us to define the geodesic convexity in labeled graphs. We follow the basic definitions taken from [1, 4–6, 9, 13, 15], which is listed in the preliminaries section.

2. Preliminaries

A shortest $u-v$ path is called a $u-v$ geodesic. The distance $d_G(u,v)$ between vertices u and v is defined as the length of a $u-v$ geodesic. The distance function $d_G : G\times G \to N$ associated to a connected graph G satisfies, for every $u,v,w \in V(G)$, the following properties:

- $d_G(u,v) \geq 0$, equality holding iff $u = v$.
- $d_G(u,v) = d_G(v,u)$.
- $d_G u,v) \leq d_G(u,w) + d_G(w,v)$.

The geodesic closed interval $I[u,v]$ is the set of all vertices in all $u-v$ geodesic including u and v. For $W \subseteq V$, the union of all geodesic closed interval $I[u,v]$ over all pairs $u,v \in W$ is called the geodetic closure of W and is denoted by $I[W]$. Any subset W of V is called geodesic convex if $I[W] = W$.

A crown garph is the corona product of $C_n \odot K_1$. The generalized friendship graph $f_{q,p}$ is a collection of p-cycles (all of order q) meeting at a common vertex. A Theta graph is a block with two non- adjacent vertices of degree 3 and all other vertices of degree 2. A vertex switching of a graph G is a graph G_v obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

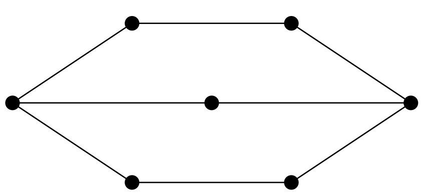


Figure 1. Theta graph

A weighted graph $G = (V,E,w)$ is one in which every edge is assigned a non negative number $w(e)$ called the weight of e. An ordinary graph is a weighted graph with unit weight assigned for all edges. Let $G(V,E,w)$ be a connected weighted graph and u,v be any two vertices of G. Then the geodesic distance between u and v is defined and denoted by $d(u,v) = min_P \sum_{e\in P} w(e)$ where P is a $u-v$ path in G and $w(e)$ is the weight associated with the edge e.

3. Geodesic Convexity in Labeled Graphs

In this section we introduce the concept of geodesic convex label and $\mathscr{L}_g$ convexity space. We define the distance between two vertices in a labeled graph in the same way as in weighted graphs. Let $G(V,E)$ be an undirected, connected graph without loops and multiple edges. A bijective function $\mathscr{L} : V(G) \to \{1,2,3,..,|V(G)|\}$ be a vertex labeling of G and it induces a function $\mathscr{L}^* : E(G) \to \{1,2,3,..,|V(G)|\}$ defined by $\mathscr{L}^*(uv) = |\mathscr{L}(u) - \mathscr{L}(v)|$. We use $\Gamma_{\mathscr{L}}$ to denote a labeled graph; $\Gamma_{\mathscr{L}} = (G,\mathscr{L})$. A labeled graph can be treated as a weighted graph, we define the distance between any two vertices in $\Gamma_{\mathscr{L}}$, by replacing $w(e)$ by $\mathscr{L}^*(e)$, $\mathscr{L}^*(e)$ is the label associated with the edge e.

Definition 3.1. *For any $u-v$ path P in $\Gamma_{\mathscr{L}}$, the path sum denoted by $\mathscr{L}(P)$ is defined as the sum of the edge labels present in the path. That is $\mathscr{L}(P) = \sum_{e\in P} \mathscr{L}^*(e)$.*

Definition 3.2. *For any two vertices u and v in $V(\Gamma_{\mathscr{L}})$, the distance between u and v denoted by $d_{\mathscr{L}}(u,v)$ is defined as $d_{\mathscr{L}}(u,v) = min_P\{\mathscr{L}(P)$ where P is a $u-v$ path in $\Gamma_{\mathscr{L}}\}$.*

Clearly the distance function $d_{\mathscr{L}}(u,v) : \Gamma_{\mathscr{L}} \times \Gamma_{\mathscr{L}} \to N$ associated to a labeled graph satisfies all the conditions of a metric. Hence for every labeled graph $\Gamma_{\mathscr{L}}$, $d_{\mathscr{L}}$ is a metric on $V(\Gamma_{\mathscr{L}})$ and the label induces a convexity $C_{\mathscr{L}}$ on $V(\Gamma_{\mathscr{L}})$ such that the vertices in any shortest path between each pair of vertices in a set $S \subset V(\Gamma_{\mathscr{L}})$ is contained in it.

Definition 3.3. *Let $\Gamma_{\mathscr{L}}$ be a labeled graph. A shortest $u-v$ path in $\Gamma_{\mathscr{L}}$ is called $(u,v)\mathscr{L}$-geodesic. For any two vertices u and v of $\Gamma_{\mathscr{L}}$ the $\mathscr{L}-$ geodesic closed interval $I_{\mathscr{L}}[u,v]$ is defined as*
$I_{\mathscr{L}}[u,v] = \{w \in V(\Gamma_{\mathscr{L}}) : d_{\mathscr{L}}(u,v) = d_{\mathscr{L}}(u,w) + d_{\mathscr{L}}(w,v)\}$.

Let $\Gamma_{\mathscr{L}}$ be a labeled graph. Let $S \subset V(\Gamma_{\mathscr{L}})$. The union of all$\mathscr{L}-$ geodesic closed interval $I_{\mathscr{L}}[u,v]$ over all pairs $u,v \in S$ is called a $\mathscr{L}-$ geodesic closure of S and is denoted by $I_{\mathscr{L}}[S]$. If $I_{\mathscr{L}}[S] = S$, we say S is $\mathscr{L}_g$ convex. Equivalently, a set S is $\mathscr{L}_g$ convex for every pair of $u,v \in S$ the interval $I_{\mathscr{L}}[u,v] \subseteq S$. For example consider C_4 with different vertex labeling.

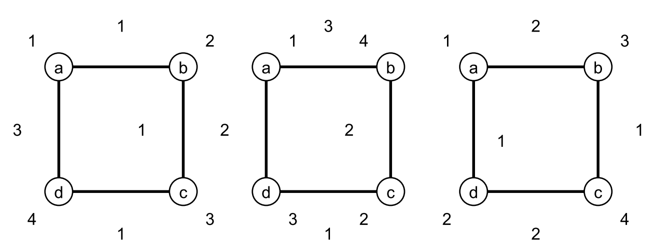


Figure 2. 3 different labelings of $C_4 : \Gamma_{\mathscr{L}_1}, \Gamma_{\mathscr{L}_2}, \Gamma_{\mathscr{L}_3}$

Geodesic convexity of the cycle C_4 is given by
$\mathscr{C} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,d\}, \{a,b,c,d\}\}$. Geodesic convex sets of $\Gamma_{\mathscr{L}_1}$, $\Gamma_{\mathscr{L}_2}$ and

Table 1. Geodesic convex sets with respect to the vertex labeling.

$\mathscr{C}_{\mathscr{L}_1}$	$\mathscr{C}_{\mathscr{L}_2}$	$\mathscr{C}_{\mathscr{L}_3}$
$\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{b,c,d\}$ and $\{a,b,c,d\}$.	$\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,d\}, \{a,d,c\}, \{b,c,d\}$ and $\{a,b,c,d\}$.	$\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,d\}$ and $\{a,b,c,d\}$

$\Gamma_{\mathscr{L}_3}$ are given in the following table. Comparing $\mathscr{C}_{\mathscr{L}_1}$, $\mathscr{C}_{\mathscr{L}_2}$ and $\mathscr{C}_{\mathscr{L}_3}$ with $\mathscr{C}$ we conclude the following:
In any graph, the empty set, the whole vertex set, every one point sets and every two point sets consisting of adjacent vertices are members of $\mathscr{C}$. Clearly, the empty set, the whole vertex set and every one point sets are in $\mathscr{C}_{\mathscr{L}}$; but every two point subsets consisting of adjacent vertices need not be in $\mathscr{C}_{\mathscr{L}}$.
Number of elements in $\mathscr{C}_{\mathscr{L}}$ may exceed the number of elements in $\mathscr{C}$. In some cases these are equal.

Definition 3.4. *An $\mathscr{L}_g$ convexity space is an ordered pair $(\Gamma_{\mathscr{L}}, \mathscr{C}_{\mathscr{L}})$ where , $\Gamma_{\mathscr{L}}$ is a labeled graph and $\mathscr{C}_{\mathscr{L}}$ is the convexity induced by the label $\mathscr{L}$.*

It is interesting to find a label in which the convex sets induced by it coincides with the geodesic convex sets. Based on this concept we defined g-convex label.

4. Geodesically Elegant graphs

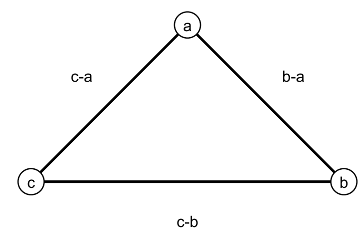
Definition 4.1. *Let $\Gamma_{\mathscr{L}}$ be a labeled graph. The label $\mathscr{L}$ is called a geodesic convex label or simply g-convex label if the convexity $\mathscr{C}_{\mathscr{L}}$ induced by the label $\mathscr{L}$ coincides with the geodesic convexity $\mathscr{C}$ on V. In other words, the $\mathscr{L}_g$ convex sets of $\mathscr{C}_{\mathscr{L}}$ are the same as the g- convex sets of $\mathscr{C}$ in G. A graph G is* ***geodesically elegant graph*** *if there exist a $g-$convex label for G.*

Remark 4.2. *: In a tree each pair of vertices are connected by a unique path. Therefore, $\mathscr{C}_{\mathscr{L}} = \mathscr{C}$ for any vertex labeling. Hence a tree T is always geodesically elegant.*

In the following proposition we characterize the necessary condition for the existence of geodesic convex label in a graph G.

Proposition 4.3. *All geodesically elegant graphs are triangle free.*

Proof. On the contrary suppose that G contains a triangle C_3 or K_3. Let us label the vertices of C_3 using the numbers a, b, and c with $a < b < c$.



Then by the triangle inequality, the two point subset $\{a,c\}$ is not convex. Hence $\mathscr{C}_{\mathscr{L}} \neq \mathscr{C}$, we conclude that if the graph G is geodesically elegant, then G is triangle free. ☐

Now we check, which of the following class of graphs are geodesically elegant.

Theorem 4.4. *The cycle C_n for all $n > 3$ is geodesically elegant.*

Proof. To prove the existence of a g-convex label, find a vertex label $\mathscr{L}$ such that the convexity induced by the label coincides with the geodesic convexity in $C_n, n > 3$.
Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n.
Define $\mathscr{L} : V(C_n) \to \{1,2,3,...,n\}$ by
Case 1 : n is even

$$\begin{gathered}\mathscr{L}(v_i) = 2i-1, 1 \leq i \leq \tfrac{n}{2} \\ \mathscr{L}(v_i) = n, \quad i = \tfrac{n}{2}+1 \\ \mathscr{L}(v_i) = \mathscr{L}(v_{i-1}) - 2, i = \tfrac{n}{2}+2 \leq i \leq n.\end{gathered}$$

Case 2 : n is odd

$$\begin{gathered}\mathscr{L}(v_i) = 2i-1, 1 \leq i \leq \tfrac{n+1}{2} \\ \mathscr{L}(v_i) = 2, \quad i = [\tfrac{n}{2}]+2 \\ \mathscr{L}(v_i) = \mathscr{L}(v_{i-1}) + 2, [\tfrac{n}{2}]+3 \leq i \leq n.\end{gathered}$$

The label $\mathscr{L}$ satisfies the conditions of a g-convex label, C_n, for all $n > 3$ is geodesically elegant. ☐

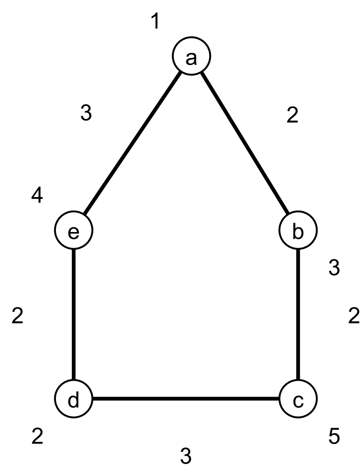


Figure 3. Geodesic convex labeling of C_5

Theorem 4.5. *The crown graph $C_n \odot K_1$ for all $n > 3$ is geodesically elegant.*

Proof. Let $\{v_i, i = 1 \text{ to } n\}$ be the vertices of the cycle C_n and $\{u_i, i = 1 \text{ to } n\}$ be the pendant vertices.
To prove the existence of a g-convex label, it is enough to find a vertex label $\mathscr{L}$ such that the convexity induced by the label coincides with the geodesic convexity in $C_n \odot K_1$.
Define $\mathscr{L} : V(C_n \odot K_1) \to \{1, 2, 3, ..., 2n\}$ by

Case 1 : n is even

$$\begin{gathered}\mathscr{L}(v_i) = 2i-1, 1 \le i \le \tfrac{n}{2}\\ \mathscr{L}(v_{n/2+1}) = n, i = \tfrac{n}{2}+1\\ \mathscr{L}(v_i) = \mathscr{L}(v_{i-1}) - 2, \tfrac{n}{2}+2 \le i \le n.\\ \mathscr{L}(u_1) = n+1\\ \mathscr{L}(u_i) = \mathscr{L}(u_{i-1}) + 2, 2 \le i \le \tfrac{n}{2}\\ \mathscr{L}(u_{\frac{n}{2}}+1) = 2n\\ \mathscr{L}(u_i) = \mathscr{L}(u_{i-1}) - 2, \quad \tfrac{n}{2}+2 \le i \le n.\end{gathered}$$

Case 2 : n is odd

$$\begin{gathered}\mathscr{L}(v_i) = 2i-1, 1 \le i \le \tfrac{n+1}{2}\\ \mathscr{L}(v_i) = 2, i = [\tfrac{n}{2}]+2\\ \mathscr{L}(v_i) = \mathscr{L}(v_{i-1}) + 2, \quad [\tfrac{n}{2}]+3 \le i \le n\\ \mathscr{L}(u_1) = n+1\\ \mathscr{L}(u_i) = \mathscr{L}(u_{i-1}) + 2, \quad 2 \le i \le \tfrac{n+1}{2}\\ \mathscr{L}(u_{\frac{n+1}{2}}+1) = n+2\\ \mathscr{L}(u_i) = \mathscr{L}(u_{i-1}) + 2, \tfrac{n+1}{2}+2 \le i \le n.\end{gathered}$$

The label $\mathscr{L}$ satisfies the conditions of a geodesic convex label and hence $C_n \odot K_1$ for all $n > 3$ is geodesically elegant. $\square$

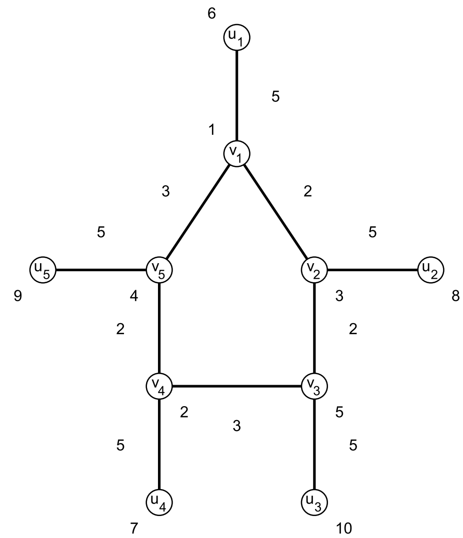


Figure 4. Geodesic convex labelings of $C_5 \odot K_1$

Theorem 4.6. *The generalized friendship graph $f_{4,n}$ is geodesically elegant.*

Proof. The generalized friendship graph $f_{4,n}$ is a collection of n cycles of order 4 meeting at a common vertex v_0.
Let $V(f_{4,n}) = \{v_0, v_1, v_2, ...v_{3n}\}$. Then $|V(f_{4,n})| = 3n+1$ in $f_{4,n}$.
Ordinary labeling of $f_{4,n}$ is shown in figure 5
Define $\mathscr{L} : V(f_{4,n}) \to \{1, 2, 3, ..., 3n+1\}$ by

$$\begin{gathered}\mathscr{L}(v_0) = 3n+1\\ \mathscr{L}(v_{3i-2}) = \mathscr{L}(v_0) - i, 1 \le i \le n\\ \mathscr{L}(v_{3n}) = \mathscr{L}(v_{3n-2}) - 1.\\ \mathscr{L}(v_{3i}) = \mathscr{L}(v_{3n}) - i, 1 \le i \le n-1\\ \mathscr{L}(v_{3n-4}) = \mathscr{L}(v_{3(n-1)}) - 1\\ \mathscr{L}(v_{3n-1}) = \mathscr{L}(v_{3n-4}) - 1\\ \mathscr{L}(v_{3i-1)} = \mathscr{L}(v_{3n-1}) - i, 1 \le i \le n-2.\end{gathered}$$

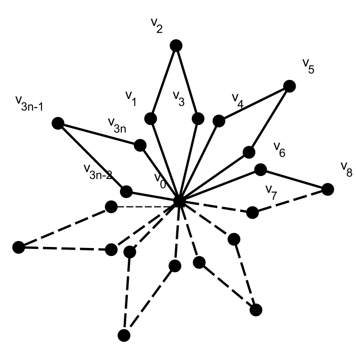


Figure 5. Ordinary labelings of $f_{4,n}$

The label $\mathscr{L}$ satisfies the conditions of a g-convex label and hence $f_{4,n}$ is geodesically elegant.

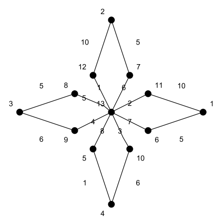


Figure 6. Geodesic convex labelings of $f_{4,4}$

$\square$

Theorem 4.7. *The Theta graph T_α is geodesically elegant.*

Proof. If $v_0, v_1, v_2, v_3, v_4, v_5, v_6$ are the vertices of the Theta graph T_α with centre v_0. We define the vertex labeling $\mathscr{L} : V(T_\alpha) \to \{1,2,3,4,5,6,7\}$ as in figure 7.
$\mathscr{C}_{\mathscr{L}}(T_\alpha) = \mathscr{C}(T_\alpha) = \{\emptyset, \{v_0\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\},$
$\{v_6\}\{v_1,v_2\}, \{v_2,v_3\}, \{v_3,v_4\}, \{v_4,v_5\}, \{v_5,v_6\}, \{v_6,v_1\},$
$\{v_1,v_0\}, \{v_0,v_4\}, \{v_1,v_2,v_3\}, \{v_2,v_3,v_4\}, \{v_4,v_3,v_5\},$
$\{v_4,v_5,v_6\}, \{v_1,v_5,v_6\}, \{v_2,v_1,v_0\}, \{v_1,v_0,v_4\}, \{v_4,v_3,v_0\},$
$\{v_4,v_5,v_0\},$
$\{v_6,v_1,v_0\}, \{v_0,v_1,v_2,v_3,v_4\}, \{v_0,v_1,v_4,v_5,v_6\}$
$\{v_0,v_1,v_2,v_3,v_4,v_5,v_6\}\}$.

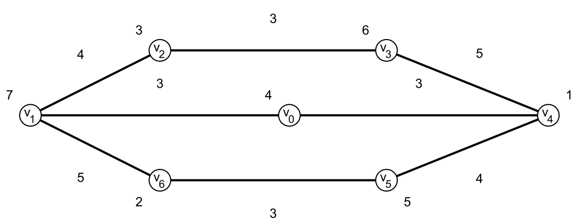


Figure 7. Geodesic convex labelings of T_α

$\square$

Theorem 4.8. *The graph obtained by switching of any vertex in a Theta graph T_α is not geodesically elegant.*

Proof. The switching of any vertex in the Theta graph T_α produces a triangle, by proposition 4.3 geodesic convex label does not exist in that graph.

Figure 8. The switching of a vertex in T_α

$\square$

We have proved that geodesically elegant graphs are triangle free, so we can't find the geodesic convex label in K_n for $n \geq 3$. In that case we can find the number of $\mathscr{L}_g$ convex sets.

Theorem 4.9. *The number of $\mathscr{L}_g$ convex sets of K_n for $n \geq 3$ with respect to any label $\mathscr{L}$ is $\frac{n^2+n+2}{2}$.*

Proof. Let $G \cong K_n$
. $V = V(G) = \{v_1, v_2, v_3, ..., v_n\}$.
Let $\mathscr{L} : V \to \{1,2,3,..,n\}$ be the labeling. Assume without loss of generality that
$\mathscr{L}(v_i) = i$, for $i = 1$ to n.
Claim : For every $i = 1,2,...,n,\ j = 0,1,2,..,n-i$
Then the set $C = \{v_i, v_{i+1}, ..., v_{i+j}\}$ is $\mathscr{L}_g$ convex.
For any $i \leq k \leq l \leq i+j$
$I_{\mathscr{L}}[v_k, v_l] = \{v_s : d_{\mathscr{L}}(v_k, v_l) = d(v_k, v_s) + d(v_s, v_l)\} = \{v_s : k \leq s \leq l\} \subset C$.
Hence C is convex.
on the other hand let W be an $\mathscr{L}_g$ convex subset of V.
Let $i = minimum\{k : v_k \in W\}$
and $j = Maximum\{k : v_k \in W\}$. Then for any s such that $i < s < j$ we have
$d_{\mathscr{L}}(v_i, v_s) = s - i$, $d_{\mathscr{L}}(v_s, v_j) = j - s$, and

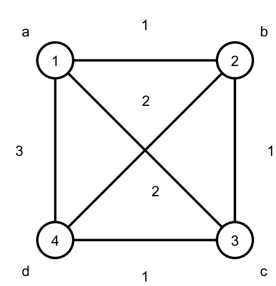
$$\begin{aligned} d_{\mathscr{L}}(v_i, v_j) &= j - i = (j-s) + (s-i) \\ &= d_{\mathscr{L}}(v_s, v_j) + d_{\mathscr{L}}(v_i, v_s). \end{aligned}$$

Therefore $v_s \in W$ forevery S such that $i \leq s \leq j$.
Hence the convex sets are precisely $\emptyset$ and those sets of the form $\{v_i, v_{i+1}, ..., v_{i+j}\}$.
Let m_i denote the number of convex sets with i vertices for $i = 0,1,2,..,n$.
Then $m_0 = 1$, $m_1 = n$
, $m_2 = n-1$,....$m_n = 1$. Hence

$$\begin{aligned} |\mathscr{C}_{\mathscr{L}}| &= 1 + n + n - 1 + ... + 1. \\ &= 1 + \tfrac{n(n+1)}{2} = \tfrac{n^2+n+2}{2} \end{aligned}$$

$\square$

Illustration:
Consider K_4



$\mathscr{L}_g$ convex sets of K_4 are $\emptyset$, one point sets, $\{a,b\}, \{b,c\}, \{c,d\}, \{b,c,d\}, \{a,b,c\}, \{a,b,c,d\}$

That is 11 $\mathscr{L}_g$convex sets.
Using $\frac{n^2+n+2}{2}$, it is 11.

5. Conclusion

In this article, the authors made an attempt to study the concept of geodesic convexity in labeled graphs and investigated which of the folllowing graphs are geodesically elegant. To investigate similar results with other labeling function and other convexities in the literature is a future area of research.

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