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Smarandache fuzzy semiring minimal-c-regular spaces

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Abstract

In this disquisition, the perceptions of $\mathscr{S}$-fuzzy-minimal-open, $\mathscr{S}$-fuzzy-minimal-closed, $\mathscr{S}$-fuzzy-maximal-open, $\mathscr{S}$-fuzzy-maximal-closed semirings are instigated and few of their attributes are contemplated. In addition, the ideas of $\mathscr{S}$-fuzzy-semiring-minimal-regular and $\mathscr{S}$-fuzzy-semiring-minimal-c-regular spaces are introduced and examined.

Keywords

$\mathscr{S}$-fuzzy-minimal-open semirings, $\mathscr{S}$-fuzzy-minimal-closed semirings, $\mathscr{S}$-fuzzy-maximal-open semirings, $\mathscr{S}$-fuzzy-maximal-closed semirings, $\mathscr{S}$-fuzzy-semiring-minimal-c-regular spaces.

AMS Subject Classification

54A40, 03E72.

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Contents

1. Introduction

A substantial number of articles on minimal and maximal open and closed sets in classical sense is found in literature due to F. Nakoaka and N. Oda in [3], [4] and [5]. Later such sets are extended to fuzzy topological spaces by B.M. Ittanagi and R.S. Wali in [1]. In [6], the perception of Smarandache fuzzy semirings was pioneered and explored. In this paper, the concepts of $\mathscr{S}$-fuzzy-semiring-minimal-regular and $\mathscr{S}$-fuzzy-semiring-minimal-c-regular spaces which are the applications of S-fuzzy minimal open semirings are initiated and their properties are analysed.

2. Preliminaries

Definition 2.1. [2] Let S be a $\mathscr{S}$-semiring. A family $\mathscr{S}$ of $\mathscr{S}$-fuzzy semirings on S is termed Smarandache fuzzy semiring structure (briefly $\mathscr{S}\mathscr{F}\mathscr{S}$-structure) on S if it satisfies the following conditions:

(i) $0_S, 1_S \in \mathscr{S}$,

(ii) If $\lambda_1, \lambda_2 \in \mathscr{S}$, then $\lambda_1 \wedge \lambda_2 \in \mathscr{S}$,

(iii) If $\lambda_i \in \mathscr{S}$ for each $i \in J$, then $\vee\lambda_i \in \mathscr{S}$.

And the ordered pair $(S, \mathscr{S})$ is termed $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Every member of $\mathscr{S}$ is termed $\mathscr{S}$-fuzzy-open-semiring and the complement of a $\mathscr{S}$-fuzzy-open-semiring is called an anti-$\mathscr{S}$-fuzzy-open-semiring (or a $\mathscr{S}$-fuzzy-closed-semiring).

The collections of all $\mathscr{S}$-fuzzy-open-semirings and $\mathscr{S}$-fuzzy-closed-semirings in $(S, \mathscr{S})$ are symbolised by $\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ and $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$ respectively.

Definition 2.2. [2] Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Let $\lambda \in I^S$. Then the $\mathscr{S}\mathscr{F}\mathscr{S}$-interior of λ is defined and symbolised as $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}int(\lambda) = \vee\{\mu : \mu \leq \lambda \text{ and } \mu \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)\}$.

Definition 2.3. [2] Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Let $\lambda \in I^S$. Then the $\mathscr{S}\mathscr{F}\mathscr{S}$-closure of λ is defined and symbolised as $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\lambda) = \wedge\{\mu : \mu \geq \lambda \text{ and } \mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)\}$.

Definition 2.4. [2] Let S be a $\mathscr{S}$-semiring. If a $\mathscr{S}$-fuzzy semiring on S is a fuzzy point x_λ, then x_λ is termed $\mathscr{S}$-fuzzy semiring point on S.

The collection of all $\mathscr{S}$-fuzzy semiring points on S is denoted by $SFSP(S)$.

Definition 2.5. [7] If A and B are any two fuzzy subsets of a set X, then "A is said to be included in B" or "A is contained in B" or "A is less then or equal to B" iff $A(x) \leq B(x)$ for all x in X and is denoted by $A \leq B$. Equivalently, $A \leq B$ iff $\mu_A(x) \leq \mu_B(X)$ for all x in X.

Definition 2.6. [1] A nonzero fuzzy open set A ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy minimal open (briefly f-minimal open) set if any fuzzy open set which is contained in A is either 0 or A.

Definition 2.7. [1] A nonzero fuzzy closed set B ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy minimal closed (briefly f-minimal closed) set if any fuzzy closed set which is contained in B is either 0 or B.

Definition 2.8. [1] A nonzero fuzzy open set A ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy maximal open (briefly f-maximal open) set if any fuzzy open set which contains A is either 1 or A.

Definition 2.9. [1] A nonzero fuzzy closed set B ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy maximal closed (briefly f-maximal closed) set if any fuzzy closed set which contains B is either 1 or B.

3. $\mathscr{S}$-Fuzzy-Semiring-Minimal-c-Regular Spaces

In this section, the notions of $\mathscr{S}\mathscr{F}$-minimal-open, $\mathscr{S}\mathscr{F}$-minimal-closed, $\mathscr{S}\mathscr{F}$-maximal-open and $\mathscr{S}\mathscr{F}$-maximal-closed semirings are propounded. In addition, the perceptions of $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-r* and $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-r* spaces are instigated and their attributes are contemplated.

Definition 3.1. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is said to be $\mathscr{S}\mathscr{F}\mathscr{S}$-structure continuous (simply $\mathscr{S}$-continuous) if for each $\lambda \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$), $f^{-1}(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$).

Definition 3.2. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-open (resp. $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-closed) if $f(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$) for every $\lambda \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$) .

Definition 3.3. A proper $\mathscr{S}$-fuzzy-open-semiring λ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-minimal-open (briefly $\mathscr{S}\mathscr{F}$-minimal-open)-semiring if any $\mathscr{S}$-fuzzy-open-semiring which is contained in λ is either 0_S or λ.

Definition 3.4. A proper $\mathscr{S}$-fuzzy-closed-semiring μ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-minimal-closed (briefly $\mathscr{S}\mathscr{F}$-minimal-closed)-semiring if any $\mathscr{S}$-fuzzy-closed-semiring which is contained in μ is either 0_S or μ.

The family of all $\mathscr{S}$-fuzzy-minimal-open (resp. $\mathscr{S}$-fuzzy-minimal-closed) semirings in $(S,\mathscr{S})$ is denoted by $SFM_iO(S)$ (resp. $SFM_iC(S)$).

Proposition 3.1. Let $(S, \mathscr{S})$ b a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space.

(i) If $\lambda \in SFM_iO(S)$ and $\mu \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$, then $\lambda \wedge \mu = 0_S$ or $\lambda < \mu$.

(ii) If $\lambda,\ \gamma \in SFM_iO(S)$, then $\lambda \wedge \gamma = 0_S$ or $\lambda = \gamma$.

Proof. The proof is apparent from Definition 3.3. ☐

Definition 3.5. A proper $\mathscr{S}$-fuzzy-open-semiring λ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-maximal-open (briefly $\mathscr{S}\mathscr{F}$-maximal-open)-semiring if any $\mathscr{S}$-fuzzy-open-semiring which contains λ is either 1_S or λ.

Definition 3.6. A proper $\mathscr{S}$-fuzzy-closed-semiring μ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-maximal-closed (briefly $\mathscr{S}\mathscr{F}$-maximal-closed)-semiring if any $\mathscr{S}$-fuzzy-closed-semiring which contains μ is either 1_S or μ.

The family of all $\mathscr{S}$-fuzzy-maximal-open (resp. $\mathscr{S}$-fuzzy-maximal-closed) semirings in $(S,\mathscr{S})$ is denoted by $SFM_aO(S)$ (resp. $SFM_aC(S)$).

Example 3.1. Let $S = \{0,1,2\}$ be a set of integers modulo 3 with respect to '$+$ and .' and hence $(S,.,+)$ is a $\mathscr{S}$-semiring. Let $\mathscr{S} = \{0_S, 1_S, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ where $\lambda_i : S \to [0,1]$ for $i = 1,2,3,4$ is defined as follows:
$\lambda_1(0) = 0.2$, $\lambda_1(1) = 0.3$, and $\lambda_1(2) = 0.4$;
$\lambda_2(0) = 1$, $\lambda_2(1) = 0.3$, and $\lambda_2(2) = 0.4$;
$\lambda_3(0) = 0.2$, $\lambda_3(1) = 1$, and $\lambda_3(2) = 0.4$;
$\lambda_4(0) = 1$, $\lambda_4(1) = 1$, and $\lambda_4(2) = 0.4$.
Evidently, $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space.
Then $SFM_iO(S) = \lambda_1$, $SFM_iC(S) = \lambda_4'$, $SFM_aO(S) = \lambda_4$ and $SFM_aC(S) = \lambda_1'$.

Proposition 3.2. Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space.

(i) If $\lambda \in SFM_aO(S)$ and $\mu \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$, then $\lambda \vee \mu = 1_S$ or $\mu < \lambda$.

(ii) If $\lambda,\ \gamma \in SFM_aO(S)$, then $\lambda \vee \gamma = 1_S$ or $\lambda = \gamma$.

Proof. The proof is apparent from Definition 3.5. ☐

Proposition 3.3. A proper $\mathscr{S}$-fuzzy-semiring λ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}$-minimal-open-semiring if and only if $(1_S - \lambda)$ is a $\mathscr{S}\mathscr{F}$-maximal-closed-semiring.

Proof. Let λ be a $\mathscr{S}\mathscr{F}$-minimal-open-semiring in $(S,\mathscr{S})$. Assume $(1_S - \lambda)$ is not a $\mathscr{S}\mathscr{F}$-maximal-closed-semiring in $(S,\mathscr{S})$. Then there exists $\mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$ such that $(1_S - \lambda) < \mu \neq 1_S$. This implies $0_S \neq (1_S - \mu) < \lambda$ and $(1_S - \mu) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$. This is a contradiction to the assumption that λ is a $\mathscr{S}\mathscr{F}$-minimal-open-semiring in $(S,\mathscr{S})$. Hence $(1_S - \lambda)$ is a $\mathscr{S}\mathscr{F}$-maximal closed semiring in $(S,\mathscr{S})$.

Conversely, let $(1_S - \lambda)$ be a $\mathscr{S}\mathscr{F}$-maximal-closed semiring in $(S,\mathscr{S})$. Assume λ is not a $\mathscr{S}\mathscr{F}$-minimal-open-semiring in $(S,\mathscr{S})$. Then there exists $\gamma \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $\gamma \neq \lambda$ such that $0_S \neq \gamma < \lambda$. This implies $(1_S - \lambda) < (1_S - \gamma) \neq 1_S$ and $(1_S - \lambda) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$. This is a contradiction to the assumption that $(1_S - \lambda)$ is a $\mathscr{S}\mathscr{F}$-maximal-closed-semiring in $(S,\mathscr{S})$. Hence λ is a $\mathscr{S}\mathscr{F}$-minimal-open-semiring in $(S,\mathscr{S})$. ☐

Proposition 3.4. A proper $\mathscr{S}$-fuzzy semiring λ of a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}$-maximal-open-semiring if and only if $(1_S-\lambda)$ is a $\mathscr{S}\mathscr{F}$-minimal-closed-semiring.

Proof. Let λ be a $\mathscr{S}\mathscr{F}$-maximal-open-semiring in $(S,\mathscr{S})$. Assume $(1_S-\lambda)$ is not a $\mathscr{S}\mathscr{F}$-minimal-closed-semiring in $(S,\mathscr{S})$. Then there exists $\mu\in\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$ such that $0_S\neq\mu<(1_S-\lambda)$. This implies $\lambda<(1_S-\mu)\neq 1_S$ and $(1_S-\mu)\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$. This is a contradiction to the assumption that λ is a $\mathscr{S}\mathscr{F}$-maximal-open-semiring in $(S,\mathscr{S})$. Hence $(1_S-\lambda)$ is a $\mathscr{S}\mathscr{F}$-minimal closed semiring in $(S,\mathscr{S})$.

Conversely, let $(1_S-\lambda)$ be a $\mathscr{S}\mathscr{F}$-minimal-closed-semiring in $(S,\mathscr{S})$. Assume λ is not a $\mathscr{S}\mathscr{F}$-maximal-open-semiring in $(S,\mathscr{S})$. Then there exists $\gamma\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $\lambda<\gamma\neq 1_S$. This implies $0_S\neq(1_S-\gamma)<(1_S-\lambda)$ and $(1_S-\gamma)\in\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$. This is a contradiction to the assumption that $(1_S-\lambda)$ is a $\mathscr{S}\mathscr{F}$-minimal-closed-semiring in $(S,\mathscr{S})$. Hence λ is a $\mathscr{S}\mathscr{F}$-maximal-open-semiring in $(S,\mathscr{S})$. □

Definition 3.7. A $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-regular (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r*) if for every $x_\lambda\in SFSP(S)$ and $\mu\in SFM_iC(S)$ such that $x_\lambda \not q\, \mu$, there exist $\gamma,\delta\in SFM_iO(S)$ such that $x_\lambda\leq\gamma$, $\mu\leq\delta$ and $\gamma \not q\, \delta$.

Definition 3.8. A $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-c-regular (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r*) if for every $x_\lambda\in SFSP(S)$ and $\mu\in SFM_iC(S)$ such that $x_\lambda \not q\, \mu$, there exist γ, $\delta\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $x_\lambda\leq\gamma$, $\mu\leq\delta$ and $\gamma \not q\, \delta$.

Proposition 3.5. If a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, then $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

Proof. Let $x_\lambda\in SFSP(S)$ and $\mu\in SFM_iC(S)$ such that $x_\lambda \not q\, \mu$. As $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, there exist $\gamma,\delta\in SFM_iO(S)$ such that $x_\lambda\leq\gamma$, $\mu\leq\delta$ and $\gamma \not q\, \delta$. Since every $\mathscr{S}\mathscr{F}$-minimal-open-semiring is a $\mathscr{S}$-fuzzy-open-semiring, γ, $\delta\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $x_\lambda\leq\gamma$, $\mu\leq\delta$ and $\gamma \not q\, \delta$. Hence $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space. □

Proposition 3.6. Let $(S,\mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Then the following statements are equivalent :

(i) $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

(ii) For every $x_\lambda\in SFSP(S)$ and $\mu\in SFM_aO(S)$ such that $x_\lambda\leq\mu$, there exists $\gamma\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $x_\lambda\leq\gamma\leq\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)\leq\mu$.

(iii) For every $x_\lambda\in SFSP(S)$ and $\eta\in SFM_iC(S)$ such that $x_\lambda \not q\, \eta$, there exists $\mu\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $x_\lambda\leq\mu$ such that $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu) \not q\, \eta$.

Proof. **(i) ⇒ (ii)** Let $x_\lambda\in SFSP(S)$ and $\mu\in SFM_aO(S)$ such that $x_\lambda\leq\mu$. Then $(1_S-\mu)\in SFM_iC(S)$ such that $x_\lambda \not q\,(1_S-\mu)$. Since $(S,\mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space, there exist γ, $\delta\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $x_\lambda\leq\gamma$, $(1_S-\mu)\leq\delta$ and $\gamma \not q\, \delta$. Now $\gamma \not q\, \delta$ implies $\gamma\leq(1_S-\delta)$. This implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)\leq\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(1_S-\delta)=1_S-\delta$. Since $(1_S-\delta)\in\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$. Hence $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)\leq(1_S-\delta)$. Also we have $(1_S-\mu)\leq\delta$, which implies $(1_S-\delta)\leq\mu$. Thus $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)\leq(1_S-\delta)\leq\mu$. Therefore $x_\lambda\leq\gamma\leq\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)\leq\mu$.
(ii) ⇒ (iii) Let $x_\lambda\in SFSP(S)$ and $\eta\in SFM_iC(S)$ such that $x_\lambda \not q\, \eta$. Then $(1_S-\eta)\in SFM_aO(S)$ such that $x_\lambda\leq(1_S-\eta)$. By (ii), there exists $\mu\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $x_\lambda\leq\mu\leq\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu)\leq(1_S-\eta)$. This implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu) \not q\, \eta$.
(iii) ⇒ (i) Let $x_\lambda\in SFSP(S)$ and $\eta\in SFM_iC(S)$ such that $x_\lambda \not q\, \eta$. By (iii), there exists $\mu\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $x_\lambda\leq\mu$ such that $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu) \not q\, \eta$. This implies $\eta\leq(1_S-\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu))$. It is apparent that $\mu \not q\,(1_S-\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\mu)))$. □

Definition 3.9. Let $(S_1,\mathscr{S}_1)$ and $(S_2,\mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f:(S_1,\mathscr{S}_1)\to(S_2,\mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-irresolute (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*) if $f^{-1}(\lambda)\in SFM_iO(S_1)$ (resp. $SFM_iC(S_1)$) for every $\lambda\in SFM_iO(S_2)$ (resp. $SFM_iC(S_2)$).

Proposition 3.7. Let $(S_1,\mathscr{S}_1)$ and $(S_2,\mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f:(S_1,\mathscr{S}_1)\to(S_2,\mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir* and $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-open function. If $(S_1,\mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space, then $(S_2,\mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

Proof. Let $y_\eta\in SFSP(S_2)$ and let $\mu\in SFM_iC(S_2)$ such that $y_\eta \not q\, \mu$. Since f is bijective, there exists $x_\lambda\in SFSP(S_1)$ such that $f(x_\lambda)=y_\eta$, which implies $x_\lambda=f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*, $f^{-1}(\mu)\in SFM_iC(S_1)$ and $y_\eta \not q\, \mu$ implies $f^{-1}(y_\eta) \not q\, f^{-1}(\mu)$. Hence $x_\lambda \not q\, f^{-1}(\mu)$. Since $(S_1,\mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space, there exist γ, $\delta\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ such that $x_\lambda\leq\gamma$, $f^{-1}(\mu)\leq\delta$ and $\gamma \not q\, \delta$.

As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-open, $f(\gamma)$, $f(\delta)\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$. Now $x_\lambda\leq\gamma$ implies $f(x_\lambda)\leq f(\gamma)$. Hence $y_\eta\leq f(\gamma)$. Also $f^{-1}(\mu)\leq\delta$ implies $\mu\leq f(\delta)$ and $\gamma \not q\, \delta$ implies $f(\gamma) \not q\, f(\delta)$. Thus for every $y_\eta\in SFSP(S_2)$ and $\mu\in SFM_iC(S_2)$ such that $y_\eta \not q\, \mu$, there exist $f(\gamma)$, $f(\delta)\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ such that $y_\eta\leq f(\gamma)$, $\mu\leq f(\delta)$ and $f(\gamma) \not q\, f(\delta)$. Hence $(S_2,\mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space. □

Definition 3.10. Let $(S_1,\mathscr{S}_1)$ and $(S_2,\mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f:(S_1,\mathscr{S}_1)\to(S_2,\mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-strongly-minimal-closed (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-c*) if $f(\lambda)\in SFM_iC(S_2)$ for every $\lambda\in SFM_iC(S_1)$.

Proposition 3.8. Let $(S_1,\mathscr{S}_1)$ and $(S_2,\mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f:(S_1,\mathscr{S}_1)\to(S_2,\mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-structure continuous and $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-c* function. If $(S_2,\mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space, then $(S_1,\mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

Proof. Let $x_\lambda\in SFSP(S_1)$ and let $\mu\in SFM_iC(S_1)$ such that $x_\lambda \not q\, \mu$. Since f is bijective, there exists $y_\eta\in SFSP(S_2)$ such that $f(x_\lambda)=y_\eta$, which implies $x_\lambda=f^{-1}(y_\eta)$. As f is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-c* function, $f(\mu)\in SFM_iC(S_2)$ and $x_\lambda \not q\, \mu$ implies $f(x_\lambda) \not q\, f(\mu)$. Hence $y_\eta \not q\, f(\mu)$. Since $(S_2,\mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space, there exist γ, $\delta\in\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ such that $y_\eta\leq\gamma$, $f(\mu)\leq\delta$ and $\gamma \not q\, \delta$.

As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure continuous, $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$. Now $y_\eta \leq \gamma$ implies $f^{-1}(y_\eta) \leq f^{-1}(\gamma)$. Hence $x_\lambda \leq f^{-1}(\gamma)$. Also $f(\mu) \leq \delta$ implies $\mu \leq f^{-1}(\delta)$ and $\gamma \not q \delta$ implies $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Thus for every $x_\lambda \in SFSP(S_1)$ and $\mu \in SFM_iC(S_1)$ such that $x_\lambda \not q \mu$, there exist $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ such that $x_\lambda \leq f^{-1}(\gamma)$, $\mu \leq f^{-1}(\delta)$ and $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space. □

Definition 3.11. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-open (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o*) if $f(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ for every $\lambda \in SFM_iO(S_1)$.

Proposition 3.9. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o* and $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir* function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

Proof. Let $y_\eta \in SFSP(S_2)$ and let $\mu \in SFM_iC(S_2)$ such that $y_\eta \not q \mu$. Since f is bijective, there exists $x_\lambda \in SFSP(S_1)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*, $f^{-1}(\mu) \in SFM_iC(S_1)$ and $y_\eta \not q \mu$ implies $f^{-1}(y_\eta) \not q f^{-1}(\mu)$. Hence $x_\lambda \not q f^{-1}(\mu)$. Since $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, there exist $\gamma, \delta \in SFM_iO(S_1)$ such that $x_\lambda \leq \gamma$, $f^{-1}(\mu) \leq \delta$ and $\gamma \not q \delta$.

As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o*, $f(\gamma)$, $f(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$. Now $x_\lambda \leq \gamma$ implies $f(x_\lambda) \leq f(\gamma)$. Hence $y_\eta \leq f(\gamma)$. Also $f^{-1}(\mu) \leq \delta$ implies $\mu \leq f(\delta)$ and $\gamma \not q \delta$ implies $f(\gamma) \not q f(\delta)$. Thus for every $y_\eta \in SFSP(S_2)$ and $\mu \in SFM_iC(S_2)$ such that $y_\eta \not q \mu$, there exist $f(\gamma)$, $f(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ such that $y_\eta \leq f(\gamma)$, $\mu \leq f(\delta)$ and $f(\gamma) \not q f(\delta)$. Hence $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space. □

Definition 3.12. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-continuous (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous) if $f^{-1}(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$ for every $\lambda \in SFM_iO(S_2)$ (resp. $SFM_iC(S_2)$).

Proposition 3.10. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous and $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-c* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space.

Proof. Let $x_\lambda \in SFSP(S_1)$ and let $\mu \in SFM_iC(S_1)$ such that $x_\lambda \not q \mu$. Since f is bijective, there exists $y_\eta \in SFSP(S_2)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-c* function, $f(\mu) \in SFM_iC(S_2)$ and $x_\lambda \not q \mu$ implies $f(x_\lambda) \not q f(\mu)$. Hence $y_\eta \not q f(\mu)$. Since $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space, there exist $\gamma, \delta \in SFM_iO(S_2)$ such that $y_\eta \leq \gamma$, $f(\mu) \leq \delta$ and $\gamma \not q \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous, $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$. Now $y_\eta \leq \gamma$ implies $f^{-1}(y_\eta) \leq f^{-1}(\gamma)$. Hence $x_\lambda \leq f^{-1}(\gamma)$. Also $f(\mu) \leq \delta$ implies $\mu \leq f^{-1}(\delta)$ and $\gamma \not q \delta$ implies $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Thus for every $x_\lambda \in SFSP(S_1)$ and $\mu \in SFM_iC(S_1)$ such that $x_\lambda \not q \mu$, there exist $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ such that $x_\lambda \leq f^{-1}(\gamma)$, $\mu \leq f^{-1}(\delta)$ and $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-r* space. □

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