Malaya Journal of Matematik, Vol. 9, No. 1, 760-764, 2021
https://doi.org/10.26637/MJM0901/0134

Applications of Smarandache fuzzy minimal open semirings

J. Mahalakshmi[1*] and M. Sudha[2]

Abstract
In this disquisition, the concepts of $\mathscr{S}$-fuzzy-minimal-open, $\mathscr{S}$-fuzzy-minimal-closed, $\mathscr{S}$-fuzzy-maximal-open, $\mathscr{S}$-fuzzy-maximal-closed semirings are instigated. Moreover, the ideas of $\mathscr{S}$-fuzzy-semiring-minimal-regular,$\mathscr{S}$-fuzzy-semiring-minimal-o-regular, $\mathscr{S}$-fuzzy-semiring-minimal-normal spaces and $\mathscr{S}$-fuzzy-semiring-minimal-c-normal spaces are introduced and examined.

Keywords
$\mathscr{S}$-fuzzy-semiring-minimal-regular spaces, $\mathscr{S}$-fuzzy-semiring-minimal-o-regular spaces, $\mathscr{S}$-fuzzy-semiring-minimal-normal spaces, $\mathscr{S}$-fuzzy-semiring-minimal-c-normal spaces.

AMS Subject Classification
54A40, 03E72.

[1,2] *Department of Mathematics, Department of Mathematics, Sri Sarada College for Women, Salem-636016, Tamil Nadu, India.*
*__Corresponding author__: [1] paapumaha13@gmail.com
Article History: Received **07** January **2021**; Accepted **08** March **2021**

Contents

1. Introduction

Numerous articles on minimal and maximal open and closed sets in classical topology is found in literature due to F. Nakoaka and N. Oda in [5], [6] and [7]. Later B.M. Ittanagi and R.S. Wali [3] extended such sets to fuzzy topological spaces. Thereafter, S. S. Benchalli, B. M. Ittanagi and R. S. Wali [2] propounded the notions of minimal T_0, minimal c-regular and minimal completely regular spaces. The perception of minimal c-normal spaces was pioneered in [1]. In this paper, some of the applications of $\mathscr{S}$-fuzzy minimal open semirings like $\mathscr{S}$-fuzzy-semiring-minimal-regular, $\mathscr{S}$-fuzzy-semiring-minimal-o-regular $\mathscr{S}$-fuzzy-semiring-minimal normal and $\mathscr{S}$-fuzzy-semiring-minimal-c-normal spaces are initiated and their properties are analysed.

2. Preliminaries

Definition 2.1. [4] Let S be a $\mathscr{S}$-semiring. A family $\mathscr{S}$ of $\mathscr{S}$-fuzzy semirings on S is termed Smarandache fuzzy semiring structure (briefly $\mathscr{S}\mathscr{F}\mathscr{S}$-structure) on S if it satisfies the following conditions:

(i) $0_S,\ 1_S \in \mathscr{S}$,

(ii) If $\lambda_1, \lambda_2 \in \mathscr{S}$, then $\lambda_1 \wedge \lambda_2 \in \mathscr{S}$,

(iii) If $\lambda_i \in \mathscr{S}$ for each $i \in J$, then $\vee\lambda_i \in \mathscr{S}$.

And the ordered pair $(S, \mathscr{S})$ is termed $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Every member of $\mathscr{S}$ is termed $\mathscr{S}$-fuzzy-open-semiring and the complement of a $\mathscr{S}$-fuzzy-open-semiring is called an anti-$\mathscr{S}$-fuzzy-open-semiring (or a $\mathscr{S}$-fuzzy-closed-semiring).

The collections of all $\mathscr{S}$-fuzzy-open-semirings and $\mathscr{S}$-fuzzy-closed-semirings in $(S, \mathscr{S})$ are symbolised by $\mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ and $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$ respectively.

Definition 2.2. [4] Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Let $\lambda \in I^S$. Then the $\mathscr{S}\mathscr{F}\mathscr{S}$-interior of λ is defined and symbolised as $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}int(\lambda) = \vee\{\mu : \mu \leq \lambda \text{ and } \mu \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)\}$.

Definition 2.3. [4] Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Let $\lambda \in I^S$. Then the $\mathscr{S}\mathscr{F}\mathscr{S}$-closure of λ is defined and symbolised as $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\lambda) = \wedge\{\mu : \mu \geq \lambda \text{ and } \mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)\}$.

Definition 2.4. [4] Let S be a $\mathscr{S}$-semiring. If a $\mathscr{S}$-fuzzy semiring on S is a fuzzy point x_λ, then x_λ is termed $\mathscr{S}$-fuzzy semiring point on S.

The collection of all $\mathscr{S}$-fuzzy semiring points on S is denoted by $SFSP(S)$.

Definition 2.5. [9] If A and B are any two fuzzy subsets of a set X, then "A is said to be included in B" or "A is contained in B" or "A is less then or equal to B" iff $A(x) \leq B(x)$ for all x in X and is denoted by $A \leq B$. Equivalently, $A \leq B$ iff $\mu_A(x) \leq \mu_B(X)$ for all x in X.

Definition 2.6. [3] A nonzero fuzzy open set A ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy minimal open (briefly f-minimal open) set if any fuzzy open set which is contained in A is either 0 or A.

Definition 2.7. [3] A nonzero fuzzy closed set B ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy minimal closed (briefly f-minimal closed) set if any fuzzy closed set which is contained in B is either 0 or B.

Definition 2.8. [3] A nonzero fuzzy open set A ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy maximal open (briefly f-maximal open) set if any fuzzy open set which contains A is either 1 or A.

Definition 2.9. [3] A nonzero fuzzy closed set B ($\neq 1$) of a fuzzy topological space (X,T) is said to be a fuzzy maximal closed (briefly f-maximal closed) set if any fuzzy closed set which contains B is either 1 or B.

3. $\mathscr{S}$-Fuzzy-Semiring-Minimal-o-Regular Spaces

In this section, the perception of $\mathscr{SFS}$-*min-o-r* spaces is pioneered and some attributes concerning this concept is explored.

Definition 3.1. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is said to be $\mathscr{SFS}$-structure continuous (simply $\mathscr{S}$-continuous) if for each $\lambda \in \mathscr{SFOS}(S_2)$ (resp. $\mathscr{SFCS}(S_2)$), $f^{-1}(\lambda) \in \mathscr{SFOS}(S_1)$ (resp. $\mathscr{SFCS}(S_1)$).

Definition 3.2. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{SFS}$-structure-open (resp. $\mathscr{SFS}$-structure-closed) if $f(\lambda) \in \mathscr{SFOS}(S_2)$ (resp. $\mathscr{SFCS}(S_2)$) for every $\lambda \in \mathscr{SFOS}(S_1)$ (resp. $\mathscr{SFCS}(S_1)$) .

Definition 3.3. A proper $\mathscr{S}$-fuzzy-open-semiring λ of a $\mathscr{SFS}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-minimal-open (briefly $\mathscr{SF}$-minimal-open)-semiring if any $\mathscr{S}$-fuzzy-open-semiring which is contained in λ is either 0_S or λ.

Definition 3.4. A proper $\mathscr{S}$-fuzzy-closed-semiring μ of a $\mathscr{SFS}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-minimal-closed (briefly $\mathscr{SF}$-minimal-closed)-semiring if any $\mathscr{S}$-fuzzy-closed-semiring which is contained in μ is either 0_S or μ.

The family of all $\mathscr{S}$-fuzzy-minimal-open (resp. $\mathscr{S}$-fuzzy-minimal-closed) semirings in $(S,\mathscr{S})$ is denoted by $SFM_iO(S)$ (resp. $SFM_iC(S)$).

Definition 3.5. A proper $\mathscr{S}$-fuzzy-open-semiring λ of a $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-maximal-open (briefly $\mathscr{SF}$-maximal-open)-semiring if any $\mathscr{S}$-fuzzy-open-semiring which contains λ is either 1_S or λ.

Definition 3.6. A proper $\mathscr{S}$-fuzzy-closed-semiring μ of a $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-maximal-closed (briefly $\mathscr{SF}$-maximal-closed)-semiring if any $\mathscr{S}$-fuzzy-closed-semiring which contains μ is either 1_S or μ.

The family of all $\mathscr{S}$-fuzzy-maximal-open (resp. $\mathscr{S}$-fuzzy-maximal-closed) semirings in $(S,\mathscr{S})$ is denoted by $SFM_aO(S)$ (resp. $SFM_aC(S)$).

Definition 3.7. A $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-regular (in short $\mathscr{SFS}$-*min-r*) if for every $x_\lambda \in SFSP(S)$ and $\mu \in SFM_iC(S)$ such that $x_\lambda \not{q}\, \mu$, there exist $\gamma, \delta \in SFM_iO(S)$ such that $x_\lambda \leq \gamma$, $\mu \leq \delta$ and $\gamma \not{q}\, \delta$.

Definition 3.8. A $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-o-regular (in short $\mathscr{SFS}$-*min-o-r*) if for every $x_\lambda \in SFSP(S)$ and $\mu \in \mathscr{SFCS}(S)$ such that $x_\lambda \not{q}\, \mu$, there exist $\gamma, \delta \in SFM_iO(S)$ such that $x_\lambda \leq \gamma$, $\mu \leq \delta$ and $\gamma \not{q}\, \delta$.

Proposition 3.1. If a $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-o-r* space, then $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-r* space.

Proof. Let $x_\lambda \in SFSP(S)$ and $\mu \in SFM_iC(S)$ such that $x_\lambda \not{q}\, \mu$. Since every $\mathscr{SF}$-minimal-closed-semiring is a $\mathscr{S}$-fuzzy-closed-semiring, $\mu \in \mathscr{SFCS}(S)$ such that $x_\lambda \not{q}\, \mu$. As $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S)$ such that $x_\lambda \leq \gamma$, $\mu \leq \delta$ and $\gamma \not{q}\, \delta$. Hence $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-r* space. □

Proposition 3.2. If a $\mathscr{SFS}$-structure space $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-o-r* space, then for every $x_\lambda \in SFSP(S)$ and $\mu \in \mathscr{SFOS}(S)$ such that $x_\lambda \leq \mu$, there exists $\gamma \in SFM_iO(S)$ such that $x_\lambda \leq \gamma \leq \mathscr{SFS}\text{-}cl(\gamma) \leq \mu$.

Proof. Let $x_\lambda \in SFSP(S)$ and $\mu \in \mathscr{SFOS}(S)$ such that $x_\lambda \leq \mu$. Then $(1_S - \mu) \in \mathscr{SFCS}(S)$ such that $x_\lambda \not{q}\, (1_S - \mu)$. Since $(S,\mathscr{S})$ is a $\mathscr{SFS}$-*min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S)$ such that $x_\lambda \leq \gamma$, $(1_S - \mu) \leq \delta$ and $\gamma \not{q}\, \delta$. Now $\gamma \not{q}\, \delta$ implies $\gamma \leq (1_S - \delta)$. This implies $\mathscr{SFS}\text{-}cl(\gamma) \leq \mathscr{SFS}\text{-}cl(1_S - \delta) = 1_S - \delta$ since $(1_S - \delta) \in \mathscr{SFCS}(S)$. Hence $\mathscr{SFS}\text{-}cl(\gamma) \leq (1_S - \delta)$. Also we have $(1_S - \mu) \leq \delta$. This implies $(1_S - \delta) \leq \mu$. Thus $\mathscr{SFS}\text{-}cl(\gamma) \leq (1_S - \delta) \leq \mu$. Therefore $x_\lambda \leq \gamma \leq \mathscr{SFS}\text{-}cl(\gamma) \leq \mu$. □

Definition 3.9. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-closed (in short $\mathscr{SFS}$-*min-c*) if $f(\lambda) \in \mathscr{SFCS}(S_2), \mathscr{S}_2)$ for every $\lambda \in SFM_iC(S_1)$.

Definition 3.10. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-irresolute (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*) if $f^{-1}(\lambda) \in SFM_iO(S_1)$ (resp. $SFM_iC(S_1)$) for every $\lambda \in SFM_iO(S_2)$ (resp. $SFM_iC(S_2)$).

Proposition 3.3. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c* and $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space.

Proof. Let $x_\lambda \in SFSP(S_1)$ and let $\mu \in SFM_iC(S_1)$ such that $x_\lambda \not q \mu$. Since f is bijective, there exists $y_\eta \in SFSP(S_2)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c*, $f(\mu) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$ and $x_\lambda \not q \mu$ implies $f(x_\lambda) \not q f(\mu)$. Hence $y_\eta \not q f(\mu)$. Since $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S_2)$ such that $y_\eta \le \gamma$, $f(\mu) \le \delta$ and $\gamma \not q \delta$.

As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*, $f^{-1}(\gamma), f^{-1}(\delta) \in SFM_iO(S_1)$. Now $y_\eta \le \gamma$ implies $f^{-1}(y_\eta) \le f^{-1}(\gamma)$. Hence $x_\lambda \le f^{-1}(\gamma)$. Also $f(\mu) \le \delta$ implies $\mu \le f^{-1}(\delta)$ and $\gamma \not q \delta$ implies $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Thus for every $x_\lambda \in SFSP(S_1)$ and $\mu \in SFM_iC(S_1)$ such that $x_\lambda \not q \mu$, there exist $f^{-1}(\gamma), f^{-1}(\delta) \in SFM_iO(S_1)$ such that $x_\lambda \le f^{-1}(\gamma)$, $\mu \le f^{-1}(\delta)$ and $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space. □

Definition 3.11. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-strongly-minimal-open (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-o*) if $f(\lambda) \in SFM_iO(S_2)$ for every $\lambda \in SFM_iO(S_1)$.

Proposition 3.4. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-structure continuous and $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-o* function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space.

Proof. Let $y_\eta \in SFSP(S_2)$ and let $\mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$ such that $y_\eta \not q \mu$. Since f is bijective, there exists $x_\lambda \in SFSP(S_1)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure continuous, $f^{-1}(\mu) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$. Also $y_\eta \not q \mu$ implies $f^{-1}(y_\eta) \not q f^{-1}(\mu)$. Hence $x_\lambda \not q f^{-1}(\mu)$.

Since $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S_1)$ such that $x_\lambda \le \gamma$, $f^{-1}(\mu) \le \delta$ and $\gamma \not q \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-o*, $f(\gamma), f(\delta) \in SFM_iO(S_2)$. Now $x_\lambda \le \gamma$ implies $f(x_\lambda) \le f(\gamma)$. Hence $y_\eta \le f(\gamma)$. Also $f^{-1}(\mu) \le \delta$ implies $\mu \le f(\delta)$ and $\gamma \not q \delta$ implies $f(\gamma) \not q f(\delta)$. Thus for every $y_\eta \in SFSP(S_2)$ and $\mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$ such that $y_\eta \not q \mu$, there exist $f(\gamma), f(\delta) \in SFM_iO(S_2)$ such that $y_\eta \le f(\gamma)$, $\mu \le f(\delta)$ and $f(\gamma) \not q f(\delta)$. Hence $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space. □

Proposition 3.5. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-closed and $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space.

Proof. Let $x_\lambda \in SFSP(S_1)$ and let $\mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$ such that $x_\lambda \not q \mu$. Since f is bijective, there exists $y_\eta \in SFSP(S_2)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure closed, $f(\mu) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_2)$ and $x_\lambda \not q \mu$ implies $f(x_\lambda) \not q f(\mu)$. Hence $y_\eta \not q f(\mu)$. Since $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S_2)$ such that $y_\eta \le \gamma$, $f(\mu) \le \delta$ and $\gamma \not q \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-ir*, $f^{-1}(\gamma), f^{-1}(\delta) \in SFM_iO(S_1)$. Now $y_\eta \le \gamma$ implies $f^{-1}(y_\eta) \le f^{-1}(\gamma)$. Hence $x_\lambda \le f^{-1}(\gamma)$. Also $f(\mu) \le \delta$ implies $\mu \le f^{-1}(\delta)$ and $\gamma \not q \delta$ implies $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Thus for every $x_\lambda \in SFSP(S_1)$ and $\mu \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$ such that $x_\lambda \not q \mu$, there exist $f^{-1}(\gamma), f^{-1}(\delta) \in SFM_iO(S_1)$ such that $x_\lambda \le f^{-1}(\gamma)$, $\mu \le f^{-1}(\delta)$ and $f^{-1}(\gamma) \not q f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space. □

Definition 3.12. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-continuous (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous) if $f^{-1}(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ (resp. $\mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$ for every $\lambda \in SFM_iO(S_2)$ (resp. $SFM_iC(S_2)$).

Proposition 3.6. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous and $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-o* function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space.

Proof. Let $y_\eta \in SFSP(S_2)$ and let $\mu \in SFM_iC(S_2)$ such that $y_\eta \not q \mu$. Since f is bijective, there exists $x_\lambda \in SFSP(S_1)$ such that $f(x_\lambda) = y_\eta$, which implies $x_\lambda = f^{-1}(y_\eta)$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-min*-continuous, $f^{-1}(\mu) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S_1)$. Also $y_\eta \not q \mu$ implies $f^{-1}(y_\eta) \not q f^{-1}(\mu)$. Hence $x_\lambda \not q f^{-1}(\mu)$.

Since $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-o-r* space, there exist $\gamma, \delta \in SFM_iO(S_1)$ such that $x_\lambda \le \gamma$, $f^{-1}(\mu) \le \delta$ and $\gamma \not q \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$*-s-min-o*, $f(\gamma), f(\delta) \in SFM_iO(S_2)$. Now $x_\lambda \le \gamma$ implies $f(x_\lambda) \le f(\gamma)$. Hence $y_\eta \le f(\gamma)$. Also $f^{-1}(\mu) \le \delta$ implies $\mu \le f(\delta)$ and $\gamma \not q \delta$ implies $f(\gamma) \not q f(\delta)$. Thus for every $y_\eta \in SFSP(S_2)$ and $\mu \in SFM_iC(S_2)$ such that $y_\eta \not q \mu$, there exist $f(\gamma), f(\delta) \in SFM_iO(S_2)$ such that $y_\eta \le f(\gamma)$, $\mu \le f(\delta)$ and $f(\gamma) \not q f(\delta)$. Hence $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-r* space. □

4. $\mathscr{S}$-Fuzzy-Semiring-Minimal-c-Normal Spaces

In this section, the ideas of $\mathscr{S}\mathscr{F}\mathscr{S}$ *-min-n* and $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-n* spaces are instigated and some of their captivating properties are examined. Furthermore, an interesting characterisation involving $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-c-n* space is obtained.

Definition 4.1. A $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-normal (in short $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-n*) if for every λ, $\mu \in SFM_iC(S)$ such that $\lambda \not q \mu$, there exist $\gamma, \delta \in SFM_iO(S)$ such that $\lambda \le \gamma$, $\mu \le \delta$ and $\gamma \not q \delta$.

Proposition 4.1. If a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S, \mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$*-min-n* space, then for every $\lambda \in SFM_iC(S)$ and $\mu \in$

$SFM_aO(S)$ such that $\lambda \leq \mu$, there exists $\gamma \in SFM_iO(S)$ such that $\lambda \leq \gamma \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mu$.

Proof. Let $\lambda \in SFM_iC(S)$ and $\mu \in SFM_aO(S)$ such that $\lambda \leq \mu$. Then $(1_S - \mu) \in SFM_iC(S)$. Hence $\lambda \not{q} (1_S - \mu)$. Since $(S, \mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-n* space, there exist $\gamma, \delta \in SFM_iO(S)$ such that $\lambda \leq \gamma$, $(1_S - \mu) \leq \delta$ and $\gamma \not{q} \delta$. Now $\gamma \not{q} \delta$ implies $\gamma \leq (1_S - \delta)$. This implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(1_S - \delta) = 1_S - \delta$ since $(1_S - \delta) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$. Hence $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq (1_S - \delta)$. Also we have $(1_S - \mu) \leq \delta$. This implies $(1_S - \delta) \leq \mu$. Thus $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq (1_S - \delta) \leq \mu$. Therefore $\lambda \leq \gamma \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mu$. □

Definition 4.2. A $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space $(S, \mathscr{S})$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-c-normal (in short $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n*) if for every λ, $\mu \in SFM_iC(S)$ such that $\lambda \not{q} \mu$, there exist γ, $\delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $\lambda \leq \gamma$, $\mu \leq \delta$ and $\gamma \not{q} \delta$.

Proposition 4.2. Let $(S, \mathscr{S})$ be a $\mathscr{S}\mathscr{F}\mathscr{S}$-structure space. Then the following statements are equivalent :

(i) $(S, \mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space.

(ii) For every $\lambda \in SFM_iC(S)$ and $\mu \in SFM_aO(S)$ such that $\lambda \leq \mu$, there exists $\gamma \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $\lambda \leq \gamma \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mu$.

(iii) For every $\lambda, \mu \in SFM_iC(S)$ such that $\lambda \not{q} \mu$, there exist $\gamma, \delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $\gamma \not{q} \delta$ such that $\lambda \leq \gamma$, $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \not{q} \mu$ and $\mu \leq \delta$, $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \not{q} \lambda$.

(iv) For every $\lambda, \mu \in SFM_iC(S)$ such that $\lambda \not{q} \mu$, there exist $\gamma, \delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $\gamma \not{q} \delta$ such that $\lambda \leq \gamma$, $\mu \leq \delta$ and $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \not{q} \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta)$.

Proof. **(i) ⇒ (ii)** Let $\lambda \in SFM_iC(S)$ and $\mu \in SFM_aO(S)$ such that $\lambda \leq \mu$. Then $(1_S - \mu) \in SFM_iC(S)$. Hence $\lambda \not{q} (1_S - \mu)$. Since $(S, \mathscr{S})$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space, there exist $\gamma, \delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $\lambda \leq \gamma$, $(1_S - \mu) \leq \delta$ and $\gamma \not{q} \delta$. Now $\gamma \not{q} \delta$ implies $\gamma \leq (1_S - \delta)$. This implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(1_S - \delta) = 1_S - \delta$. Since $(1_S - \delta) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$. Hence $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq (1_S - \delta)$. Also we have $(1_S - \mu) \leq \delta$. This implies $(1_S - \delta) \leq \mu$. Thus $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}(\gamma) \leq (1_S - \delta) \leq \mu$. Therefore $\lambda \leq \gamma \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq \mu$.
(ii) ⇒ (iii) Let $\lambda, \mu \in SFM_iC(S)$ with $\lambda \not{q} \mu$. This implies $\lambda \leq (1_S - \mu)$, where $(1_S - \mu) \in SFM_aO(S)$. By (ii), there exists $\gamma \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ such that $\lambda \leq \gamma \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq (1_S - \mu)$. Now $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \leq (1_S - \mu)$ implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma) \not{q} \mu$. Let $\delta = 1_S - \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)$. Then $\mu \leq \delta \leq (1_S - \gamma) \leq (1_S - \lambda)$. Since $(1_S - \gamma) \in \mathscr{S}\mathscr{F}\mathscr{C}\mathscr{S}(S)$, $\mu \leq \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \leq (1_S - \gamma) \leq (1_S - \lambda)$. Now $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \leq (1_S - \lambda)$ implies $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \not{q} \lambda$ and it is apparent that $\gamma \not{q} \delta$.
(iii) ⇒ (iv) Let $\lambda, \mu \in SFM_iC(S)$ with $\lambda \not{q} \mu$. By (iii), there exist γ, $\delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S)$ with $\gamma \not{q} \delta$ such that $\lambda \leq \gamma$, $\mu \leq \delta$, $\mu \leq (1_S - \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma))$ and $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \leq (1_S - \lambda)$. It is apparent that $\mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\delta) \not{q} \mathscr{S}\mathscr{F}\mathscr{S}\text{-}cl(\gamma)$.
(iv) ⇒ (i) The proof is apparent. □

Proposition 4.3. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-ir* and $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-open function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space.

Proof. Let $\lambda, \mu \in SFM_iC(S_2)$ such that $\lambda \not{q} \mu$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-ir*, $f^{-1}(\lambda), f^{-1}(\mu) \in SFM_iC(S_1)$. Also $f^{-1}(\lambda) \not{q} f^{-1}(\mu)$. Since $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space, there exist $\gamma, \delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ such that $f^{-1}(\lambda) \leq \gamma$, $f^{-1}(\mu) \leq \delta$ and $\gamma \not{q} \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-open, $f(\gamma)$, $f(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$. Now $f^{-1}(\lambda) \leq \gamma$ implies $\lambda \leq f(\gamma)$, $f^{-1}(\mu) \leq \delta$ implies $\mu \leq f(\delta)$ since f is bijective and also $\gamma \not{q} \delta$ implies $f(\gamma) \not{q} f(\delta)$. Thus for every $\lambda, \mu \in SFM_iC(S_2)$ such that $\lambda \not{q} \mu$, there exist $f(\gamma)$, $f(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ such that $\lambda \leq f(\gamma)$, $\mu \leq f(\delta)$ and $f(\gamma) \not{q} f(\delta)$. Hence $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space. □

Definition 4.3. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-strongly-minimal-closed (in short $\mathscr{S}\mathscr{F}\mathscr{S}$-*s-min-c*) if $f(\lambda) \in SFM_iC(S_2)$ for every $\lambda \in SFM_iC(S_1)$.

Proposition 4.4. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-continuous and $\mathscr{S}\mathscr{F}\mathscr{S}$-*s-min-c* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space.

Proof. Let $\lambda, \mu \in SFM_iC(S_1)$ such that $\lambda \not{q} \mu$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-*s-min-c*, $f(\lambda), f(\mu) \in SFM_iC(S_2)$. Also $f(\lambda) \not{q} f(\mu)$.

Since $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space, there exist $\gamma, \delta \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ such that $f(\lambda) \leq \gamma$, $f(\mu) \leq \delta$ and $\gamma \not{q} \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-structure-continuous, $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$. Now $f(\lambda) \leq \gamma$ implies $\lambda \leq f^{-1}(\gamma)$, $f(\mu) \leq \delta$ implies $\mu \leq f^{-1}(\delta)$ since f is bijective and also $\gamma \not{q} \delta$ implies $f^{-1}(\gamma) \not{q} f^{-1}(\delta)$. Thus for every $\lambda, \mu \in SFM_iC(S_1)$ such that $\lambda \not{q} \mu$, there exist $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_1)$ such that $\lambda \leq f^{-1}(\gamma)$, $\mu \leq f^{-1}(\delta)$ and $f^{-1}(\gamma) \not{q} f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space. □

Definition 4.4. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. A function $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ is termed $\mathscr{S}$-fuzzy-semiring-minimal-open (in short $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-o*) if $f(\lambda) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$ for every $\lambda \in SFM_iO(S_1)$

Proposition 4.5. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{S}\mathscr{F}\mathscr{S}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-o* and $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-ir* function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-n* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-c-n* space.

Proof. Let $\lambda, \mu \in SFM_iC(S_2)$ such that $\lambda \not{q} \mu$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-ir*, $f^{-1}(\lambda), f^{-1}(\mu) \in SFM_iC(S_1)$. Also $f^{-1}(\lambda) \not{q} f^{-1}(\mu)$. Since $(S_1, \mathscr{S}_1)$ is a $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-n* space, there exist $\gamma, \delta \in SFM_iO(S_1)$ such that $f^{-1}(\lambda) \leq \gamma$, $f^{-1}(\mu) \leq \delta$ and $\gamma \not{q} \delta$. As f is $\mathscr{S}\mathscr{F}\mathscr{S}$-*min-o*, $f(\gamma)$, $f(\delta) \in \mathscr{S}\mathscr{F}\mathscr{O}\mathscr{S}(S_2)$. Since

f is bijective $f^{-1}(\lambda) \leq \gamma$ implies $\lambda \leq f(\gamma)$, $f^{-1}(\mu) \leq \delta$ implies $\mu \leq f(\delta)$ and also $\gamma \not{q}\, \delta$ implies $f(\gamma) \not{q}\, f(\delta)$. Thus for every $\lambda, \mu \in SFM_iC(S_2)$ such that $\lambda \not{q}\, \mu$, there exist $f(\gamma)$, $f(\delta) \in \mathscr{SFOS}(S_2)$ such that $\lambda \leq f(\gamma)$, $\mu \leq f(\delta)$ and $f(\gamma) \not{q}\, f(\delta)$. Hence $(S_2, \mathscr{S}_2)$ is a $\mathscr{SFS}$*-min-c-n* space. □

Proposition 4.6. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{SFS}$*-s-min-o* and $\mathscr{SFS}$*-min-ir* function. If $(S_1, \mathscr{S}_1)$ is a $\mathscr{SFS}$*-min-n* space, then $(S_2, \mathscr{S}_2)$ is a $\mathscr{SFS}$*-min-n* space.

Proof. The proof is similar to that of Proposition 4.5. □

Proposition 4.7. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{SFS}$*-min*-continuous and $\mathscr{SFS}$*-s-min-c* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{SFS}$*-min-n* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{SFS}$*-min-c-n* space.

Proof. Let $\lambda, \mu \in SFM_iC(S_1)$ such that $\lambda \not{q}\, \mu$. As f is $\mathscr{SFS}$*-s-min-c*, $f(\lambda), f(\mu) \in SFM_iC(S_2)$. Also $f(\lambda) \not{q}\, f(\mu)$. Since $(S_2, \mathscr{S}_2)$ is a $\mathscr{SFS}$*-min-n* space, there exist $\gamma, \delta \in SFM_iO(S_2)$ such that $f(\lambda) \leq \gamma$, $f(\mu) \leq \delta$ and $\gamma \not{q}\, \delta$. As f is $\mathscr{SFS}$*-min*-continuous, $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{SFOS}(S_1)$. Since f is bijective, $f(\lambda) \leq \gamma$ implies $\lambda \leq f^{-1}(\gamma)$, $f(\mu) \leq \delta$ implies $\mu \leq f^{-1}(\delta)$ and also $\gamma \not{q}\, \delta$ implies $f^{-1}(\gamma) \not{q}\, f^{-1}(\delta)$. Thus for every $\lambda, \mu \in SFM_iC(S_1)$ such that $\lambda \not{q}\, \mu$, there exist $f^{-1}(\gamma)$, $f^{-1}(\delta) \in \mathscr{SFOS}(S_1)$ such that $\lambda \leq f^{-1}(\gamma)$, $\mu \leq f^{-1}(\delta)$ and $f^{-1}(\gamma) \not{q}\, f^{-1}(\delta)$. Hence $(S_1, \mathscr{S}_1)$ is a $\mathscr{SFS}$*-min-c-n* space. □

Proposition 4.8. Let $(S_1, \mathscr{S}_1)$ and $(S_2, \mathscr{S}_2)$ be any two $\mathscr{SFS}$-structure spaces. Let $f : (S_1, \mathscr{S}_1) \to (S_2, \mathscr{S}_2)$ be a bijective, $\mathscr{SFS}$*-min-ir* and $\mathscr{SFS}$*-s-min-c* function. If $(S_2, \mathscr{S}_2)$ is a $\mathscr{SFS}$*-min-n* space, then $(S_1, \mathscr{S}_1)$ is a $\mathscr{SFS}$*-min-n* space.

Proof. The proof is similar to that of Proposition 4.7. □

References

[1] S. S. Benchalli, B. M. Ittanagi and R. S. Wali, On minimal normal and minimal compact spaces, *J. Comput. Math. Sci.*, 3(2012), 280-286.

[2] S. S. Benchalli, B. M. Ittanagi and R. S. Wali, *On minimal separation axioms in topological spaces,* 3(2012), 98-104.

[3] B.M. Ittanagi and R.S. Wali, On fuzzy minimal open and fuzzy maximal open sets in fuzzy topological spaces, *Int. J. Mathematical Sciences and Applications*, 1(2011), 1023-1037.

[4] J. Mahalakshmi and M. Sudha, Smarandache fuzzy semiring para-nearly compact spaces, *The Journal of Fuzzy Mathematics,* 28(2020), 523-543.

[5] F. Nakaoka and N. Oda, Some applications of minimal open sets, *Int. J. Math. Math. Sci.,* 27(2001), 471-476.

[6] F. Nakaoka and N. Oda, Some properties of maximal open sets, *Int. J. Math. Math. Sci.,* 21(2003), 1331-1340.

[7] F. Nakaoka and N. Oda, Minimal closed sets and maximal closed sets, *Int. J. Math. Sci.*, (2006), 1-8.

[8] W. B. Vasantha Kandasamy, Smarandache Fuzzy Algebra, *American Research Press*, Rehoboth, 2003.

[9] L.A.Zadeh, Fuzzy sets, *Information and control*, 8 (1965), 338-353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆

ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

⋆⋆⋆⋆⋆⋆⋆⋆⋆