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Some degree based connectivity indices of the polygonal cylinders of a graph

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Abstract
The polygonal cylinder $C_{m,n}$ is a graph obtained from Cartesian product of paths P_m and P_n and using topological identification of vertices and edges of two opposite sides $P_m \times P_n$. We have redefined the polygonal cylinder $C_{m,n}$ by using Cartesian product of $C_{m-1} \times P_n$. In this paper, we have introduced a closed polygonal cylinder $C_{[m,n]}$, is a graph obtained from Cartesian product of cycle C_m and P_n. Further more we have obtained first and second Zagreb, F-index, first and second hyper-Zagreb, harmonic, Randić, sum-connectivity and atom-bond connectivity indices of polygonal cylinder and closed polygonal cylinder of a graph.

Keywords
Polygonal cylinder, Zagreb index, hyper-Zagreb index, Randić index, sum-connectivity index and ABC index.

AMS Subject Classification
05C05, 05C07.

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1. Introduction

The topological indices are graph invariants which are numerical values associated with molecular graphs. In mathematical chemistry, molecular descriptors play a leading role specifically in the field of QSPR/QSAR modelling. Among them, an outstanding area is preserved for the topological indices which are well-known in graph invariant. A real valued mapping considering graph as argument is called a graph invariant, if it gives same value to graphs which are isomorphic. The order and size of a graph are examples of two graph invariants. The topological indices were initiated when the eminent chemist H. Wiener found the first topological index, known as Wiener index, in 1947 while searching boiling point of alkanes, amidst the topological indices invented on initial stage, the Zagreb indices belong to the well-known and well researched molecular descriptors.

2. Preliminaries

Let G be a finite, undirected graph without loops and multiple edges on n vertices and m edges and is called (n,m) graph. We denote vertex set and edge set of graph G as $V(G)$ and $E(G)$, respectively. For a graph G, the degree of a vertex V is the number of edges incident to V and is denoted by $d_G(v)$. For unexplained graph terminology and notation refer [9, 12].

Present days, topological indices are extensively used in mathematical chemistry. In the literature many researchers are defined degree based topological indices. Among them, first and second Zagreb index were defined by Gutman and Trinajstić [5] in 1972 as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2,$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

The first Zagreb index [13] can also be expressed as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

Another degree based topological index is forgotten topological index or F-index was introduced by Furtula and Gutman [4] is defined as

$$F(G)=\sum_{v\in V(G)} d_G(u)^3=\sum_{uv\in E(G)}[d_G(u)^2+d_G(v)^2].$$

The first hyper-Zagreb index was introduced by Shirdel et al. in [16] which is defined as

$$HM_1(G)=\sum_{uv\in E(G)}[d_G(u)+d_G(v)]^2.$$

The second hyper-Zagreb index was introduced by Farahani et al. in [2] which is defined as

$$HM_2(G)=\sum_{uv\in E(G)}[d_G(u)d_G(v)]^2.$$

The Randić index or product connectivity index of a graph G was proposed by Randić in [15] is defined as

$$R(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

The harmonic index of a graph G was introduced by Fajtlowicz in [3] is defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}.$$

The sum-connectivity index of a graph G was defined in [20] as

$$X(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}.$$

The atom-bond connectivity index, which is defined in [1] as

$$ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}.$$

Definition 2.1. [9] *The Cartesian product $G_1\times G_2$, consider any two points $u=(u_1,u_2)$ and $v=(v_1,v_2)$ in $V=V_1\times V_2$. Then u and v are adjacent in $G_1\times G_2$ whenever $[u_1=v_1$ and $u_2\ adj\ v_2]$ or $[u_2=v_2$ and $u_1\ adj\ v_1]$.*

Recently, Nizami et al. [14] formed new graph which is in the form of polygonal cylinder shape. The new graph is obtained by using Cartesian product of paths P_m and P_n, where $m\geq 4$ and $n\geq 2$. The polygonal cylinder defined as below.

Definition 2.2. [14] *Consider the Cartesian product $P_m\times P_n$ of paths $P_m, m\geq 4$, and $P_n, n\geq 2$, with vertices $u_1,u_2,\ldots,u_m$ and $v_1,v_2,\ldots,v_n$, respectively. Identify the vertices $(u_1,v_1),(u_1,v_2),\ldots,(u_1,v_n)$ with the vertices $(u_m,v_1),(u_m,v_2),\ldots,(u_m,v_n)$, respectively, and identify the edge $((u_1,v_i),(u_1,v_{i+1}))$ with the edge $((u_m,v_i),(u_m,v_{i+1}))$, where $1\leq i\leq n-1$. It is denoted as $C_{m,n}$.*

We have redefined the above said polygonal cylinder $C_{m,n}$ as follows.

Definition 2.3. *The polygonal cylinder $C_{m,n}$ is a graph obtained from a Cartesian product $C_{m-1}\times P_n$ of two graphs of cycle $C_{m-1}, m\geq 5$ and path $P_n, n\geq 2$, with vertices $u_1,u_2,\ldots,u_{m-1}$ and $v_1,v_2,\ldots,v_n$, respectively.*

Further we have introduced new graph structure, that is closed polygonal cylinder. It is denoted as $C_{[m,n]}$ and defined as follows.

Definition 2.4. *The closed polygonal cylinder $C_{[m,n]}$ is a graph formed by Cartesian product of $C_m\times P_n$ of cycle $C_m, m\geq 3$, and path $P_n, n\geq 3$, with vertices $u_1,u_2,\ldots,u_m$ and $v_1,v_2,\ldots,v_n$, respectively. Identify the vertices (u_i,v_1) is adjacent to (u_{i+2},v_1) and (u_i,v_n) is adjacent to (u_{i+2},v_n) where $1\leq i\leq m-3$.*

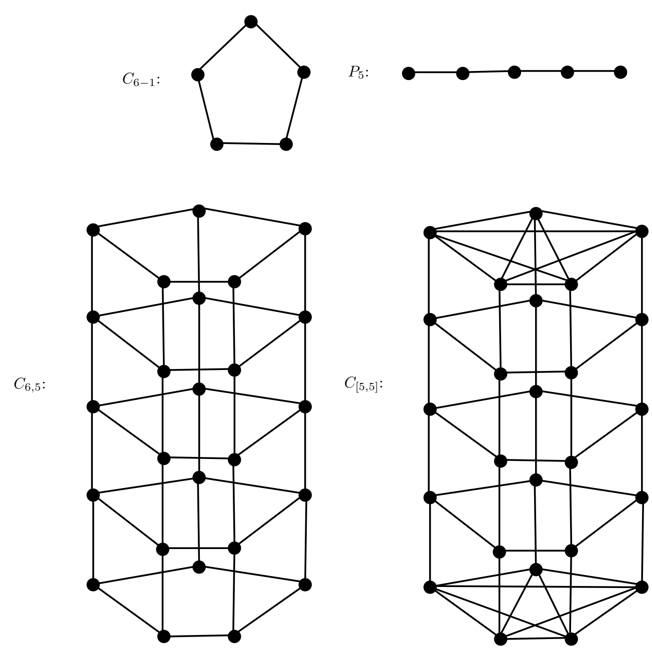

Figure 1. Graphs C_{6-1}, P_5 and its polygonal cylinder $C_{6,5}$ and closed polygonal cylinder $C_{[5,5]}$.

3. Main Results

In this section, we have obtained degree based connectivity indices of the polygonal cylinder and closed polygonal cylinder of a graph.

Now we computed degree based connectivity indices of the polygonal cylinder of a graph.

Theorem 3.1. *The first Zagreb index of polygonal cylinder of a graph G is*

$$M_1(C_{m,n}) = 2(m-1)(8n-7).$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G. By algebraic method, we get $|V(G)| = n(m-1)$ and $|E(G)| = (m-1)(2n-1)$. We have two partitions of the vertex set $V(G)$ as follows:
$V_3 = \{v \in V(G) : d_G(v) = 3\}; |V_3| = 2(m-1)$, and
$V_4 = \{v \in V(G) : d_G(v) = 4\}; |V_4| = (n-2)(m-1)$.
Also we have three partitions of the edge set $E(G)$ as follows:
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}; |E_6| = 2(m-1)$,
$E_7 = \{uv \in E(G) : d_G(u) = 3 \textit{ and } d_G(v) = 4\}$;
$|E_7| = 2(m-1)$, and
$E_8 = \{uv \in E(G) : d_G(u) = 4 \textit{ and } d_G(v) = 4\}$;
$|E_8| = (m-1)(2n-5)$.

Now

$$\begin{aligned} M_1(G) &= \sum_{v\in V(G)} d_G(v)^2 \\ &= \sum_{v\in V_3} 3^2 + \sum_{v\in V_4} 4^2 \\ &= 9\times 2(m-1) + 16\times(m-1)(n-2) \\ &= 18(m-1) + 16(m-1)(n-2) \\ &= 2(m-1)(8n-7). \end{aligned}$$

□

Theorem 3.2. *The second Zagreb index of polygonal cylinder of a graph G is*

$$M_2(C_{m,n}) = 2(m-1)(16n-19).$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} M_2(G) &= \sum_{uv\in E(G)} d_G(u)d_G(v) \\ &= \sum_{uv\in E_6} 3\times 3 + \sum_{uv\in E_7} 3\times 4 + \sum_{uv\in E_8} 4\times 4 \\ &= 9\times 2(m-1) + 12\times 2(m-1) \\ &\quad +16\times(m-1)(2n-5) \\ &= 18(m-1) + 24(m-1) + 16(m-1)(2n-5) \\ &= 2(m-1)(16n-19). \end{aligned}$$

□

Theorem 3.3. *The F-index of polygonal cylinder of a graph G is*

$$F(C_{m,n}) = 2(m-1)(32n-37).$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} F(G) &= \sum_{v\in V(G)} d_G(v)^3 \\ &= \sum_{v\in V_3} 3^3 + \sum_{v\in V_4} 4^3 \\ &= 27\times 2(m-1) + 64\times(m-1)(n-2) \\ &= 54(m-1) + 64(m-1)(n-2) \\ &= 2(m-1)(32n-37). \end{aligned}$$

□

Theorem 3.4. *The first hyper Zagreb index of polygonal cylinder of a graph G is*

$$HM_1(C_{m,n}) = 2(m-1)(64n-75).$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} HM_1(G) &= \sum_{uv\in E(G)} [d_G(u)+d_G(v)]^2 \\ &= \sum_{uv\in E_6} [3+3]^2 + \sum_{uv\in E_7} [3+4]^2 + \sum_{uv\in E_8} [4+4]^2 \\ &= 36\times 2(m-1) + 49\times 2(m-1) \\ &\quad +64\times(m-1)(2n-5) \\ &= 72(m-1) + 98(m-1) + 64(m-1)(2n-5) \\ &= 2(m-1)(64n-75). \end{aligned}$$

□

Theorem 3.5. *The second hyper Zagreb index of polygonal cylinder of a graph G is*

$$HM_2(C_{m,n}) = 2(m-1)(256n-415).$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} HM_2(G) &= \sum_{uv\in E(G)} [d_G(u)d_G(v)]^2 \\ &= \sum_{uv\in E_6} [3\times 3]^2 + \sum_{uv\in E_7} [3\times 4]^2 + \sum_{uv\in E_8} [4\times 4]^2 \\ &= 81\times 2(m-1) + 144\times 2(m-1) \\ &\quad +256\times(m-1)(2n-5) \\ &= 162(m-1) + 288(m-1) + 256(m-1)(2n-5) \\ &= 2(m-1)(256n-415). \end{aligned}$$

□

Theorem 3.6. *The harmonic index of polygonal cylinder of a graph G is*

$$H(C_{m,n}) = \frac{(m-1)(42n-1)}{84}.$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} H(G) &= \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)} \\ &= \sum_{uv\in E_6} \frac{2}{3+3} + \sum_{uv\in E_7} \frac{2}{3+4} + \sum_{uv\in E_8} \frac{2}{4+4} \\ &= \frac{2}{6}\times 2(m-1) + \frac{2}{7}\times 2(m-1) \\ &\quad + \frac{2}{8}\times (m-1)(2n-5) \\ &= (m-1)\left[\frac{4}{6}+\frac{4}{7}+\frac{2}{8}(2n-5)\right] \\ &= \frac{(m-1)(42n-1)}{84}. \end{aligned}$$

□

Theorem 3.7. *The Randić index of polygonal cylinder of a graph G is*

$$R(C_{m,n}) = \frac{(m-1)(6n+4\sqrt{3}-7)}{12}.$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} R(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} \\ &= \sum_{uv\in E_6} \frac{1}{\sqrt{3\times 3}} + \sum_{uv\in E_7} \frac{1}{\sqrt{3\times 4}} + \sum_{uv\in E_8} \frac{1}{\sqrt{4\times 4}} \\ &= \frac{1}{3}\times 2(m-1) + \frac{1}{2\sqrt{3}}\times 2(m-1) \\ &\quad + \frac{1}{4}\times (m-1)(2n-5) \\ &= (m-1)\left[\frac{8+4\sqrt{3}+6n-15}{12}\right] \\ &= \frac{(m-1)(6n+4\sqrt{3}-7)}{12}. \end{aligned}$$

□

Theorem 3.8. *The sum connectivity index of polygonal cylinder of a graph G is*

$$X(C_{m,n}) = \frac{(m-1)(42n-1)}{2\sqrt{21}}.$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} X(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}} \\ &= \sum_{uv\in E_6} \frac{1}{\sqrt{3+3}} + \sum_{uv\in E_7} \frac{1}{\sqrt{3+4}} + \sum_{uv\in E_8} \frac{1}{\sqrt{4+4}} \\ &= \frac{1}{\sqrt{6}}\times 2(m-1) + \frac{1}{\sqrt{7}}\times 2(m-1) \\ &\quad + \frac{1}{\sqrt{8}}\times (m-1)(2n-5) \\ &= (m-1)\left[\frac{208+84n-210}{\sqrt{336}}\right] \\ &= \frac{(m-1)(42n-1)}{2\sqrt{21}}. \end{aligned}$$

□

Theorem 3.9. *The atom-bond connectivity index of polygonal cylinder of a graph G is*

$$ABC(C_{m,n}) = \frac{(m-1)(16+2\sqrt{15}+6\sqrt{6}n-15\sqrt{6})}{12}.$$

Proof. Let $G = C_{m,n}$, where $C_{m,n}$ is a polygonal cylinder of a graph G.

Now

$$\begin{aligned} ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\ &= \sum_{uv\in E_6} \sqrt{\frac{3+3-2}{3\times 3}} + \sum_{uv\in E_7} \sqrt{\frac{3+4-2}{3\times 4}} \\ &\quad + \sum_{uv\in E_8} \sqrt{\frac{4+4-2}{4\times 4}} \\ &= \frac{2}{3}\times 2(m-1) + \sqrt{\frac{5}{12}}\times 2(m-1) \\ &\quad + \frac{\sqrt{6}}{4}\times (m-1)(2n-5) \\ &= (m-1)\left[\frac{16+2\sqrt{15}+\sqrt{6}(6n-15)}{12}\right] \\ &= \frac{(m-1)(16+2\sqrt{15}+6\sqrt{6}n-15\sqrt{6})}{12}. \end{aligned}$$

□

Next, we have computed degree based connectivity indices of the closed polygonal cylinder of a graph G.

Theorem 3.10. *The first Zagreb index of closed polygonal cylinder of a graph G is*

$$M_1(C_{[m,n]}) = 2m(m^2+8n-16).$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G. By algebraic method, we get $|V(G)| = mn$ and $|E(G)| = m(m-4)+2mn$. We have two partitions of the vertex set $V(G)$ as follows:
$V_4 = \{v \in V(G) : d_G(v) = 4\}; |V_4| = m(n-2)$, and
$V_m = \{v \in V(G) : d_G(v) = m\}; |V_m| = 2m$.
Also we have three partitions of the edge set $E(G)$ as follows:
$E_8 = \{uv \in E(G) : d_G(u) = d_G(v) = 4\}; |E_8| = 2mn - 5m$,
$E_{4+m} = \{uv \in E(G) : d_G(u) = 4 \text{ and } d_G(v) = m\}$;
$|E_{4+m}| = 2m$, and
$E_{2m} = \{uv \in E(G) : d_G(u) = d_G(v) = m\}; |E_{2m}| = m(m-1)$.

Now

$$\begin{aligned} M_1(G) &= \sum_{v\in V(G)} d_G(v)^2 \\ &= \sum_{v\in V_4} 4^2 + \sum_{v\in V_m} m^2 \\ &= 16 \times m(n-2) + m^2 \times 2m \\ &= 16m(n-2) + 2m^3 \\ &= 2m(m^2+8n-16). \end{aligned}$$

□

Theorem 3.11. *The second Zagreb index of closed polygonal cylinder of a graph G is*

$$M_2(C_{[m,n]}) = m^4 - m^3 + 8m^2 - 80m + 32mn.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned} M_2(G) &= \sum_{uv\in E(G)} d_G(u)d_G(v) \\ &= \sum_{uv\in E_8} 4\times 4 + \sum_{uv\in E_{4+m}} 4\times m + \sum_{uv\in E_{2m}} m\times m \\ &= 16\times(2mn-5m) + 4m\times 2m + m^2\times m(m-1) \\ &= 32mn - 80m + 8m^2 + m^4 - m^3 \\ &= m^4 - m^3 + 8m^2 - 80m + 32mn. \end{aligned}$$

□

Theorem 3.12. *The F-index of closed polygonal cylinder of a graph G is*

$$F(C_{[m,n]}) = 2m(m^3+32n-64).$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned} F(G) &= \sum_{v\in V(G)} d_G(v)^3 \\ &= \sum_{v\in V_4} 4^3 + \sum_{v\in V_m} m^3 \\ &= 64\times m(n-2) + m^3\times 2m \\ &= 64m(n-2) + 2m^4 \\ &= 2m(m^3+32n-64). \end{aligned}$$

□

Theorem 3.13. *The first hyper Zagreb index of closed polygonal cylinder of a graph G is*

$$HM_1(C_{[m,n]}) = 4m^4 - 2m^3 + 16m^2 - 288m + 128mn.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned} HM_1(G) &= \sum_{uv\in E(G)} [d_G(u)+d_G(v)]^2 \\ &= \sum_{uv\in E_8} [4+4]^2 + \sum_{uv\in E_{4+m}} [4+m]^2 + \sum_{uv\in E_{2m}} [m+m]^2 \\ &= 64\times(2mn-5m) + (4+m)^2\times 2m \\ &\quad +4m^2\times m(m-1) \\ &= 128mn - 320m + 32m + 2m^3 + 16m^2 + 4m^4 - 4m^3 \\ &= 4m^4 - 2m^3 + 16m^2 - 288m + 128mn. \end{aligned}$$

□

Theorem 3.14. *The second hyper Zagreb index of closed polygonal cylinder of a graph G is*

$$HM_2(C_{[m,n]}) = m^6 - m^5 + 32m^3 - 1280m + 512mn.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned} HM_2(G) &= \sum_{uv\in E(G)} [d_G(u)d_G(v)]^2 \\ &= \sum_{uv\in E_8} [4\times 4]^2 + \sum_{uv\in E_{4+m}} [4\times m]^2 + \sum_{uv\in E_{2m}} [m\times m]^2 \\ &= 256\times(2mn-5m) + 16m^2\times 2m \\ &\quad +m^4\times m(m-1) \\ &= 512mn - 1280m + 32m^3 + m^6 - m^5 \\ &= m^6 - m^5 + 32m^3 - 1280m + 512mn. \end{aligned}$$

□

Theorem 3.15. *The harmonic index of closed polygonal cylinder of a graph G is*

$$H(C_{[m,n]}) = \frac{2m^2n - m^2 + 8mn + 8m - 16}{4m+16}.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned}
H(G) &= \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)} \\
&= \sum_{uv\in E_8} \frac{2}{4+4} + \sum_{uv\in E_{4+m}} \frac{2}{4+m} + \sum_{uv\in E_{2m}} \frac{2}{m+m} \\
&= \frac{1}{4}\times(2mn-5m) + \frac{2}{4+m}\times 2m + \frac{1}{m}\times m(m-1) \\
&= \frac{2m^3n - m^3 + 8m^2n + 8m^2 - 16m}{4m(m+4)} \\
&= \frac{2m^2n - m^2 + 8mn + 8m - 16}{4m+16}.
\end{aligned}$$

□

Theorem 3.16. *The Randić index of closed polygonal cylinder of a graph G is*

$$R(C_{[m,n]}) = \frac{m(2n-1)+4(\sqrt{m}-1)}{4}.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned}
R(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} \\
&= \sum_{uv\in E_8} \frac{1}{\sqrt{4\times 4}} + \sum_{uv\in E_{4+m}} \frac{1}{\sqrt{4\times m}} + \sum_{uv\in E_{2m}} \frac{1}{\sqrt{m\times m}} \\
&= \frac{1}{4}\times(2mn-5m) + \frac{1}{2\sqrt{m}}\times 2m + \frac{1}{m}\times m(m-1) \\
&= \frac{2mn + 4\sqrt{m} - m - 4}{4} \\
&= \frac{m(2n-1)+4(\sqrt{m}-1)}{4}.
\end{aligned}$$

□

Theorem 3.17. *The sum connectivity index of closed polygonal cylinder of a graph G is*
$X(C_{[m,n]}) = \frac{2m(n+\sqrt{m})\sqrt{m+4}-\sqrt{m^2+4m}(2+5\sqrt{m})}{2\sqrt{2m+8}} + \frac{2m\sqrt{2}}{\sqrt{2m+8}}.$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned}
X(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}} \\
&= \sum_{uv\in E_8} \frac{1}{\sqrt{4+4}} + \sum_{uv\in E_{4+m}} \frac{1}{\sqrt{4+m}} + \sum_{uv\in E_{2m}} \frac{1}{\sqrt{m+m}} \\
&= \frac{1}{2\sqrt{2}}\times(2mn-5m) + \frac{1}{\sqrt{4+m}}\times 2m \\
&\quad + \frac{1}{\sqrt{2m}}\times m(m-1) \\
&= \frac{\sqrt{m+4}\Big(2m(n+\sqrt{m}) - \sqrt{m}(2+5\sqrt{m})\Big) + 4m\sqrt{2}}{2\sqrt{2m+8}} \\
&= \frac{2m(n+\sqrt{m})\sqrt{m+4} - \sqrt{m^2+4m}(2+5\sqrt{m})}{2\sqrt{2m+8}} \\
&\quad + \frac{2m\sqrt{2}}{\sqrt{2m+8}}.
\end{aligned}$$

□

Theorem 3.18. *The atom-bond connectivity index of closed polygonal cylinder of a graph G is*

$$ABC(C_{[m,n]}) = \frac{\sqrt{6}m(2n-5)}{4} + (m-1)\sqrt{2m-2} + \sqrt{m^2+2m}.$$

Proof. Let $G = C_{[m,n]}$, where $C_{[m,n]}$ is a closed polygonal cylinder of a graph G.

Now

$$\begin{aligned}
ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= \sum_{uv\in E_8} \sqrt{\frac{4+4-2}{4\times 4}} + \sum_{uv\in E_{(4+m)}} \sqrt{\frac{4+m-2}{4\times m}} \\
&\quad + \sum_{uv\in E_{2m}} \sqrt{\frac{m+m-2}{m\times m}} \\
&= \frac{\sqrt{6}}{4}\times(2mn-5m) + \sqrt{\frac{m+2}{4m}}\times 2m \\
&\quad + \sqrt{\frac{2m-2}{m^2}}\times m(m-1) \\
&= \frac{\sqrt{6}(2mn-5m)+4\sqrt{m(m+2)}}{4} \\
&\quad + (m-1)\sqrt{2(m-1)} \\
&= \frac{\sqrt{6}m(2n-5)}{4} + (m-1)\sqrt{2m-2} \\
&\quad + \sqrt{m^2+2m}.
\end{aligned}$$

□

4. Conclusion

In this paper, we can give explicit formulae for first and second Zagreb, F-index, first and second hyper-Zagreb, harmonic, Randić, sum-connectivity and atom-bond connectivity indices of polygonal cylinder and closed polygonal cylinder of a graph.

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