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Single component Darcy-Benard surface tension driven convection of couple stress fluid in a composite layer

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Abstract

The onset of surface tension driven convection is analyzed in a composite system comprising, an incompressible couple stress fluid-saturated porous layer over which lies a layer of the same fluid. The lower surface of the porous layer and upper surface of fluid layer are free and at the interface of the system normal velocity, tangential velocity, normal stress and tangential stress are assumed to be continuous. An eigenvalue problem is solved exactly and an analytical expression for the thermal Marangoni number is obtained for adiabatic-adiabatic and adiabatic-isothermal thermal boundary conditions. The effect of variation of different dimensionless parameters such as couple stress parameters, thermal diffusivity ratio, viscosity ratio, porosity and wave number on the onset of Marangoni convection is investigated, as a function of depth ratio.

Keywords

Couple stress fluid, Surface tension driven convection, Composite layer, Isothermal and Adiabatic thermal boundaries.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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1. Introduction

Investigation on convection driven by buoyancy force in couple stress fluids has been studied extensively over the decades because of its different features and its wide range of applications in engineering techniques and industries like solidification of liquid crystals, extraction of crude oil in the petroleum industry and exotic lubrication, etc. Bio-Fluids such as human blood, animal blood, synthetic fluids and rheological fluids are few examples for couple stress fluids. Convection driven by surface tension in fluids is called Marangoni Convection. Surface tension monotonically decreases with a rise in temperature thus the onset of convection is faster. Modelling of flow problems through the composite layer has great practical and theoretical significance. Primarily it consists of a fluid layer (Region -1) and a saturated porous layer (Region- 2) the combination of both the regions has extensive applications in engineering, crystal growth industries, medical technology, flame spreading over a pool of liquid fuel and in petroleum reservoirs, etc. Flame spread is powered by the convective flows in the liquid-phase. This flow is mainly governed by surface tension, viscous forces and partially by gravity. In Petroleum reservoir, oil is recovered from underground from the porous oil-bearing rocks. Insulation of high temperatures in porous oil-bearing rocks reduces the surface tension of the crude oil and thus quickly flows into the drilled well making the extraction of oil from the well smoother.

Pearson [1958] described about formation of convection cells due to surface tension in a fluid layer. Nield [1965, 77, 98] studied about the effect of buoyancy and surface tension in the formation of convection cells and on the onset of convection in a fluid layer overlying a layer of a porous medium. He has also analyzed the effect of surface tension on the onset of natural convection in a saturated porous medium. Straughan [2001] has studied surface tension driven convection in a fluid overlying a porous layer. Rudraiah *et al.* [1998] analyzed on the effect of Brinkman boundary layer on the onset of Marangoni convection in a fluid saturated porous layer. The theory of couple stress was first introduced by Stokes [1966] which explains the classical theory of polar effect in the presence of couple stress, body couple, and non-symmetric tensors. The effects of couple stress in a liquid have no microstructure therefore, the kinematic energy of spin density and angular momentum are not considered and the governing equations of couple stress are thereby determined completely by the velocity field and hence the equation is similar to the Navier Stokes equation. Sharma and Shivani [2001] investigated on couple stress fluid heated from below in a porous medium and found that the couple stress in fluid delays the onset of convection but the permeability of the porous medium quickens the onset of convection. Sunil *et al.* [2002] have studied the global stability for thermal convection in a couple-stress fluid heated from below and found couple-stress fluids are thermally more stable than the ordinary viscous fluids. Malashetty [2011] has studied the onset of convection in couple stress fluid-saturated porous layers by using a thermal non-equilibrium model. Srinivasacharya *et al.* [2011] studied the steady flow of incompressible couple stress fluid flow between parallel porous plates maintained at constant but different temperatures with the assumption that there is a constant suction at upper plate and a constant injection at the lower plate. Waqar Khan *et al.* [2014] investigated exact solutions of the couple stress fluid motion for different flow situations. Taslim et al. [1989] studied on thermal stability of horizontally superposed porous and fluid layers. Cieszko *et al.* [1999] studied on derivation of matching conditions at the surface between fluid saturated porous solid and bulk fluid. Shivakumara *et al.* [2006] studied the onset of surface-tension-driven convection in a two-layer system comprising an incompressible fluid-saturated porous layer over which lies a layer of the same fluid. At the interface of the system both Beavers-Joseph and the Jones slip conditions are considered. Sumithra *et al.* [2018] have investigated on single component Marangoni convection in a composite layer where both the lower and upper boundaries are adiabatic and isothermal. Sumithra R and Shyamala V [2020] investigated on Darcy-Benard Marangoni convection in a composite Layer comprising of couple stress fluid. It is observed that the effect of couple stress parameter for fluid layer is prominent, for porous layer dominant composite system.

In this article, the objective is to evaluate the problem of single component Darcy-Benard surface tension driven convection in a composite layer consisting of couple stress fluid, bounded by free – free boundaries. The eigenvalue of surface tension driven convection is solved by using the exact method. Thermal Marangoni numbers are obtained for two types of thermal boundaries, type (a): Adiabatic - Adiabatic, both the boundaries of the composite layer are adiabatic, type (b):Adiabatic-Isothermal, the upper boundary of the fluid layer is adiabatic and the lower boundary of the porous layer is isothermal. The upper free surface contributes to the onset of Marangoni convection for both the thermal boundary conditions.

2. Nomenclature

$\overrightarrow{q}(u,v,w)$ - velocity vector
P- Pressure
ρ_0 - Reference density
t - Time
T- Temperature
K - Permeability of the porous medium
ϕ- Porosity of the porous media
σ_t- Surface tension
C_p - Specific heat
μ- Co-efficient viscosity of fluid
κ & κ_m - Thermal diffusivity of the fluid and porous layers respectively
μ_m- Effective viscosity of the fluid in the porous layer
μ' & μ'_m - Couple stress viscosity of the fluid in fluid and porous layers respectively
A_h-Ratio of heat capacities $\left(\frac{(\rho_0 C_p)_m}{(\rho C_p)_f}\right)$
a & a_m - Non-dimensional horizontal wave numbers fluid and porous layers respectively
n & n_m - Frequencies of fluid and porous layers respectively
d & d_m - height of the fluid and porous layers respectively
Λ_f - Couple stress parameter for fluid layer $(\mu'/\mu d^2)$
Λ_m - Couple stress parameter for porous layer $(\mu'_m/\mu d_m^2)$
θ & θ_m- Dimensionlass temperature
W & W_m -Dimensionless vertical velocity
$\widehat{T}$- Thermal diffusivity ratio (κ_m/κ)
$\widehat{\mu}$ - Viscosity ratio (μ_m/μ)
$\widehat{d}$ - Depth ratio (d_m/d)
f- refers to fluid layer
m- refers to porous layer
b- refers to basic state

3. Mathematical formulation

Consider an infinite horizontal layer of a couple stress fluid in a composite layer. A composite layer has two regions: region-1 consists of clear couple stress fluid and region-2 consists of couple stress fluid saturated porous layer.

The depth of region-1 is $0 \le z \le d$ and region-2 is $-d_m \le z_m \le 0$ and also at the interface $z = 0 = z_m$. A Cartesian coordinate system is taken at the interface of the porous and fluid layer as X-axis and Z- axis vertically upwards. Let the temperature gradient $\Delta T = T_u - T_l$ where T_u is the temperature

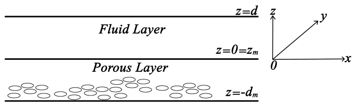

Figure 1. Physical Configuration

at $z = d$ and T_l is the temperature at $z_m = -d_m$.

The governing equations of fluid and porous layers are respectively.

$$\nabla.\vec{q} = 0 \quad (3.1)$$

$$\rho_0\,[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}] = -\nabla P + \mu\nabla^2\vec{q} - \mu^{'}\nabla^4\vec{q} \quad (3.2)$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T \quad (3.3)$$

$$\nabla_m.\vec{q_m} = 0 \quad (3.4)$$

$$\rho_0\,[\frac{1}{\phi}\frac{\partial \vec{q_m}}{\partial t_m} + \frac{1}{\phi^2}(\vec{q_m}.\nabla_m)\vec{q_m}] = -\nabla_m P_m - \frac{\mu}{K}\vec{q_m} + \frac{\mu_m^{'}}{K}\nabla_m^2\vec{q_m} \quad (3.5)$$

$$Ah\frac{\partial T_m}{\partial t_m} + (\vec{q_m}.\nabla m)T_m = \kappa_m\nabla_m^2 T_m \quad (3.6)$$

The basic state solution of the composite system is obtained for the quiescent flow where temperature and pressure are functions of z only. To investigate the stability of the basic solution an infinitesimal disturbances are introduced to quiescent state. On substituting the basic and infinitesimal disturbances in the governing equations respectively and also taking curl twice to the resulting equations, the pressure terms are eliminated from the momentum equations of the fluid and porous layers. The vertical component of the composite system is retained. The resulting variables are non-dimensionalized by using the following scales for fluid and porous layers separately.

$$(x,y,z) = d(x^*,y^*,z^*), \quad (x_m,y_m,z_m) = d_m(x_m^*,y_m^*,z_m^*)$$

$$(u,v,w) = \frac{\kappa}{d}(u^*,v^*,w^*), \quad (u_m,v_m,w_m) = \frac{\kappa_m}{d_m}(u_m^*,v_m^*,w_m^*)$$

$$\theta = (T_0 - T_u)\theta^*, \quad \theta_m = (T_0 - T_u)\theta_m^* \quad \nabla = \frac{\nabla^*}{d} \quad \nabla_m = \frac{\nabla_m^*}{d_m}$$

$$t = \frac{d^2}{\kappa}t^*, \quad t_m = \frac{d_m^2}{\kappa_m}t_m^*, \quad P = \frac{\mu\kappa}{d^2}P^*, \quad P_m = \frac{\mu_m\kappa_m}{d_m^2}P_m^*$$

The resulting non-dimensionalized equations are subjected to normal mode expansion on dependent variables in the fluid and porous layers respectively as,

$$\begin{bmatrix} W \\ \theta \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \end{bmatrix} f(x,y)e^{nt} \quad (3.7)$$

$$\begin{bmatrix} W_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \theta_m(z_m) \end{bmatrix} f_m(x_m,y_m)e^{n_m t_m} \quad (3.8)$$

where $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$. Considering the dimensional horizontal wave numbers are same for the fluid and porous layers, we have $\dfrac{a}{d} = \dfrac{a_m}{d_m}$ and hence $a_m = \widehat{d}a$. The Ordinary differential equations obtained considering that the onset of convection only in the form of steady convection with the frequencies $n = n_m = 0$ are

In $0 \le z \le 1$

$$(\Lambda_f(D^2 - a^2) - 1)(D^2 - a^2)^2 W = 0 \quad (3.9)$$

$$(D^2 - a^2)\theta + W = 0 \quad (3.10)$$

In $-1 \le z_m \le 0$

$$(D_m^2 - a_m^2)(1 - \Lambda_m(D_m^2 - a_m^2))W_m = 0 \quad (3.11)$$

$$(D_m^2 - a_m^2)\theta_m + W_m = 0 \quad (3.12)$$

4. Boundary Conditions

Boundary conditions are chosen in composite system based on its physical configuration that is, boundary conditions on upper fluid layer, lower porous layer and at the interface. The interface boundary conditions have a great effect on the prediction of convective stability in a composite layer. The interface effect also determines the flow pattern, temperature distributions and heat transfer rates. Equations (3.9) and (3.11) are to be solved subjected to the following appropriate velocity boundary conditions:

Velocity boundary Conditions:

$$\begin{aligned}
&D^2W(1) + Ma^2\theta(1) = 0 \\
&W(1) = 0, \quad D^3W(1) - 3a^2DW(1) = 0, \\
&\widehat{T}W(0) = W_m(0), \quad \widehat{T}\widehat{d}DW(0) = \widehat{\mu}D_mW_m(0) \\
&\widehat{T}\widehat{d}^2(D^2 + a^2)W(0) = \widehat{\mu}(D_m^2 + a_m^2)W_m(0) \\
&\widehat{T}\widehat{d}^3\beta^2(D^3W(0) - 3a^2DW(0)) = -D_mW_m(0) \\
&\quad + \widehat{\mu}\beta^2(D_m^3W_m(0) - 3a_m^2D_mW_m(0)) \\
&W_m(-1) = 0, \quad D_m^2W_m(-1) = 0 \\
&D_m^3W(-1) - 3a_m^2D_mW_m(-1) = 0 \qquad (4.1)
\end{aligned}$$

Where,

$$M = \frac{-(T_0 - T_u)d}{\mu K}\frac{\partial\sigma}{\partial T} \textit{ is the thermal Marangoni number}$$

Temperature boundary Conditions:

Type (a) A-A, Adiabatic-Adiabatic temperature boundary conditions:

$$D\theta(1)=0; \qquad \theta(0)=\widehat{T}\theta_m(0);$$
$$D\theta(0)=D_m\theta_m(0); \qquad D_m\theta_m(-1)=0 \tag{4.2}$$

Type (b) A-I, Adiabatic-Isothermal temperature boundary conditions:

$$D\theta(1)=0; \qquad \theta(0)=\widehat{T}\theta_m(0);$$
$$D\theta(0)=D_m\theta_m(0); \qquad \theta_m(-1)=0 \tag{4.3}$$

5. Exact method of solution

From equations (3.9) and (3.11), the vertical velocity distributions W and W_m are determined as,

$$W(z)=[a_1\cosh(az)+a_2\sinh(az)+a_3z\cosh(az)+ a_4z\sinh(az)+a_5\cosh(\delta z)+a_6\sinh(\delta z)] \tag{5.1}$$

$$W_m(z_m)=[a_{m1}cosh(a_mz_m)+a_{m2}sinh(a_mz_m)+ a_{m3}cosh(\delta_mz_m)+a_{m4}sinh(\delta_mz_m)] \tag{5.2}$$

where,
a_i $(i=1,2,3,4,5,6)$ and a_{mi} $(i=1,2,3,4)$ are arbitrary constants and $\delta=\sqrt{a^2+\frac{1}{\Lambda_f}}$, $\delta_m=\sqrt{a_m^2+\frac{1}{\Lambda_m}}$.

The expressions for $W(z)$ and $W_m(z_m)$ are appropriately written as,

$$W(z)=a_1[\cosh(az)+A_1\sinh(az)+A_2z\cosh(az)+ A_3z\sinh(az)+A_4\cosh(\delta z)+A_5\sinh(\delta z)] \tag{5.3}$$

$$W_m(z_m)=a_1[A_{m1}\cosh(a_mz_m)+A_{m2}\sinh(a_mz_m) +A_{m3}\cosh(\delta_mz_m)+A_{m4}\sinh(\delta_mz_m)] \tag{5.4}$$

Where $A_1,A_2,A_3,A_4,A_5,A_{m1},A_{m2},A_{m3},A_{m4}$ are constants determined by solving the boundary conditions (4.1) and they are as follows:

$$A_{m4}=\frac{-H_5}{\nabla_9+\nabla_{10}}, \qquad A_{m3}=\tanh(\delta_m)A_{m4}$$

$$A_{m2}=\nabla_{11}A_{m3}+\nabla_{12}A_{m4}$$

$$A_{m1}=\frac{\sinh(a_m)A_{m2}-\cosh(\delta_m)A_{m3}+\sinh(\delta_m)A_{m4}}{\cosh(a_m)}$$

$$A_5=E_1A_{m1}+E_2A_{m2}+E_3A_{m3}+E_4A_{m4}+E_5$$

$$A_4=\frac{A_{m1}+A_{m3}}{\widehat{T}}-1$$

$$A_3=B_1A_{m1}+B_2A_{m3}+B_3$$

$$A_2=F_1A_{m1}+F_2A_{m2}+F_3A_{m3}+F_4A_{m4}+F_5$$

$$A_1=G_1A_{m1}+G_2A_{m2}+G_3A_{m3}+G_4A_{m4}+G_5$$

where,

$$B_1=\frac{2\widehat{\mu}{a_m}^2}{2a\widehat{T}\widehat{d}^2}-\frac{(a^2+\delta^2)}{2a\widehat{T}}$$

$$B_2=\frac{\widehat{\mu}(\alpha^2+{a_m}^2)}{2a\widehat{T}\widehat{d}^2}-\frac{(a^2+\delta^2)}{2a\widehat{T}}, \quad B_3=\frac{(\delta^2-a^2)}{2a}$$

$$C_1=2a^3B_1\cosh(a)-\frac{(\delta^3-3a^2\delta)}{T}\sinh(\delta)$$

$$C_2=2a^3B_2\cosh(a)-\frac{(\delta^3-3a^2\delta)}{T}\sinh(\delta)$$

$$C_3=\left[2a^3B_3\cosh(a)+2a^3\sinh(a)+(\delta^3-3a^2\delta)\sinh(\delta)\right]$$

$$D_1=-B_1\sinh(a)-\frac{\cosh(\delta)}{\widehat{T}}$$

$$D_2=-B_2\sinh(a)-\frac{\cosh(\delta)}{\widehat{T}}$$

$$D_3=-B_3\sinh(a)+\cosh(\delta)-\cosh(a)$$

$$\nabla_1=2a^2\cosh(a)-2a^3\sinh(a)$$

$$\nabla_2=2a^2\delta\cosh(a)+(\delta^3-3a^2\delta)\cosh(\delta)$$

$$\nabla_3=\frac{2a^2\widehat{\mu}a_m\cosh(a)}{\widehat{T}\widehat{d}} \qquad \nabla_4=\frac{2a^2\widehat{\mu}\delta_m\cosh(a)}{\widehat{T}\widehat{d}}$$

$$\nabla_5=\left[(\delta^3-3a^2\delta)\sinh(a)\cosh(\delta)+2a^3\cosh(a)\sinh(\delta)\right]$$

$$\nabla_6=C_1\sinh(a)+2D_1a^3\cosh(a)$$

$$\nabla_7=C_2\sinh(a)+2D_2a^3\cosh(a)$$

$$\nabla_8=C_3\sinh(a)+2D_3a^3\cosh(a)$$

$$\nabla_9=(\nabla_{11}\tanh(\delta_m)+\nabla_{12})(H_1\tanh(a_m)+H_2)$$

$$\nabla_{10}=H_3\tanh(\delta_m)+H_4, \qquad \nabla_{11}=\frac{\nabla_{21}-\nabla_{22}}{2{a_m}^3}$$

$$\nabla_{21}=(3{a_m}^2\delta_m-{\delta_m}^3)\sinh(\delta_m)\cosh(a_m)$$

$$\nabla_{22}=2{a_m}^3\sinh(a_m)\cosh(\delta_m)$$

$$\nabla_{12}=\frac{-\nabla_{23}-\nabla_{24}}{2{a_m}^3}$$

$$\nabla_{23}=(3{a_m}^2\delta_m-{\delta_m}^3)\cosh(a_m)\cosh(\delta_m)$$

$\nabla_{24} = 2{a_m}^3 \sinh(a_m)\sinh(\delta_m)$

$$E_1 = \frac{\nabla_1\nabla_6 - 2a^3C_1}{\nabla_1\nabla_5 - 2a^3\nabla_2}, \quad E_2 = \frac{-2a^3\nabla_3}{\nabla_1\nabla_5 - 2a^3\nabla_2}$$
$$E_3 = \frac{\nabla_1\nabla_7 - 2a^3C_2}{\nabla_1\nabla_5 - 2a^3\nabla_2}, \quad E_4 = \frac{-2a^3\nabla_4}{\nabla_1\nabla_5 - 2a^3\nabla_2}$$
$$E_5 = \frac{\nabla_1\nabla_8 - 2a^3C_3}{\nabla_1\nabla_5 - 2a^3\nabla_2}, \quad F_1 = \frac{C_1 - E_1\nabla_2}{\nabla_1}$$
$$F_2 = \frac{\nabla_2 - E_2\nabla_2}{\nabla_1}, \quad F_3 = \frac{C_2 - E_3\nabla_2}{\nabla_1}$$
$$F_4 = \frac{\nabla_4 - E_4\nabla_2}{\nabla_1}, \quad F_5 = \frac{C_3 - E_5\nabla_2}{\nabla_1}$$
$$G_1 = \frac{-F_1 - E_1\delta}{a}, \quad G_2 = \frac{\widehat{\mu} a_m}{\widehat{T}\widehat{d}a} - \frac{(F_2 + \delta E_2)}{a}$$
$$G_3 = \frac{-F_3 - E_3\delta}{a}, \quad G_4 = \frac{\widehat{\mu}\delta_m}{\widehat{T}\widehat{d}a} - \frac{(F_4 + \delta E_4)}{a}$$
$$G_5 = \frac{-F_5 - E_5\delta}{a}$$
$$H_1 = \widehat{T}\widehat{d^3}\beta^2[-2a^3G_1 + E_1(\delta^3 - 3a^2\delta)]$$
$$H_2 = \left[\widehat{T}\widehat{d^3}\beta^2[-2a^3G_2 + E_2(\delta^3 - 3a^2\delta)] + 2\widehat{\mu}\beta^2{a_m}^3 + a_m\right]$$
$$H_3 = \widehat{T}\widehat{d^3}\beta^2[-2a^3G_3 + E_3(\delta^3 - 3a^2\delta)]$$
$$H_4 = \widehat{T}\widehat{d^3}\beta^2[-2a^3G_4 + E_4(\delta^3 - 3a^2\delta)] - ({\delta_m}^3 - 3{a_m}^2\delta_m)]\widehat{\mu}\beta^2 + \delta_m$$
$$H_5 = \widehat{T}\widehat{d^3}\beta^2[-2a^3G_5 + E_5(\delta^3 - 3a^2\delta)]$$

The temperature distributions are obtained by solving the equations (3.10) and (3.12) and are as follows

$$\theta(z) = a_1[A_6\cosh(az) + A_7\sinh(az) - h(z)] \quad (5.5)$$
$$\theta_m(z_m) = a_1[A_{m5}\cosh(a_m z_m) + A_{m6}\sinh(a_m z_m) - h_m(z_m)] \quad (5.6)$$

Where

$$h(z) = \frac{z\sinh(az)}{2a} + A_1\frac{z\cosh(az)}{2a} + \frac{A_2}{4a}\left[z^2\sinh(az) + \frac{z\cosh(az)}{a}\right] + A_4\frac{\cosh(\delta z)}{\delta^2 - a^2} + \frac{A_3}{4a}\left[z^2\cosh(az) - \frac{z\sinh(az)}{a}\right] + A_5\frac{\sinh(\delta z)}{\delta^2 - a^2}$$

$$h_m(z_m) = A_{m1}\frac{z_m\sinh(a_m z_m)}{2a_m} + A_{m2}\frac{z_m\cosh(a_m z_m)}{2a_m} + A_{m3}\frac{\cosh(\delta_m z_m)}{((\delta_m)^2 - {a_m}^2)} + A_{m4}\frac{\sinh(\delta_m z_m)}{((\delta_m)^2 - {a_m}^2)}$$

These temperature distributions are solved with respect to two types of thermal boundary conditions.

Thermal Marangoni numbers

Thermal Marangoni numbers M_1 and M_2 respectively for type(a) and type(b) are obtained by taking the corresponding temperature distributions and using the boundary condition involving both the velocity and temperature of (4.1) and are found to be,

$$M_1 = \frac{-(N_1 + N_2 + N_3 + N_4)}{a^2(N_5 + N_6 + N_7 + N_8)} \quad (5.7)$$
$$M_2 = \frac{-(N_1 + N_2 + N_3 + N_4)}{a^2(N_9 + N_{10} + N_7 + N_8)} \quad (5.8)$$

where,

$$N_1 = \cosh(a)[a^2 + a^2A_2 + 2aA_3]$$
$$N_2 = \sinh(a)[a^2A_1 + 2aA_2 + a^2A_3]$$
$$N_3 = \delta^2\cosh(\delta)A_4, \quad N_4 = \delta^2\sinh(\delta)A_5$$
$$N_5 = \cosh(a)\left[A_6 - \frac{A_1}{2a} + \frac{A_2}{4a^2} - \frac{A_3}{4a}\right]$$
$$N_6 = \sinh(a)\left[A_7 - \frac{1}{2a} - \frac{A_2}{4a} + \frac{A_3}{4a^2}\right]$$
$$N_7 = \frac{-\cosh(\delta)}{(\delta^2 - a^2)}A_4, \quad N_8 = \frac{-\sinh(\delta)}{(\delta^2 - a^2)}A_5$$
$$N_9 = \cosh(a)\left[A_8 - \frac{A_1}{2a} + \frac{A_2}{4a^2} - \frac{A_3}{4a}\right]$$
$$N_{10} = \sinh(a)\left[A_9 - \frac{1}{2a} - \frac{A_2}{4a} + \frac{A_3}{4a^2}\right]$$
$$A_6 = \frac{I_5}{I_6}, \quad A_7 = \frac{I_1 - a\sinh(a)A_6}{a\cosh(a)}$$
$$A_8 = \frac{I_1 - a\cosh(a)A_9}{a\sinh(a)}, \quad A_9 = \frac{I_8}{I_9}$$
$$I_1 = \cosh(a)\left[\frac{1}{2} + \frac{A_1}{2a} - \frac{(a^2-1)}{4a^2}A_2 + \frac{A_3}{4a}\right] + \sinh(a)\left[\frac{1}{2a} + \frac{A_1}{2} + \frac{A_2}{4a} - \frac{(a^2-1)}{4a^2}A_3\right] + \frac{\delta\sinh(\delta)}{(\delta^2 - a^2)}A_4 + \frac{\delta\cosh(\delta)}{(\delta^2 - a^2)}A_5$$
$$I_2 = \frac{\widehat{T}A_4}{\delta^2 - a^2} - \frac{A_{m3}}{{\delta_m}^2 - {a_m}^2}$$
$$I_3 = \left[\frac{A_1}{2a} - \frac{A_2}{4a^2} + \frac{\delta A_5}{(\delta^2 - a^2)} - \frac{A_{m2}}{2a_m} - \frac{\delta_m A_{m4}}{((\delta_m)^2 - {a_m}^2)}\right]$$

$$I_4 = \sinh a_m \left[\frac{A_{m2}}{2} - \frac{A_{m1}}{2a_m}\right] - \cosh a_m \left[\frac{A_{m1}}{2} - \frac{A_{m2}}{2a_m}\right] - \frac{\delta_m \sinh(\delta_m)}{((\delta_m)^2 - a_m{}^2)} A_{m3} + \frac{\delta_m \cosh(\delta_m)}{((\delta_m)^2 - a_m{}^2)} A_{m4}$$

$$I_5 = \cosh(a_m) I_1 + a_m \sinh(a_m)\cosh(a) I_2 - \cosh(a)\cosh(a_m) I_3 - \cosh(a) I_4$$

$$I_6 = a\sinh(a) cosh(a_m) + \widehat{T} a_m \cosh(a)\sinh(a_m)$$

$$I_7 = \sinh(a_m)\left[\frac{A_{m1}}{2a_m}\right] - \cosh(a_m)\left[\frac{A_{m2}}{2a_m}\right] + \frac{\cosh(\delta_m)}{((\delta_m)^2 - a_m{}^2)} A_{m3} - \frac{\sinh(\delta_m)}{(\delta_m{}^2 - a_m{}^2)} A_{m4}$$

$$I_8 = \sinh(a_m) I_1 + a_m \cosh(a)\cosh(a_m) I_2 - \cosh(a)\sinh(a_m) I_3 + a_m \cosh(a) I_7$$

$$I_9 = a\sinh(a) sinh(a_m) + \widehat{T} a_m \cosh(a)\cosh(a_m)$$

6. Results and Discussions

The onset of surface tension driven convection in a two layer system consisting of a fluid layer overlying a porous layer saturated by couple stress fluid is investigated theoretically. The eigenvalue problem is solved by exact method and an analytical expression for the Marangoni number is obtained for two types of temperature boundary conditions, viz. (i) lower free surface of the porous layer and upper free surface of the fluid layer are adiabatic, and (ii) lower free surface of the porous layer is isothermal and upper free surface of the fluid layer is adiabatic. Thermal Marangoni number compares the rate at which the thermal energy is transported by flow to the rate at which thermal energy diffuses. The thermal Marangoni numbers M_1 for Type(a) and M_2 for Type(b) are presented and compared in the following figures.

The thermal Marangoni numbers which are obtained as functions of depth ratio for the fixed values of $a = 0.5$, $\widehat{T} = 1.0$, $\beta = 0.01$, $\widehat{\mu} = 1.0$, $\Lambda_f = 1$ and $\Lambda_m = 1$. The depth ratio which is ratio of porous and fluid layer thickness has a significant role on the flow in the composite layer system. The smaller values of depth ratio mean the system is fluid layer dominant composite layer and larger values of depth ratio mean the system is porous layer dominant composite layer, and here dominance is in terms of thickness of the layers. In the figure the line curve represents M_1 for thermal Marangoni number for adiabatic-adiabatic thermal boundary condition and dashed curve represents M_2 for thermal Marangoni number for isothermal-adiabatic thermal boundary condition.

The curves in figure (2) shows the evolution of M_1 and M_2 with depth ratios for three values of couple stresses in fluid layer $\Lambda_f = 0.01$, 0.03 and 0.05 when $a = 0.5$, $\widehat{T} = 1$,

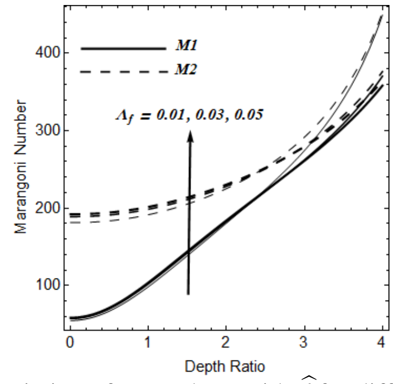

Figure 2. Variation of M_1 and M_2 with $\widehat{d}$ for different values of Λ_f

$\beta = 0.01$, $\widehat{\mu} = 1$ and $\Lambda_m = 0.1$. It is observed that increase in $\widehat{d}$ increases the value of thermal Marangoni number M_1 and M_2. Also starting value of thermal Marangoni number for lower depth ratio is different for both the cases $M_1 < M_2$ and the curves merge for $\widehat{d} > 3$ and thereafter $M_1 = M_2$ for the corresponding values of Λ_f in each of the cases. Also, with increase in couple stresses in fluid layer increases Marangoni number in both cases and this implies the variation in couple stresses enhances the onset of convection and reinforce the stability of the system.

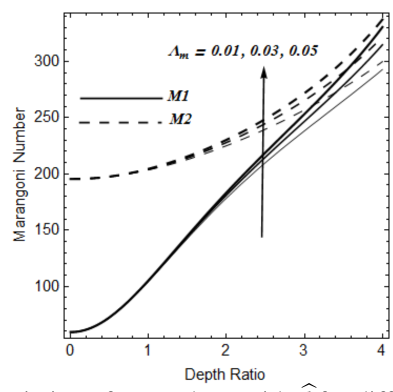

Figure 3. Variation of M_1 and M_2 with $\widehat{d}$ for different values of Λ_m

The curves in figure (3) show the evolution of M_1 and M_2 with depth ratios for three values of couple stress in porous layer $\Lambda_m = 0.01$, 0.03 and 0.05 when $a = 0.5$, $\widehat{T} = 1$, $\beta = 0.01$, $\widehat{\mu} = 1$ and $\Lambda_f = 0.1$. It is observed that increase in $\widehat{d}$ increases the value of thermal Marangoni number M_1 and M_2. Also starting value of thermal Marangoni number for lower depth ratio is different for both the types $M_1 < M_2$ and the curves merge for the depth ratio $\widehat{d} < 2.5$ with further increase in $\widehat{d}$ the curves branch out and thereafter $M_1 = M_2$ for the corresponding values of Λ_m . Also, with increase in couple stresses in porous layer increases thermal Marangoni number in both types and that is variation in couple stresses enhances the onset of convection, thus the system is stable for both the types of thermal boundary conditions.

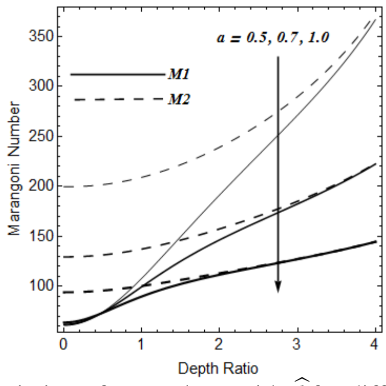

Figure 4. Variation of M_1 and M_2 with $\widehat{d}$ for different values of a

The curves in figure (4) show the evolution of M_1 and M_2 with depth ratio for three values of wave number $a = 0.5$, 0.7 and 1.0 when $\Lambda_m = 1$, $\Lambda_f = 1$, $\widehat{T} = 1$, $\beta = 0.01$ and $\widehat{\mu} = 1$. It is observed that increase in $\widehat{d}$ increases the value of thermal Marangoni numbers M_1 and M_2 . It is seen that the curves are merged for $\widehat{d} < 1$ in M_1, implies that the size of the convection cells are same in fluid dominated composite layer. And with $\widehat{d} >> 1$ the curves are branched out and M_1 is highest for lowest value of a . In the case of adiabatic- isothermal thermal boundary condition the starting value at lower depth ratio is with greater variation. Also it is observed the curves merge at a critical depth ratio for each value of a in both the cases respectively. Hence $M_1 = M_2$ after the critical depth ratio implies the system is most stable when a is the highest as the size of convection cells are smaller also a gradual increase in Marangoni number for both the types. The size of convection cells reduce with increase in the value of a. Thus the size of convection cells reduce in porous layer dominated composite system with increase in a but the size of convection cells increase in fluid dominated composite system for lower values of a.

The curves in figure (5) show the evolution of M_1 and M_2 with depth ratios for three values of viscosity ratio $\widehat{\mu} = 1.0$, 1.5 and 2.0 when $\Lambda_m = 1$, $\Lambda_f = 1$, $a = 0.5$, $\widehat{T} = 1$ and $\beta = 0.01$. It is observed that Marangoni number increases with increase in viscosity ratio also the curves are merged in both the cases for $\widehat{d} < 3$. The curves branch out and merge with the each other as the depth ratio increases when $\widehat{d} > 3$ for each values of $\widehat{\mu}$ respectively in both the types. The onset of convection is delayed by increase in viscosity ratio and hence the system is stabilized.

The curves in figure (6) show the evolution of M_1 and M_2 with depth ratios for three values of porous parameter $\beta = 0.001$, 0.003 and 0.005 when $\Lambda_m = 1$, $\Lambda_f = 1$, $a = 0.5$, $\widehat{T} = 1.0$ and $\widehat{\mu} = 1.0$. It is seen that the curves are merged for both the cases up to $\widehat{d} < 2.5$ and after which the curves

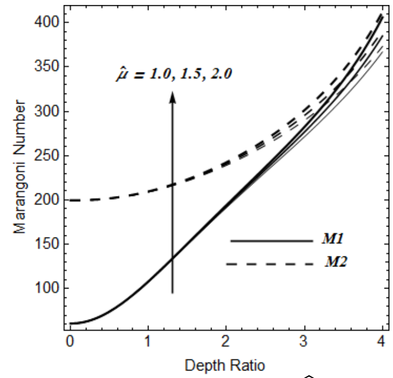

Figure 5. Variation of M_1 and M_2 with $\widehat{d}$ for different values of μ

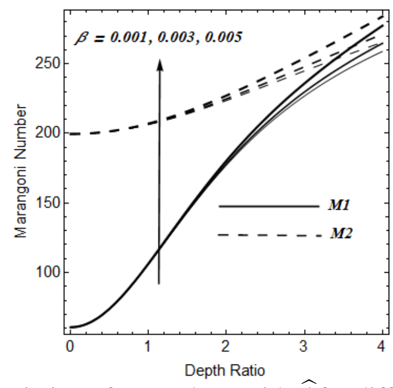

Figure 6. Variation of M_1 and M_2 with $\widehat{d}$ for different values of β

branch out. With increase in β increases Marangoni number for both the cases thus making the system stable. The onset of convection is faster in a porous medium.

The curves in figure (7) show the evolution of M_1 and M_2 with depth ratios for three values of thermal diffusivity ratio $\widehat{T} = 0.8$, 0.9 and 1.0 when $\Lambda_m = 1$, $\Lambda_f = 1$, $a = 0.5$, $\beta = 0.01$ and $\widehat{\mu} = 1.0$. It is observed that Marangoni number increases with increase in thermal diffusivity ratio up to a critical depth ratio and later decreases for higher value vice-versa. The starting points of curves for both M_1 and M_2 is different but merges for $\widehat{d} > 4$ implies the Marangoni number is same for both the types with further increase in depth ratio. The onset of convection is delays for higher values of diffusivity ratio and hence stabilizes the system.

From the figures envisaged, it is evident that it is possible to control the Marangoni convection effectively in a composite system by appropriately choosing the values of depth ratio, couple stress parameter, permeability, thermal diffusivity ratio, viscosity ratio and wave number.

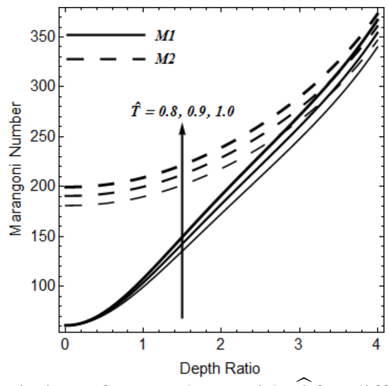

Figure 7. Variation of M_1 and M_2 with $\widehat{d}$ for different values of $\widehat{T}$

7. Conclusion

Darcy-Benard surface tension driven convection in a composite system saturated with couple stress fluid is studied analytically. The following are the results can be drawn from the above plotted graphs:

1. For all non-dimensional parameters variation the thermal Marangoni numbers $M_1 > M_2$ for up to a critical depth ratio $\widehat{d}$ and as $\widehat{d}$ increases the difference in M_1 and M_2 decreases.

2. Marangoni number is equal after a critical depth ratio for both the thermal boundary conditions.

3. Increase in couple stress parameter values in both the layers increases thermal marangoni number that is the presence of couple stresses is to reinforce stability on the composite system.

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