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A general neighborhood redefined Zagreb index on Boron Nanotubes

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Abstract

Mathematical chemistry has widened the scope of research by giving potential applications in almost every field. The topological index is a part of mathematical chemistry, where the chemical structure is modelled as a graph and a numerical invariant can be determined. This numeric quantity explains the chemical and bio-activities of the chemical structure for further research analysis. In this work, the general neighborhood redefined Zagreb index is defined and computed for boron triangular nanotubes and boron-α nanotubes. Nine degree-based indices viz., first Zagreb, second Zagreb, hyper Zagreb, first NDe, second NDe, third NDe, fourth NDe, redefined first Zagreb and redefined second Zagreb indices are derived from the general neighborhood redefined Zagreb index obtained for the said chemical structure.

Keywords

Topological indices, $NReZ_{(\beta,\gamma)}$-index, boron triangular nanotube, boron-α nanotube.

AMS Subject Classification

05C07, 05C10

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Contents

1. Introduction

Chemical graph theory is a study of chemical compounds with simple graph, multiple edges and finite graphs. The graph G consist of vertices and edges such that vertices represents the atoms and the edges represents the link connecting the vertices. The expansion of research in mathematical chemistry has resulted because of its potential applications in chemistry, pharmacy as drug design is dependent on the numerical invariant, called the topological index. The chemical graph theory provides the study of characteristics of chemical compounds[8, 19, 20, 23]. The mathematical chemistry provides several topological indices used in QSAR/QSPR studies [4, 5, 15, 17]. There are various kinds of topological indices that are used to determine the characteristics of chemical compounds. The numerical invariant that is determined in the study of mathematical chemistry is used to check the toxicity of the chemical compound. This study is referred as QSTR(Quantitative Structure Toxicity Relationship).

Consider a graph $G=(V(G),E(G))$, where $V(G)$ represents vertices and $E(G)$ represents edges of G. The $d_G(b)$ is the degree of a vertex, is the combined sum of vertices and edges depending on the sum of neighborhood degree. The group of all vertices close to b are often called as open neighborhood of b $(N_G(b))$. The closed neighborhood of b is the set $N_G[b]=N_G(a)\cup\{b\}$. The set $N_G[b]$ is the set of closed neighborhood vertices of b. Let $D_G(b)=d_G(b)+\sum_{a\in N_G(b)} d_G(b)$ be the degree sum of closed neighborhood vertices of b [12, 22].

Adnan Aslam et al., in [1], H. Ali et al., in [3] and S. Akther et al., in [2] discussed some topological indices for chemical structures such as boron triangular nanotubes and benzenoid structures. Recently Jia-Bao Liu et al., in [11] obtained the expressions for topological aspects of Boron Nanotubes. Motivated by the work on nanostructures, we define a novel general neighborhood redefined Zagreb index($NReZ_{(\beta,\gamma)}$) and compute the values of neighborhood viz., first Zagreb, second Zagreb, hyper Zagreb, first NDe, second NDe, third NDe, fourth NDe, redefined first Zagreb and redefined second Zagreb indices using $NReZ_{(\beta,\gamma)}$ for nanostructures such as boron triangular and boron-α nanotubes.

Definition 1.1. *Sourav Mondal et al., [13] introduced neighborhood version of first Zagreb index, second Zagreb index, first NDe index, second NDe index, third NDe index, fourth NDe index, harmonic index, hyper Zagreb index and third NDe index and are defined as*

$$NM_1(G)=\sum_{a\in V(G)} S_G(a)^2=\sum_{ab\in E(G)} S_G(a)+S_G(b)$$

$$NM_2(G)=\sum_{ab\in E(G)} S_G(a)\times S_G(b)$$

$$ND_1(G)=\sum_{ab\in E(G)} \sqrt{S_G(a)\times S_G(b)}$$

$$ND_2(G)=\sum_{ab\in E(G)} \frac{1}{\sqrt{S_G(a)+S_G(b)}}$$

$$ND_3(G)=\sum_{ab\in E(G)} [S_G(a)\times S_G(b)][S_G(a)+S_G(b)]$$

$$ND_4(G)=\sum_{ab\in E(G)} \frac{1}{\sqrt{S_G(a)\times S_G(b)}}$$

$$HM_N(G)=\sum_{ab\in E(G)} [S_G(a)+S_G(b)]^2$$

Definition 1.2. *Shanmukha et al., [18] introduced neighborhood version of the redefined first and second Zagreb indices and are defined as*

$$NReZ_1(G)=\sum_{ab\in E(G)} \frac{[S_G(a)+S_G(b)]}{[S_G(a)\times S_G(b)]}$$

$$NReZ_2(G)=\sum_{ab\in E(G)} \frac{[S_G(a)\times S_G(b)]}{[S_G(a)+S_G(b)]}$$

Motivated by the work in the paper entitled "Beyond the Zagreb indices" [9] and [7, 10, 14] on Zagreb indices, defined a degree based general neighborhood redefined Zagreb index $NReZ_{(\beta,\gamma)}$. It is stated as,

$$NReZ_{(\beta,\gamma)}(G)=\sum_{ab\in E(G)} (S_G(a)\times S_G(b))^{\beta}\times(S_G(a)+S_G(b))^{\gamma}$$

where $\beta,\gamma\in R$.
The below Table 1 depicts the relationships of $NReZ_{(\beta,\gamma)}$-index with other topological indices defined.

Table 1. Relationships between $NReZ_{(\beta,\gamma)}$-index and other topological indices.

Topological index	$NReZ_{(\beta,\gamma)}-index$
Neighborhood first Zagreb index	$NReZ_{(0,1)}(G)$
Neighborhood second Zagreb index	$NReZ_{(1,0)}(G)$
Neighborhood Hyper Zagreb index	$NReZ_{(0,2)}(G)$
First NDe index	$NReZ_{(\frac{1}{2},0)}(G)$
Second NDe index	$NReZ_{(0,\frac{1}{2})}(G)$
Third NDe index	$NReZ_{(1,1)}(G)$
Fourth NDe index	$NReZ_{(-\frac{1}{2},0)}(G)$
Neighborhood redefined first Zagreb index	$NReZ_{(-1,1)}(G)$
Neighborhood redefined second Zagreb index	$NReZ_{(1,-1)}(G)$

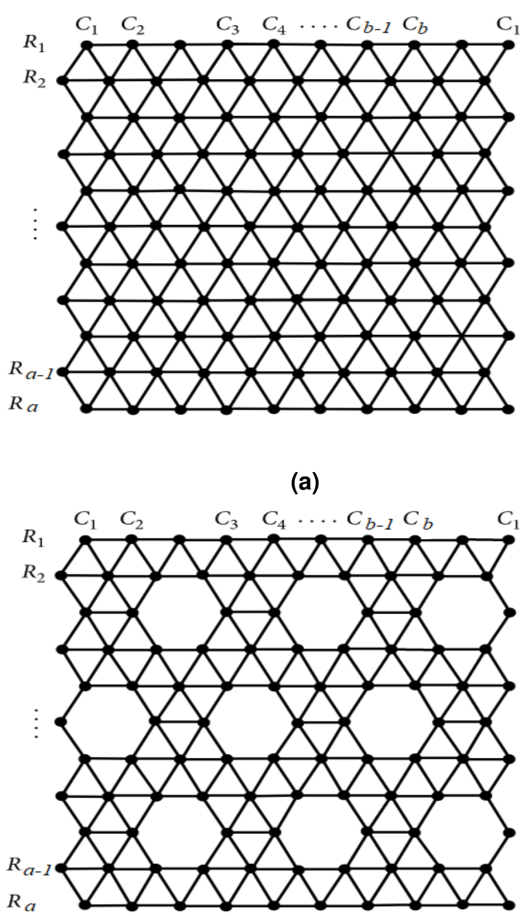
2. Materials and Methods

Our results consist of degree based topological indices of two nanostructures. First, the general neighborhood redefined Zagreb index is defined, then by substituting some specific values to the parameters β and γ, neighborhood degree based topological indices are calculated. To calculate the results combinatorial computing, vertex and edge partition, degree counting methods and also graph tools are used.

3. Boron Nanotubes

Boron nanotubes were discovered experimentally in the mid-'90s. Boron nanotubes are similar to that of carbon nanotubes structurally, except for carbon atoms that are alternately replaced by nitrogen and boron atoms. The properties of both

differ, boron is an electrical insulator whereas the carbon nanotube may be metallic or semiconductor. The layered boron is thermally and chemically stable than that of carbon nanotubes. Because of its unique physical and chemical properties, boron has a very wide range of commercial and scientific applications. Boron nanotubes have wide applications in polymers, metals, ceramics because of its properties like stiffness and chemical stability. Also it is used in treatment of cancer.
A $2D-$ sheet of boron triangular nanotube is obtained by placing an atom in the middle of the hexagon and joining all the vertices of the hexagon with the newly added vertex. It looks like an array of triangles that are placed linearly row and column-wise which is illustrated in Figure 1(a). Boron-α nanotube is a mixture of hexagons and triangles as illustrated in Figure 1(b).



(b)

Figure 1. $2D$ sheet of (a) boron triangular nanotube $BT[a,b]$ (b) boron-α nanotube $BA[a,b]$.

4. Main Results

The molecular graphs of boron triangular and boron-α nanotubes are denoted by $BT[a,b]$ and $BA[a,b]$ respectively as shown in Figure 1. Here a represents the cardinality of rows where as b represents the cardinality of columns in a $2D$ sheet of $BT[a,b]$ and $BA[a,b]$. We categorize the Boron-α nanotubes into two classes with respect to a. These classes are denoted by $BA(X)[a,b]$ and $BA(Y)[a,b]$ for $a \equiv 2\ mod\ 3$ and $a \equiv 0\ mod\ 3$, respectively. The order and size of $BT[a,b]$, $BA[a,b]$ are given in Table 2.

Table 2. The vertex and edge partitions of boron nanotubes.

Molecular graph	*Order*	*Size*
$BT[a,b]$	$\frac{3ab}{2}$	$\frac{3b(3a-2)}{2}$
$BA(X)[a,b]$	$\frac{b(4a+1)}{3}$	$\frac{b(7a-2)}{2}$
$BA(Y)[a,b]$	$\frac{4ab}{3}$	$\frac{b(7a-4)}{2}$

4.1 Boron Triangular Nanotube $BT[a,b]$

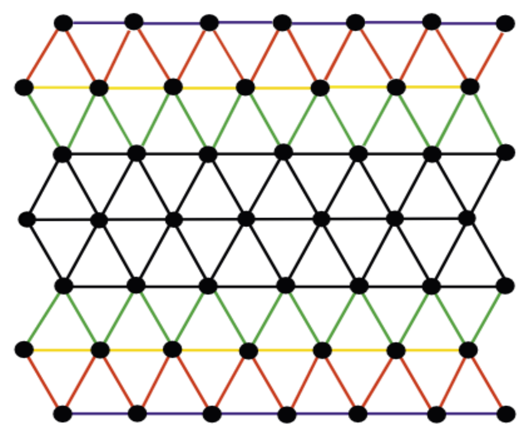


Figure 2. The Boron triangular nanotube BT[7,4].

From the Figure 2, the Boron triangular nanotube BT[a,b] has five different types of edges, whose edge partitions with respect to degree sum of neighbor vertices are depicted in Table 3.

Table 3. The edge partitions of $BT[7,4]$ with respect to degree sum of neighbor vertices.

$(S_G(a),\ S_G(b)),\ ab \in E(G)$	*No. of Edges*
$E_1(20,20)$	$3b$
$E_2(20,32)$	$6b$
$E_3(32,32)$	$3b$
$E_4(32,36)$	$6b$
$E_5(36,36)$	$\frac{3b(3a-14)}{2}$

Theorem 4.1. *The $NReZ_{(\beta,\gamma)}$-index of Boron triangular nanotube $BT[a,b]$ for $a \geqslant 3$ and b is even then*

$$\begin{aligned} NReZ_{(\beta,\gamma)}(BT[a,b]) &= 3b[(400)^{\beta} \times (40)^{\gamma}] \\ &+ 6b[(640)^{\beta} \times (52)^{\gamma}] + 3b[(1024)^{\beta} \times (64)^{\gamma}] \\ &+ 6b[(1152)^{\beta} \times (68)^{\gamma}] + \frac{3b(3a-14)}{2}[(1296)^{\beta} \times (72)^{\gamma}] \end{aligned}$$

Proof. The Table 3 gives the tabulated data of the number and types of edges for the structure $BT[a,b]$. Considering all the types of edges and its count in the definition of the general neighborhood redefined Zagreb index, the result is proved.

$$\begin{aligned}
&NReZ_{(\beta,\gamma)}(G) = \\
&\quad \sum_{ab\in E(G)} (S_G(a)\times S_G(b))^{\beta} \times (S_G(a)+S_G(b))^{\gamma} \\
&= \sum_{ab\in E_1(BT[a,b])} (20\times 20)^{\beta} \times (20+20)^{\gamma} \\
&\quad + \sum_{ab\in E_2(BT[a,b])} (20\times 32)^{\beta} \times (20+32)^{\gamma} \\
&\quad + \sum_{ab\in E_3(BT[a,b])} (32\times 32)^{\beta} \times (32+32)^{\gamma} \\
&\quad + \sum_{ab\in E_4(BT[a,b])} (32\times 36)^{\beta} \times (32+36)^{\gamma} \\
&\quad + \sum_{ab\in E_5(BT[a,b])} (36\times 36)^{\beta} \times (36+36)^{\gamma}
\end{aligned}$$

$$\begin{aligned}
&NReZ_{(\beta,\gamma)}(BT[a,b]) = 3b[(400)^{\beta} \times (40)^{\gamma}] \\
&+6b[(640)^{\beta} \times (52)^{\gamma}] + 3b[(1024)^{\beta} \times (64)^{\gamma}] \\
&+6b[(1152)^{\beta} \times (68)^{\gamma}] + \frac{3b(3a-14)}{2}[(1296)^{\beta} \times (72)^{\gamma}].
\end{aligned}$$

□

Using the general neighborhood redefined Zagreb index, the other topological indices are calculated as shown below. For β and γ of general neighborhood redefined Zagreb index, values are given as per the Table 1. For the set of each values of β and γ, different topological indices are calculated using the result of theorem 4.1.

Corollary 4.1.1. *Let $G \cong BT[a,b]$ be the boron triangular nanotube, for $a \geqslant 3$ and b is even, then*

a. $NM_1(G) = 324ab - 480b$,

b. $NM_2(G) = 5832ab - 12192b$,

c. $HM_N(G) = 23328ab - 47808b$,

d. $ND_1(G) = 162ab - \frac{61141}{250}b$,

e. $ND_2(G) = \frac{4773}{125}ab - \frac{42473}{1000}b$,

f. $ND_3(G) = 419904ab - 1045248b$,

g. $ND_4(G) = \frac{1}{8}ab + \frac{37}{500}b$,

h. $NReZ_1(G) = \frac{1}{4}ab + \frac{325}{2000}b$,

i. $NReZ_2(G) = 81ab - \frac{124507}{1000}b$.

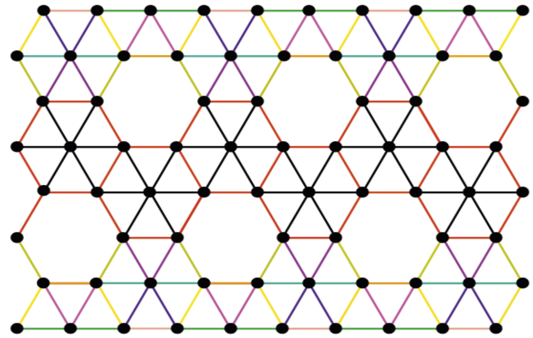


Figure 3. The Boron-α nantube BA(X)[8,6].

4.2 Boron-α Nanotube $BA(X)[a,b]$

From the Figure 3, the Boron-α nanotube $BA(X)[a,b]$ has eleven different types of edges, whose edge partitions with respect to degree sum of neighbor vertices are depicted in Table 4.

Table 4. The edge partitions of BA(X)[8,6] with respect to degree sum of neighbor vertices.

$(S_G(a),\ S_G(b)),\ ab \in E(G)$	*No. of Edges*
$E_1(18,19)$	$2b$
$E_2(18,24)$	$2b$
$E_3(19,19)$	b
$E_4(19,24)$	$2b$
$E_5(19,28)$	$2b$
$E_6(24,24)$	b
$E_7(24,27)$	$2b$
$E_8(24,28)$	$2b$
$E_9(27,27)$	$\frac{b(3a-14)}{2}$
$E_{10}(27,28)$	$2b$
$E_{11}(27,30)$	$2b(a-5)$

Theorem 4.2. *The $NReZ_{(\beta,\gamma)}$-index of Boron-α nanotube $BA(X)[a,b]$ for $a \geqslant 3$ and b is even then*

$$\begin{aligned}
&NReZ_{(\beta,\gamma)}(BA(X)[a,b]) = 2b[(342)^{\beta} \times (37)^{\gamma}] \\
&+2b[(432)^{\beta} \times (42)^{\gamma}] + b[(361)^{\beta} \times (38)^{\gamma}] \\
&+2b[(456)^{\beta} \times (43)^{\gamma}] + 2b[(532)^{\beta} \times (47)^{\gamma}] \\
&+b[(576)^{\beta} \times (48)^{\gamma}] + 2b[(648)^{\beta} \times (51)^{\gamma}] \\
&+2b[(672)^{\beta} \times (52)^{\gamma}] + \frac{b(3a-14)}{2}[(729)^{\beta} \times (54)^{\gamma}] \\
&+2b[(756)^{\beta} \times (55)^{\gamma}] + 2b(a-5)[(810)^{\beta} \times (57)^{\gamma}].
\end{aligned}$$

Proof. The Table 4 gives the tabulated data of the number and types of edges for the structure $BA(X)[a,b]$. Considering all the types of edges and its count in the definition of the general neighborhood redefined Zagreb index, the result is proved.

$$\begin{aligned}
&NReZ_{(\beta,\gamma)}(G) = \\
&\quad \sum_{ab\in E(G)} (S_G(a)\times S_G(b))^{\beta} \times (S_G(a)+S_G(b))^{\gamma} \\
&\quad = \sum_{ab\in E_1(BA(X)[a,b])} (18\times 19)^{\beta} \times (18+19)^{\gamma} \\
&\quad + \sum_{ab\in E_2(BA(X)[a,b])} (18\times 24)^{\beta} \times (18+24)^{\gamma} \\
&\quad + \sum_{ab\in E_3(BA(X)[a,b])} (19\times 19)^{\beta} \times (19+19)^{\gamma} \\
&\quad + \sum_{ab\in E_4(BA(X)[a,b])} (19\times 24)^{\beta} \times (19+24)^{\gamma} \\
&\quad + \sum_{ab\in E_5(BA(X)[a,b])} (19\times 28)^{\beta} \times (19+28)^{\gamma} \\
&\quad + \sum_{ab\in E_6(BA(X)[a,b])} (24\times 24)^{\beta} \times (24+24)^{\gamma} \\
&\quad + \sum_{ab\in E_7(BA(X)[a,b])} (24\times 27)^{\beta} \times (24+27)^{\gamma} \\
&\quad + \sum_{ab\in E_8(BA(X)[a,b])} (24\times 28)^{\beta} \times (24+28)^{\gamma} \\
&\quad + \sum_{ab\in E_9(BA(X)[a,b])} (27\times 27)^{\beta} \times (27+27)^{\gamma} \\
&\quad + \sum_{ab\in E_{10}(BA(X)[a,b])} (27\times 28)^{\beta} \times (27+28)^{\gamma} \\
&\quad + \sum_{ab\in E_{11}(BA(X)[a,b])} (27\times 30)^{\beta} \times (27+30)^{\gamma} \\
&NReZ_{(\beta,\gamma)}(BA(X)[a,b]) = 2b[(342)^{\beta}\times(37)^{\gamma}] \\
&\quad + 2b[(432)^{\beta}\times(42)^{\gamma}] + b[(361)^{\beta}\times(38)^{\gamma}] \\
&\quad + 2b[(456)^{\beta}\times(43)^{\gamma}] + 2b[(532)^{\beta}\times(47)^{\gamma}] \\
&\quad + b[(576)^{\beta}\times(48)^{\gamma}] + 2b[(648)^{\beta}\times(51)^{\gamma}] \\
&\quad + 2b[(672)^{\beta}\times(52)^{\gamma}] + \frac{b(3a-14)}{2}[(729)^{\beta}\times(54)^{\gamma}] \\
&\quad + 2b[(756)^{\beta}\times(55)^{\gamma}] + 2b(a-5)[(810)^{\beta}\times(57)^{\gamma}].
\end{aligned}$$

□

Using the general neighborhood redefined Zagreb index, the other topological indices are calculated as shown below. For β and γ of general neighborhood redefined Zagreb index, values are given as per the Table 1. For the set of each values of β and γ, different topological indices are calculated using the result of theorem 4.2.

Corollary 4.2.1. *Let $G \cong BA(X)[a,b]$ be the boron-α nanotube, for $a \geqslant 3$ and b is even, then*

a. $NM_1(G) = 195ab - 208b$,

b. $NM_2(G) = \frac{5427ab}{2}ab - 4590b$,

c. $HM_N(G) = 10872ab - 18112b$,

d. $ND_1(G) = \frac{97421}{1000}ab - \frac{5273}{50}b$,

e. $ND_2(G) = \frac{278}{5}ab - 126b$,

f. $ND_3(G) = 151389ab - 325932b$,

g. $ND_4(G) = \frac{629}{5000}ab - \frac{97}{1000}b$,

h. $NReZ_1(G) = \frac{63}{250}ab + \frac{1}{5}b$,

i. $NReZ_2(G) = \frac{48671}{1000}ab - \frac{53451}{1000}b$.

4.3 Boron-α Nanotube $BA(Y)[a,b]$

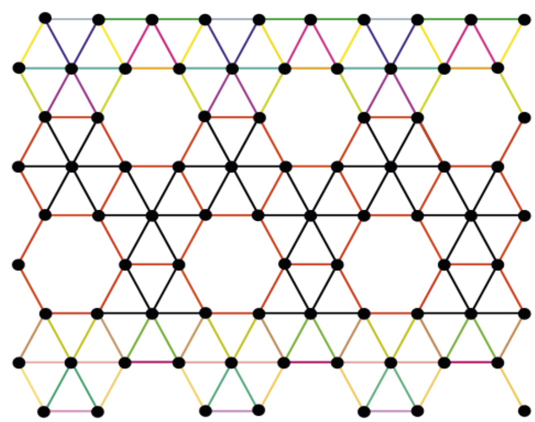


Figure 4. The Boron-α nantube BA(Y)[9,6].

From the Figure 4, the Boron-α nanotube $BA(Y)[a,b]$ has nineteen different types of edges, whose edge partitions with respect to degree sum of neighbor vertices are depicted in Table 5.

Theorem 4.3. *The $NReZ_{(\beta,\gamma)}$-index of Boron-α nanotube $BA(Y)[a,b]$ for $a \geqslant 3$ and b is even then*

$$\begin{aligned}
&NReZ_{(\beta,\gamma)}(BA(Y)[a,b]) = \frac{b}{2}[(196)^{\beta}\times(28)^{\gamma}] \\
&\quad + b[(350)^{\beta}\times(39)^{\gamma}] + b[(364)^{\beta}\times(40)^{\gamma}] \\
&\quad + b[(342)^{\beta}\times(37)^{\gamma}] + b[(432)^{\beta}\times(42)^{\gamma}]
\end{aligned}$$

$$+\frac{b}{2}[(361)^{\beta}\times(38)^{\gamma}]+b[(456)^{\beta}\times(43)^{\gamma}]$$
$$+b[(532)^{\beta}\times(47)^{\gamma}]+\frac{b}{2}[(576)^{\beta}\times(48)^{\gamma}]$$
$$+b[(648)^{\beta}\times(51)^{\gamma}]+b[(672)^{\beta}\times(52)^{\gamma}]$$
$$+\frac{b}{2}[(625)^{\beta}\times(50)^{\gamma}]+b[(650)^{\beta}\times(51)^{\gamma}]$$
$$+b[(675)^{\beta}\times(52)^{\gamma}]+b[(750)^{\beta}\times(55)^{\gamma}]$$
$$+b[(702)^{\beta}\times(53)^{\gamma}]+\frac{b(3a-14)}{2}[(729)^{\beta}\times(54)^{\gamma}]$$
$$+b[(756)^{\beta}\times(55)^{\gamma}]+2b(a-5)[(810)^{\beta}\times(57)^{\gamma}].$$

Table 5. The edge partitions of BA(Y)[9,6] with respect to degree sum of neighbor vertices.

$(S_G(a),\ S_G(b)),\ ab\in E(G)$	$No.\ of\ Edges$
$E_1(14,14)$	$\frac{b}{2}$
$E_2(14,25)$	b
$E_3(14,26)$	b
$E_4(18,19)$	b
$E_5(18,24)$	b
$E_6(19,19)$	$\frac{b}{2}$
$E_7(19,24)$	b
$E_8(19,28)$	b
$E_9(24,24)$	$\frac{b}{2}$
$E_{10}(24,27)$	b
$E_{11}(24,28)$	b
$E_{12}(25,25)$	$\frac{b}{2}$
$E_{13}(25,26)$	b
$E_{14}(25,27)$	b
$E_{15}(25,30)$	b
$E_{16}(26,27)$	b
$E_{17}(27,27)$	$\frac{b(3a-14)}{2}$
$E_{18}(27,28)$	b
$E_{19}(27,30)$	$2b(a-5)$

Proof. The Table 5 gives the tabulated data of the number and types of edges for the structure $BA(Y)[a,b]$. Considering all the types of edges and its count in the definition of the general neighborhood redefined Zagreb index, the result is proved.

$$NReZ_{(\beta,\gamma)}(G)=$$
$$\sum_{ab\in E(G)}(S_G(a)\times S_G(b))^{\beta}\times(S_G(a)+S_G(b))^{\gamma}$$
$$=\sum_{ab\in E_1(BA(Y)[a,b])}(14\times14)^{\beta}\times(14+14)^{\gamma}$$
$$+\sum_{ab\in E_2(BA(Y)[a,b])}(14\times25)^{\beta}\times(14+25)^{\gamma}$$
$$+\sum_{ab\in E_3(BA(Y)[a,b])}(14\times26)^{\beta}\times(14+26)^{\gamma}$$
$$+\sum_{ab\in E_4(BA(Y)[a,b])}(18\times19)^{\beta}\times(18+19)^{\gamma}$$
$$+\sum_{ab\in E_5(BA(Y)[a,b])}(18\times24)^{\beta}\times(18+24)^{\gamma}$$
$$+\sum_{ab\in E_6(BA(Y)[a,b])}(19\times19)^{\beta}\times(19+19)^{\gamma}$$
$$+\sum_{ab\in E_7(BA(Y)[a,b])}(19\times24)^{\beta}\times(19+24)^{\gamma}$$
$$+\sum_{ab\in E_8(BA(Y)[a,b])}(19\times28)^{\beta}\times(19+28)^{\gamma}$$
$$+\sum_{ab\in E_9(BA(Y)[a,b])}(24\times24)^{\beta}\times(24+24)^{\gamma}$$
$$+\sum_{ab\in E_{10}(BA(Y)[a,b])}(24\times27)^{\beta}\times(24+27)^{\gamma}$$
$$+\sum_{ab\in E_{11}(BA(Y)[a,b])}(24\times28)^{\beta}\times(24+28)^{\gamma}$$
$$+\sum_{ab\in E_{12}(BA(Y)[a,b])}(25\times25)^{\beta}\times(25+25)^{\gamma}$$
$$+\sum_{ab\in E_{13}(BA(Y)[a,b])}(25\times26)^{\beta}\times(25+26)^{\gamma}$$
$$+\sum_{ab\in E_{14}(BA(Y)[a,b])}(25\times27)^{\beta}\times(25+27)^{\gamma}$$
$$+\sum_{ab\in E_{15}(BA(Y)[a,b])}(25\times30)^{\beta}\times(25+30)^{\gamma}$$
$$+\sum_{ab\in E_{16}(BA(Y)[a,b])}(26\times27)^{\beta}\times(26+27)^{\gamma}$$
$$+\sum_{ab\in E_{17}(BA(Y)[a,b])}(27\times27)^{\beta}\times(27+27)^{\gamma}$$
$$+\sum_{ab\in E_{18}(BA(Y)[a,b])}(27\times28)^{\beta}\times(27+28)^{\gamma}$$
$$+\sum_{ab\in E_{19}(BA(Y)[a,b])}(27\times30)^{\beta}\times(27+30)^{\gamma}$$
$$NReZ_{(\beta,\gamma)}(BA(Y)[a,b])=\frac{b}{2}[(196)^{\beta}\times(28)^{\gamma}]$$
$$+b[(350)^{\beta}\times(39)^{\gamma}]+b[(364)^{\beta}\times(40)^{\gamma}]$$
$$+b[(342)^{\beta}\times(37)^{\gamma}]+b[(432)^{\beta}\times(42)^{\gamma}]$$
$$+\frac{b}{2}[(361)^{\beta}\times(38)^{\gamma}]+b[(456)^{\beta}\times(43)^{\gamma}]$$
$$+b[(532)^{\beta}\times(47)^{\gamma}]+\frac{b}{2}[(576)^{\beta}\times(48)^{\gamma}]$$
$$+b[(648)^{\beta}\times(51)^{\gamma}]+b[(672)^{\beta}\times(52)^{\gamma}]$$
$$+\frac{b}{2}[(625)^{\beta}\times(50)^{\gamma}]+b[(650)^{\beta}\times(51)^{\gamma}]$$
$$+b[(675)^{\beta}\times(52)^{\gamma}]+b[(750)^{\beta}\times(55)^{\gamma}]$$
$$+b[(702)^{\beta}\times(53)^{\gamma}]+\frac{b(3a-14)}{2}[(729)^{\beta}\times(54)^{\gamma}]$$
$$+b[(756)^{\beta}\times(55)^{\gamma}]+2b(a-5)[(810)^{\beta}\times(57)^{\gamma}].$$

□

Using the general neighborhood redefined Zagreb index, the other topological indices are calculated as shown below. For β and γ of general neighborhood redefined Zagreb index, values are given as per the Table 1. For the set of each values of β and γ, different topological indices are calculated using the result of theorem 4.3.

Corollary 4.3.1. *Let $G \cong BA(Y)[a,b]$ be the boron-α nanotube, for $a \geqslant 3$ and b is even, then*

a. $NM_1(G) = 195ab - 178b$,

b. $NM_2(G) = \frac{5427}{2}ab - 4995b$,

c. $HM_N(G) = 10872ab - 16244b$,

d. $ND_1(G) = \frac{97421}{1000}ab - \frac{12689}{100}b$,

e. $ND_2(G) = \frac{278}{5}ab - \frac{26827}{200}b$,

f. $ND_3(G) = 151389ab - 338312b$,

g. $ND_4(G) = \frac{629}{5000}ab - \frac{283}{5000}b$,

h. $NReZ_1(G) = \frac{63}{250}ab + \frac{63}{500}b$,

i. $NReZ_2(G) = \frac{48671}{1000}ab - \frac{16149}{250}b$.

5. Conclusion

This work is concentrated on two important chemical structures viz., boron nanotubes and boron-α nanotubes for which the nine indices viz., first Zagreb, second Zagreb, hyper Zagreb, first NDe, second NDe, third NDe, fourth NDe, redefined first Zagreb, and redefined second Zagreb indices are derived from the general neighborhood redefined Zagreb index. These results can be used for further studies in chemistry for studying the chemical and biological properties of the compounds.

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