



Connectedness of spherical fuzzy graph

T. Lavanya^{1*} and D. Amsaveni²

Abstract

Connectivity plays a vital role in network theory. The level sets are very useful when analyzing the connections in the network design. Spherical fuzzy graph has an important tool with their quad attributes to deal the uncertainty efficiently. In this study, the connectedness of spherical fuzzy graph are elucidated with examples and some of their properties are demonstrated. Also the level sets for spherical fuzzy graphs are defined and discussed.

Keywords

Spherical fuzzy bridge, (a,b,c)-cut, weak spherical fuzzy graph, partial spherical fuzzy graph, full spherical fuzzy graph.

AMS Subject Classification

03E72, 03F55.

^{1,2}Department of Mathematics, Sri Sarada College for Women, Salem-636016, Tamil Nadu, India.

*Corresponding author: ¹tplaly@gmail.com; ²damsaveni2020@gmail.com

Article History: Received 30 November 2020; Accepted 03 March 2021

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1. Introduction

Graph theory has many application in the real time scenarios and the concepts of connectivity plays an important role in the networking and circuits. The interconnection of fuzzy concepts and graph theoretic approach are very useful for the world with their numerous application. In this connection many authors studied about the fuzzy graph but initially the concept proposed by Kaufmann[2] based on Zadeh's fuzzy relations [3]. Rosenfeld[4] introduced the several basic graph-theoretic concepts such as bridges, cut nodes, connectedness, trees and cycles. The generalization of fuzzy graph is the intuitionistic fuzzy graph initiated by Atanasova[5]. Cen Zuo[7] introduced the picture fuzzy graphs. Ashraf[6] introduced the new set as spherical fuzzy set with application in the decision making with multi-criterian and the spherical fuzzy graph defined by Muhammad Akram[1]. In this paper, the strength of connectedness is defined and the level sets are discussed in the spherical fuzzy graph with some of their properties.

2. Preliminaries

Definition 2.1. [6] Spherical fuzzy set (SFS), S on a universe \mathbb{X} is an object of the form, $P = \{(p, \alpha_P(p), \gamma_P(p), \beta_P(p)) | p \in \mathbb{X}\}$ where, $\alpha_P(p) \in [0, 1]$ is called the "degree of positive membership of p in \mathbb{X} ", $\gamma_P(p) \in [0, 1]$ is called the "degree of neutral membership of p in \mathbb{X} ", $\beta_P(p) \in [0, 1]$ is called the "degree of negative membership of p in \mathbb{X} ", and where $\alpha_P(p), \gamma_P(p), \beta_P(p)$ satisfy the following condition,

$$\forall p \in \mathbb{X}, \alpha_P^2(p) + \gamma_P^2(p) + \beta_P^2(p) \leq 1,$$

Then for $p \in \mathbb{X}$, $\pi_P(p) = \sqrt{1 - \alpha_P^2(p) - \gamma_P^2(p) - \beta_P^2(p)}$ is called the degree of refusal membership of p in \mathbb{X} .

Definition 2.2. [1] Let \mathbb{V} be the non-empty set possess spherical fuzzy graph (SFG) is $\mathbb{G} = (P, Q)$ here P and Q are spherical fuzzy set and spherical fuzzy relation on \mathbb{V} in such

$$\alpha_Q(p, q) \leq \min\{\alpha_P(p), \alpha_P(q)\};$$

$$\gamma_Q(p, q) \leq \min\{\gamma_P(p), \gamma_P(q)\};$$

$$\beta_Q(p, q) \leq \max\{\beta_P(p), \beta_P(q)\}.$$

and $0 \leq \alpha_Q^2(p, q) + \gamma_Q^2(p, q) + \beta_Q^2(p, q) \leq 1 \forall p, q \in \mathbb{V}$. The spherical fuzzy vertex set P is of \mathbb{G} and the spherical edge set Q is of \mathbb{G} . The insubstantial edge is $Q(p, q) = 0 \forall (p, q) \in \mathbb{V} \times \mathbb{V} - \mathbb{E}$. The spherical fuzzy digraph has no symmetry relation in \mathbb{G} .

3. Properties of connectedness in SFG

Definition 3.1. In a SFG, $G = (P, Q)$, a path \wp is defined as sequence of distinct vertices $p_0, p_1, p_2, \dots, p_n$ such that $(\alpha_Q(p_{i-1}, p_i), \gamma_Q(p_{i-1}, p_i), \beta_Q(p_{i-1}, p_i)), i = 1, 2, 3, \dots, l$ where, l represents the length of the path in SFG and it satisfies either one of the condition is

$\alpha_Q(p, q) > 0; \gamma_Q(p, q) > 0; \beta_Q(p, q) > 0$ for some p, q in P
 $\alpha_Q(p, q) > 0; \gamma_Q(p, q) > 0; \beta_Q(p, q) = 0$ for some p, q in P
 $\alpha_Q(p, q) > 0; \gamma_Q(p, q) = 0; \beta_Q(p, q) > 0$ for some p, q in P
 $\alpha_Q(p, q) = 0; \gamma_Q(p, q) > 0; \beta_Q(p, q) > 0$ for some p, q in P
 $\alpha_Q(p, q) > 0; \gamma_Q(p, q) = 0; \beta_Q(p, q) = 0$ for some p, q in P
 $\alpha_Q(p, q) = 0; \gamma_Q(p, q) > 0; \beta_Q(p, q) = 0$ for some p, q in P
 $\alpha_Q(p, q) = 0; \gamma_Q(p, q) = 0; \beta_Q(p, q) > 0$ for some p, q in P
 and $\alpha_Q(p, q) = 0; \gamma_Q(p, q) = 0; \beta_Q(p, q) = 0$ for some p, q in P that is, there is no edge between p and q . Otherwise there exists an edge between p and q .

Definition 3.2. Take $G = (P, Q)$ be a SFG. If two vertices p and q are connected by a path of length l in G such as $\wp: p_0, p_1, p_2, \dots, p_{l-1}, p_l$ then $\alpha_Q(p, q), \gamma_Q(p, q), \beta_Q(p, q)$ for some p, q in P are described as follows,

$\alpha_Q^l(p, q) = \alpha_Q(p, p_1) \wedge \alpha_Q(p_1, p_2) \wedge \dots \wedge \alpha_Q(p_{l-1}, q)$
 $\gamma_Q^l(p, q) = \gamma_Q(p, p_1) \wedge \gamma_Q(p_1, p_2) \wedge \dots \wedge \gamma_Q(p_{l-1}, q)$
 $\beta_Q^l(p, q) = \beta_Q(p, p_1) \vee \beta_Q(p_1, p_2) \vee \dots \vee \beta_Q(p_{l-1}, q)$
 consider $\alpha_Q^\infty(p, q), \gamma_Q^\infty(p, q), \beta_Q^\infty(p, q)$ is the strength of connectedness between two nodes p and q of a SFG G .

Definition 3.3. If $p, q \in P$ in SFG then $\alpha_Q^\infty(p, q), \gamma_Q^\infty(p, q), \beta_Q^\infty(p, q)$ are defined as
 α - strength of connectedness between p and q as
 $\alpha_Q^\infty(p, q) = \sup\{\alpha_Q^l(p, q) | l = 1, 2, 3, \dots\}$
 γ - strength of connectedness between p and q as
 $\gamma_Q^\infty(p, q) = \sup\{\gamma_Q^l(p, q) | l = 1, 2, 3, \dots\}$
 β - strength of connectedness between p and q as
 $\beta_Q^\infty(p, q) = \sup\{\beta_Q^l(p, q) | l = 1, 2, 3, \dots\}$ where \inf and \sup is used to find the minimum and maximum of the grade of membership value.

Definition 3.4. Let $G = (P, Q)$ be a spherical fuzzy graph. Then, G is said to be connected spherical fuzzy graph if for every vertices $p, q \in P$, $\alpha_Q^\infty(p, q) > 0$ or $\gamma_Q^\infty(p, q) > 0$ or $\beta_Q^\infty(p, q) < 1$.

Definition 3.5. Let $G=(P, Q)$ be a spherical fuzzy graph. Then, a path \wp of G between any two vertices is called the strongest path if its strength equals the strength of connectedness of $\alpha_Q^\infty(p, q), \gamma_Q^\infty(p, q)$ and $\beta_Q^\infty(p, q)$, all the values lie in the same edge.

Example 3.6. Let the vertex set $\mathbb{V} = \{p, q, r, s, t\}$ and the edge set $\mathbb{E} = \{pq, pr, qr, rs, qs, ps, st\}$ in $\mathbb{G}^* = (V, E)$. Take the spherical fuzzy set $P = (\alpha_P, \gamma_P, \beta_P)$ in \mathbb{V} and the spherical

fuzzy edge set in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$$\begin{aligned} (\alpha_P(p), \gamma_P(p), \beta_P(p)) &= (0.7, 0.5, 0.3), \\ (\alpha_P(q), \gamma_P(q), \beta_P(q)) &= (0.5, 0.8, 0.9), \\ (\alpha_P(r), \gamma_P(r), \beta_P(r)) &= (0.9, 0.8, 0.1), \\ (\alpha_P(s), \gamma_P(s), \beta_P(s)) &= (0.6, 0.7, 0.5), \\ (\alpha_P(t), \gamma_P(t), \beta_P(t)) &= (0.8, 0.9, 0.4). \end{aligned}$$

and

$$\begin{aligned} (\alpha_Q(pq), \gamma_Q(pq), \beta_Q(pq)) &= (0.4, 0.3, 0.8), \\ (\alpha_Q(pr), \gamma_Q(pr), \beta_Q(pr)) &= (0.7, 0.4, 0.3), \\ (\alpha_Q(ps), \gamma_Q(ps), \beta_Q(ps)) &= (0.6, 0.6, 0.2), \\ (\alpha_Q(qs), \gamma_Q(qs), \beta_Q(qs)) &= (0.5, 0.6, 0.7), \\ (\alpha_Q(qr), \gamma_Q(qr), \beta_Q(qr)) &= (0.4, 0.8, 0.9), \\ (\alpha_Q(rs), \gamma_Q(rs), \beta_Q(rs)) &= (0.6, 0.7, 0.5), \\ (\alpha_Q(st), \gamma_Q(st), \beta_Q(st)) &= (0.5, 0.7, 0.1), \end{aligned}$$

Consider the connected spherical fuzzy graph as defined in the above spherical fuzzy graph example and the $p-t$ paths are

$P_1 : p-s-t$ with membership value $(0.5, 0.6, 0.2)$,
 $P_2 : p-q-s-t$ with membership value $(0.4, 0.3, 0.8)$,
 $P_3 : p-r-s-t$ with membership value $(0.5, 0.4, 0.5)$,
 $P_4 : p-r-q-s-t$ with membership value $(0.4, 0.4, 0.9)$,
 $P_5 : p-q-r-s-t$ with membership value $(0.4, 0.3, 0.9)$,

then the strength of connectedness of the paths $p-t$ are

$$\begin{aligned} \alpha_Q^\infty(p, q) &= \sup\{0.5, 0.4, 0.5, 0.4, 0.4\} = 0.5, \\ \gamma_Q^\infty(p, q) &= \sup\{0.6, 0.3, 0.4, 0.4, 0.3\} = 0.6, \\ \beta_Q^\infty(p, q) &= \inf\{0.2, 0.8, 0.5, 0.9, 0.9\} = 0.2. \end{aligned}$$

In the SFG G , the strength of connectedness between two vertices p and q is $(0.5, 0.6, 0.2)$ and $p-s-t$ is the strongest path.

Definition 3.7. Let P be a SFS of a universal set \mathbb{X} . Then (a, b, c) - cut of P is a crisp subset $\mathcal{C}_{(a,b,c)}(P)$ of the SFS, P is given by $\mathcal{C}_{(a,b,c)}(P) = \{p : p \in \mathbb{X} | \alpha_P(p) \geq a, \gamma_P(p) \geq b, \beta_P(p) \leq c\}$ where $a, b, c \in [0, 1]$ with $a^2 + b^2 + c^2 \leq 1$. That is,

$$\begin{aligned} a_{PM} &= \{p \in \mathbb{X} : \alpha_P(p) \geq a\} \\ b_{PNU} &= \{p \in \mathbb{X} : \gamma_P(p) \geq b\} \\ a_{PNM} &= \{p \in \mathbb{X} : \beta_P(p) \leq c\} \end{aligned}$$

are a, b and c - cut of membership, neutral and nonmembership respectively of a SFG G , where a_{PM}, b_{PNU} and a_{PNM} indicates membership function, Neutral membership function and non-membership function respectively.

Definition 3.8. Let P be a SFS then height for membership value is defined as $H(P_M) = \sup\{\alpha_P(p)\}$, height for neutral value is defined as $H(P_{NU}) = \sup\{\gamma_P(p)\}$ and height for non-membership value is defined as $H(P_{NM}) = \inf\{\beta_P(p)\}$.

Definition 3.9. The SFS P is normal if there is atleast one $p \in P$ such that $\alpha_P(x) = 1$



Definition 3.10. Let P be a SFS then the depth for membership value is defined as $D(P_M) = \inf_{p \in P} \{\alpha_P(p)\}$, depth for neutral value is defined as $D(P_{NU}) = \inf_{p \in P} \{\gamma_P(p)\}$ and depth for non-membership value is defined as $D(P_{NM}) = \sup_{p \in P} \{\beta_P(p)\}$.

Definition 3.11. The support of P is defined by

$$P^* = (\alpha_{P^*}, \gamma_{P^*}, \beta_{P^*}) = \{p \in P \mid \alpha_P(p) > 0, \gamma_P(p) > 0, \beta_P(p) < 1\}.$$

The support of Q is defined by

$$Q^* = (\alpha_{Q^*}, \gamma_{Q^*}, \beta_{Q^*}) = \{(p, q) \in Q \mid \alpha_Q(p, q) > 0, \gamma_Q(p, q) > 0, \beta_Q(p, q) < 1\}.$$

Definition 3.12. Let $G^* = (P^*, Q^*)$. For $a, b, c \in [0, 1]$,

$$P^{(a,b,c)} = \{p \in P \mid \alpha_P(p) \geq a, \gamma_P(p) \geq b, \beta_P(p) \leq c\}$$

is called (a, b, c) -level subset of P and

$$Q^{(a,b,c)} = \{(p, q) \in Q \mid \alpha_Q(p, q) \geq a, \gamma_Q(p, q) \geq b, \beta_Q(p, q) \leq c\}$$

is called (a, b, c) -level subset of Q .

Here $G^{(a,b,c)} = (P^{(a,b,c)}, Q^{(a,b,c)})$.

Definition 3.13. A bridge (p, q) in G is said to be α -bridge, γ -bridge, β -bridge, if deleting (p, q) reduces the α - strength of connectedness, γ - strength of connectedness and β - strength of connectedness between some pair of vertices respectively. A bridge (p, q) is said to be fuzzy bridge if it is α -bridge, γ -bridge and β -bridge.

Definition 3.14. Let $(p, q) \in Q$. The edge is called bridge if (p, q) is a bridge of $G^* = (P^*, Q^*)$.

Definition 3.15. Let $(p, q) \in Q$. The edge (p, q) is called spherical fuzzy bridge if $\alpha'_Q < \alpha_Q, \gamma'_Q < \gamma_Q$ and $\beta'_Q > \beta_Q$ for some $(p, q) \in Q$, where α', γ' and β' are α, γ and β restricted to $V \times V - \{(p, q), (q, p)\}$.

Definition 3.16. Let $(p, q) \in Q$. The edge (p, q) is called a weak spherical fuzzy bridge if there exists $(a, b, c) \in (\mathbf{0}, H(Q)]$ such that (p, q) is a bridge of $G^{(a,b,c)}$.

Definition 3.17. Let $(p, q) \in Q$. The edge (p, q) is called a partial spherical fuzzy bridge if there exists $(a, b, c) \in (D(Q), H(Q)] \cup \{H(Q)\}$.

Definition 3.18. Let $(p, q) \in Q$. The edge (p, q) is called a full spherical fuzzy bridge for $G^{(a,b,c)}$ for all $(a, b, c) \in (\mathbf{0}, H(Q)]$.

Example 3.19. Let the vertex set $\mathbb{V} = \{u, v, w, x\}$ and the edge set $\mathbb{E} = \{uv, vw, xw, uw\}$ in $G^* = (V, E)$. Take the spherical fuzzy set $P = (\alpha_P, \gamma_P, \beta_P)$ in \mathbb{V} and the spherical fuzzy edge set in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$$(\alpha_P(u), \gamma_P(u), \beta_P(u)) = (0.5, 0.6, 0.2),$$

$$(\alpha_P(v), \gamma_P(v), \beta_P(v)) = (0.3, 0.7, 0.5),$$

$$(\alpha_P(w), \gamma_P(w), \beta_P(w)) = (0.7, 0.2, 0.7),$$

$$(\alpha_P(x), \gamma_P(x), \beta_P(x)) = (0.5, 0.4, 0.7)$$

and

$$(\alpha_Q(uv), \gamma_Q(uv), \beta_Q(uv)) = (0.3, 0.6, 0.5),$$

$$(\alpha_Q(vw), \gamma_Q(vw), \beta_Q(vw)) = (0.3, 0.2, 0.7),$$

$$(\alpha_Q(xw), \gamma_Q(xw), \beta_Q(xw)) = (0.5, 0.2, 0.7),$$

$$(\alpha_Q(uw), \gamma_Q(uw), \beta_Q(uw)) = (0.4, 0.2, 0.6)$$

In the SFG G , we have $H(Q) = (0.5, 0.6, 0.5)$ and $D(Q) = (0.3, 0.2, 0.7)$. Thus $(a, b, c) \in (\mathbf{0}, H(Q)]$ means that $(a, b, c) \in (0, 0.3] \times (0, 0.2] \times (0, 0.5]$. For $0 < a \leq 0.3, 0 < b \leq 0.2, 0 < c \leq 0.5$, $G^{(a,b,c)} = (P, \{(uv)\})$. Hence we conclude that (u, v) is not a bridge, but it is a weak spherical fuzzy bridge and not a partial and full spherical fuzzy bridge.

Proposition 3.20. Let (p, q) be a bridge in G^* . Then (p, q) is spherical fuzzy bridge iff $\alpha'_Q < \alpha_Q, \gamma'_Q < \gamma_Q$ and $\beta'_Q > \beta_Q$ for some $(p, q) \in Q$.

Proposition 3.21. (p, q) is a spherical fuzzy bridge iff (p, q) is not the weakest bridge of any cycle.

Proposition 3.22. The (p, q) is a spherical fuzzy bridge iff (p, q) is a bridge for G^* and $\alpha_Q(p, q) = H(\alpha_Q), \gamma_Q(p, q) = H(\gamma_Q)$ and $\beta_Q(p, q) = H(\beta_Q)$.

Proof. Suppose that (p, q) is a full spherical fuzzy bridge. Then (p, q) is a bridge for $G^{(a,b,c)}$ for all $(a, b, c) \in (\mathbf{0}, H(Q)] = (0, H(\alpha_Q)] \times (0, H(\gamma_Q)] \times (0, H(\beta_Q)]$. Hence $(p, q) \in Q^{H(Q)}$ and so $\alpha_Q(p, q) = H(\alpha_Q), \gamma_Q(p, q) = H(\gamma_Q)$, and $\beta_Q(p, q) = H(\beta_Q)$. Since (p, q) is a bridge for $G^{(a,b,c)}$ for all $(a, b, c) \in (\mathbf{0}, H(Q)] = (0, H(\alpha_Q)] \times (0, H(\gamma_Q)] \times (0, H(\beta_Q)]$, it follows that (p, q) is a bridge for G^* since $V = P^{D(Q)}$ and $E = Q^{H(Q)}$. Conversely, suppose that (p, q) is a bridge for G^* and $\alpha_Q(p, q) = H(\alpha_Q), \gamma_Q(p, q) = H(\gamma_Q)$ and $\beta_Q(p, q) = H(\beta_Q)$. Then $(p, q) \in Q^{(a,b,c)}$ for all $(a, b, c) \in (\mathbf{0}, H(Q)]$. Thus since also (p, q) is a bridge for G^* , (p, q) is a bridge for $G^{(a,b,c)}$ for all $(a, b, c) \in (\mathbf{0}, H(Q)]$ since each $G^{(a,b,c)}$ is a subgraph of G^* . Hence (p, q) is a full spherical fuzzy bridge. \square

Proposition 3.23. Suppose that (p, q) is not contained in a cycle of G^* . Then the following conditions are equivalent:

1. $\alpha_Q(p, q) = H(\alpha_Q), \gamma_Q(p, q) = H(\gamma_Q)$ and $\beta_Q(p, q) = H(\beta_Q)$.
2. (p, q) is a partial spherical fuzzy bridge.
3. (p, q) is a full spherical fuzzy bridge.

Proof. Since (p, q) is not contained in a cycle of G^* , (p, q) is a bridge of G^* . Hence by the above proposition, (1) \iff (3). Clearly, (3) \iff (2). Suppose that (2) holds. Then (p, q) is a bridge for $G^{(a,b,c)}$ for all $(a, b, c) \in (D(Q), H(Q)]$ and so $(p, q) \in Q^{H(Q)}$. Thus $\alpha_Q(p, q) = H(\alpha_Q), \gamma_Q(p, q) = H(\gamma_Q)$ and $\beta_Q(p, q) = H(\beta_Q)$; that is, (1) holds. \square

Proposition 3.24. If (p, q) is a bridge, the (p, q) is a weak spherical fuzzy bridge and spherical fuzzy bridge.



Proposition 3.25. *If (p,q) is a spherical fuzzy bridge if and only if the edge (p,q) is a weak spherical fuzzy bridge.*

Proof. Suppose that (p,q) is a weak spherical fuzzy bridge. Then there exist $(a,b,c) \in (\mathbf{0}, H(Q)]$ such that (p,q) is a bridge for $G^{(a,b,c)}$. Hence the edge (p,q) removed then it disconnects $G^{(a,b,c)}$. Thus any path from p to q in G has an edge (x,y) with $\alpha_Q(x,y) < a, \gamma_Q(x,y) < b, \beta_Q(x,y) > c$. Thus the removal of (p,q) results in $\alpha_Q^{\infty}(p,q) < a \leq \alpha_Q^{\infty}(p,q), \gamma_Q^{\infty}(p,q) < b \leq \gamma_Q^{\infty}(p,q), \beta_Q^{\infty}(p,q) > c \geq \beta_Q^{\infty}(p,q)$. Hence (p,q) is spherical fuzzy bridge.

Conversely, suppose that (p,q) is spherical fuzzy bridge. Then there exist (x,y) such that removal of (p,q) results is $\alpha_Q^{\infty}(p,q) < \alpha_Q^{\infty}(p,q), \gamma_Q^{\infty}(p,q) < \gamma_Q^{\infty}(p,q), \beta_Q^{\infty}(p,q) > \beta_Q^{\infty}(p,q)$. Hence the edge (x,y) is on every strongest path connecting x and y and in fact $\alpha_Q(x,y) \geq, \gamma_Q(x,y) \geq$ and $\beta_Q(x,y) \leq$ this value. Thus there does not exist a path (other than (p,q)) connecting p and q in $G^{(\alpha_Q(p,q), \gamma_Q(p,q), \beta_Q(p,q))}$, else this other path without (p,q) would be of strength $\geq \alpha_Q(p,q), \geq \gamma_Q(p,q)$ and $\leq \beta_Q(p,q)$ and would be part of a path connecting x and y of strongest length, contrary to the fact that (p,q) is on every such path. Hence (p,q) is a bridge of $G^{(\alpha_Q(p,q), \gamma_Q(p,q), \beta_Q(p,q))}$, and $0 < \alpha_Q(p,q) \leq H(\alpha_Q), 0 < \gamma_Q(p,q) \leq H(\gamma_Q), 0 < \beta_Q(p,q) \leq H(\beta_Q)$. Thus $\alpha_Q(p,q), \gamma_Q(p,q)$ and $\beta_Q(p,q)$ are desired (a,b,c) . \square

4. Conclusion

In the network theory each edge is assigned by a weight. The weight of a graph such as path and cycle is elucidated as the minimum of its edges. The maximum of weights of all paths between two vertices is elucidated as the strength of connectedness between the vertices. The reduction in the strength of connectedness is more relevant than the whole disconnection of the graph. Total weighted graph has both vertex set and the edge set are weighted. Fuzzy graph has numerous applications in the real time situation and their extensions of intuitionistic, pythagorean and picture fuzzy graphs are filed by some of its insufficient conditions. The spherical fuzzy graph has quad attributes are sufficient to deal the uncertainty greatly. We have investigated the strength of connectedness of the spherical fuzzy graph using level sets.

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 ISSN(P):2319 – 3786
 Malaya Journal of Matematik
 ISSN(O):2321 – 5666

