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Heat and mass lines visualization techniques for heat and mass flow in double-diffusive natural convection utilizing Cu-water nanofluid

Durgesh Kushawaha[1], Sushil Yadav[2], Manoj Kumar[3*] and Anu Jain[4]

Abstract
Most of the recent works indicate that the isotherms, isosolutes, and streamlines are commonly used to study the thermosolute natural convection phenomena. Although the fluid flow is effectively illustrated by the use of streamlines (to visualize fluid flow), isotherms (to visualize temperature distribution), and isosolutes (to visualize solute distribution) cannot represent the 'heat flow' and 'solute flow' as the isotherms and isosolutes indicate only the spatial distribution of temperature and solute respectively. A visualization tool is important to understand the heat and solute flow during thermosolute convection. In addition, it is not easy to visualize convection heat and solute transfers due to the presence of both conduction and convection. In this work, two dimensional laminar thermosolute natural convection has been numerically investigated in a differentially heated and soluted square enclosure in the presence of cu-water nanofluid. To enhancing the heat transfer, nanofluids are quite helpful. The left and right vertical walls are considered as higher and lower temperatures and solute, respectively whereas the horizontal walls are being kept adiabatic and non-diffusive. The mathematical formulation of heat and mass functions have been done, and based on these functions, the heat and mass lines contours have been drawn to investigate the behavior of heat and solute in the cavity. The flow governing equations have been solved using a finite difference method together with successive over-relaxation (SOR) technique after converted into the vorticity-stream function form. A comparison of isotherms with heatlines and isosolute with masslines has been done in detail.

Keywords
Double-diffusive convection, Heat function, Mass function, Nanofluid.

[1] *Department of Mathematics, Delhi University, Delhi, India.*
[2] *Department of Mathematics, Maharaja Agrasen College, Delhi University, Delhi, India.*
[3] *Department of Mathematics, Deshbandhu college, Delhi University, Delhi, India.*
[4] *Department of Mathematics, Lakshmibai College, Delhi University, Delhi, India.*
*__Corresponding author__: [3] mkumar@db.du.ac.in
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Contents

1. Introduction

Many natural and engineering transport phenomenons are induced by the combined action of the buoyancy forces from both thermal and solute (mass or concentration) processes and know as thermosolute or double-diffusive process. Alternatively, the thermosolute natural convection is a process of buoyancy-driven flows governed by the combined temperature and solute gradients. In other words, the buoyancy force is developed by combined concentration and temperature gradients. Compared to traditional fluids used in heat and mass transfer applications, such as water, mineral oil, and ethylene glycol, which have very low thermal conductivity, nanofluids have attracted great interest from researchers due to their potential in the enhancement of heat transfer. Nanofluid is a mixed liquid containing a suspension of metallic or nonmetallic nano-sized solid particles and base fluid.

To visualize the fluid flow, authors have used stream-line techniques. Heat and mass transfers displayed by the isotherms and isoconcentrations[2, 3, 8, 9]. But those aren't enough to do the tasks because isotherm and isosolutes are the lines joining the points which have equal temperature and concentration (solute). From the definition of isotherms and isosolutes, we can conclude that both fail to induce the energy and solute distribution in the enclosures. So we need something similar to the streamlines for better visualization of heat and mass transfers in the enclosures. The limited use of heat and mass lines techniques are noticed so far. We are listing some research articles in which authors have used these techniques. The heat and mass lines visualization tools have been used by Bondareva et al. [6] and Hu et al. [7].

2. Problem description and formulation

In the present study, laminar thermosolute natural convection has been studied. The steady incompressible flow is considered in a square enclosure. We are considered the base fluid as water $\mathrm{Pr}=6.2$ and Cu are taken as nanoparticles in the enclosure. The left wall maintains higher temperature (T_k) and solute (S_k) while the right wall is kept at lower temperature (T_i) and solute (S_t) such that $T_k > T_t$ and $S_k > S_i$. The horizontal walls are adiabatic and non- diffusive. The thermosolute natural convection effect is modeled by using Boussinesq approximation. Effect of Joule heating, thermal radiation, chemical reaction, and viscous dissipation are assumed to be neglected. Furthermore, the cross-diffusion effects are supposed to be negligible. The flow governing equations have been solved using a finite difference method together with successive over-relaxation (SOR) technique after converted into the vorticity-stream function form. The influence of Rayleigh number and volume fraction of nanoparticle on isotherms, isosolutes, heat and mass functions, overall heat and solute transfer have been analyzed.

For better display of heat transport in enclosures filled with fluids, the heatline technique introduced by Kimura and Bejan [1] first, which gives better insight over isotherms approach. Later on, this visualization technique was adopted and extended for heat and mass transfer by Trevisan and Bejan [5] and introduced mass function and massline analogy to visualize mass transfer in a fluid flow which gives comparatively better insight to mass transport characteristics of fluid flow than isoconcentrations (isosolutes) approach.

Heat function [1, 4] in the dimensional form is denoted as h and defined as

$$\frac{\partial h}{\partial y} = (\rho c_p)_{dj}\, u\,(T^* - T_i^*) - k_{tj'}\frac{\partial T^*}{\partial x} \tag{2.1}$$

(Net energy flow in x -direction)

$$-\frac{\partial h}{\partial x} = (\rho c_v)_{ej}\, v\,(T^* - T_i^*) - k_{v'}\frac{\partial T^*}{\partial y}(2) \tag{2.2}$$

(Net energy flow in y -direction) The non-dimensional form of equations (2.1) and (2.2) are as follows:

$$\frac{\partial H}{\partial Y} = UT - \frac{\partial T}{\partial X} \tag{2.3}$$

$$-\frac{\partial H}{\partial X} = VT - \frac{\partial T}{\partial Y} \tag{2.4}$$

where H is the dimensionless heat function, and it is represented as

$$H = \frac{h}{k_{tj}\left(T_k^* - T_t^*\right)}.$$

The manipulation of eqns. (2.3) and (2.4) yields the following 2^{nd} order partial differential equation (a Poisson equation) for the heat function.

$$\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} = \frac{\partial (UT)}{\partial Y} - \frac{\partial (VT)}{\partial X} \tag{2.5}$$

We can get the dimensionless heat function H in the inner region of the enclosure considered by solving eqn. (2.5) using the appropriate boundary conditions. The drawing of isolines of the heat function provides heatlines.

Mass function [4, 5] in the dimensional form is denoted as m and defined as

$$\frac{\partial m}{\partial y} = \rho_{t'}\, u\,(S^* - S_i^*) - \rho_{s'} D\frac{\partial S^*}{\partial x} \tag{2.6}$$

(Net solute flow in x -direction)

$$-\frac{\partial m}{\partial x} = \rho_{v'}\, v\,(S^* - S_i^*) - \rho_{t'} D\frac{\partial S^*}{\partial y} \tag{2.7}$$

(Net solute flow in y -direction) The non-dimensional form of equations (2.6) and (2.7) are as follows:

$$\frac{\partial M}{\partial Y} = US - \frac{1}{\mathrm{Le}}\frac{\partial S}{\partial X} \tag{2.8}$$

$$-\frac{\partial M}{\partial X} = VS - \frac{1}{\mathrm{Le}}\frac{\partial S}{\partial Y} \tag{2.9}$$

where M is the dimensionless mass function, and it is represented as

$$M = \frac{m}{\mathrm{Le}\,\rho_{ej} D\left(S_k^* - S_i^\circ\right)}.$$

The manipulation of eqns. (2.8) and (2.9) yields the following 2^{nd} order partial differential equation (a Poisson equation) for the mass function.

$$\frac{\partial^2 M}{\partial X^2} + \frac{\partial^2 M}{\partial Y^2} = \frac{\partial (US)}{\partial Y} - \frac{\partial (VS)}{\partial X} \tag{2.10}$$

We can get the dimensionless mass function M in the inner region of the enclosure considered by solving eqn. (2.10) using the appropriate boundary conditions. The drawing of isolines of the mass function provides masslines.

3. Results and discussion

From Figs. 1 and 2, we depict that the isotherms and isosolutes for base fluid and nanofluid represent temperature and solute along a particular line. But no information about the direction of heat and solute flow inside the enclosure. Fig. 1(c) and Fig. 2(c) display the heatlines at $Ra = 2 \times 10^3$ and $Ra = 2 \times 10^5$ for base fluid and nanofluid. As we can see some heatlines start from left wall and end at right one and having positive signs, those are responsible for direct heat transfer from hot to the cold wall. Some are closed curves inside the enclosure with negative signs yield the thermal mixing inside the enclosure. With the help of both we can better understand the heat flow inside the enclosure. With the rise of Rayleigh number the direct heat transfer dominates the thermal mixing. Furthermore, both direct heat transfer and thermal mixing enhancing utilizing nanofluid over classical fluid. From Table 1, we may observe the same. This happens due to the higher thermal conductivity of nanofluid over base fluid.

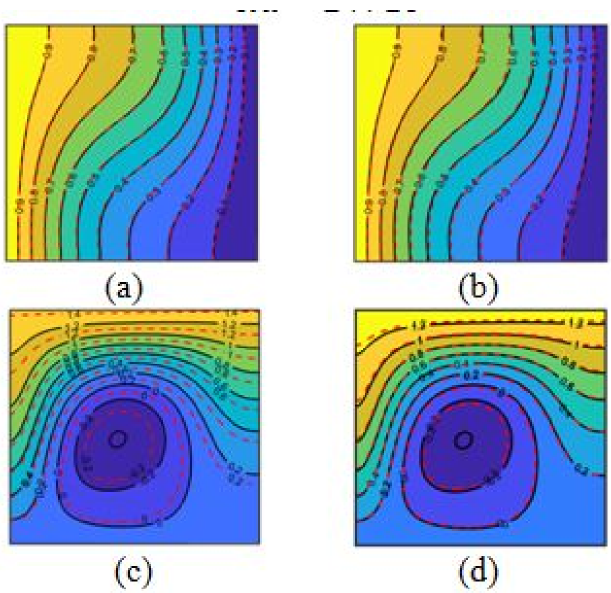

Figure 1. Isotherms, isosolutes, heatlines, and masslines for base fluid (solid line) and nanofluid (dashed) at $Ra = 2 \times 10^3$, $Le = 2, N = 1$.

Fig. 1(d) and Fig. 2(d) display the masslines at $Ra = 2 \times 10^3$ and $Ra = 2 \times 10^5$ for base fluid and nanofluid. Same terminologies are adopted for masslines as we used for heatlines. As the Rayleigh number raises, both direct solute transfer and solute mixing strengthens. Furthermore,direct solute transfer falls whereas solutemixing enhancing utilizing nanofluid over classical fluid. The same can be observed from Table 1.

From Table 2, we may conclude that the overall heat transfer from the hot wall to the fluid is enhancing with the increases of Rayleigh number. This happens due to the domination of buoyancy forceon viscous force when Rayleigh number increases. Moreover, for all the Rayleigh overall heat transfer strengthens utilizing nanofluid over base fluid due the higher thermal conductivity of nanofluid.The overall solute transfer from higher solute wall to the fluid increases with the increment in the Rayleigh number where there is a fall when we use nanofluid; this can be observed from Table 3.

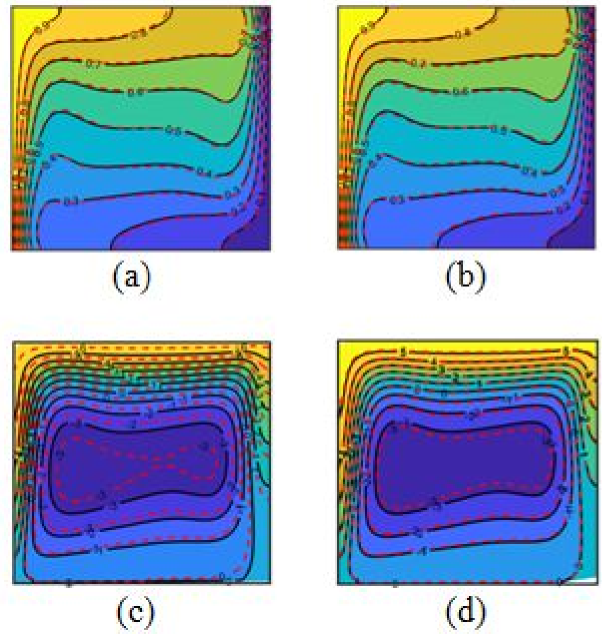

Figure 2. Isotherms, isosolutes, heatlines, and masslines for base fluid (solid line) and nanofluid (dashed) at $Ra = 2 \times 10^5$, $Le = 2, N = 1$.

Table 1: Comparison of direct heat and solute transfer and thermal and solute mixing for base fluid and nanofluid for various Rayleigh numbers.

Ra	2×10^3	2×10^5
Base fluid		
H_{min}	−0.643	−3.371
H_{max}	1.557	6.681
M_{min}	−0.308	−2.369
M_{max}	2.225	8.919
Nanofluid		
H_{min}	−0.585	−2.721
H_{max}	1.681	7.197
M_{min}	−0.272	−2.105
M_{max}	2.171	8.742

Table 2: Comparison of overall heat transfer for base fluid and nanofluid.

Ra	$Nu_{2vg}(bf)$	$Nu_{2vg}(nf)$	% change
2×10^3	1.558	1.683	8.02
2×10^5	6.686	7.202	7.72

Table 3: Comparison of overall solute transfer for base fluid and nanofluid.

Ra	$Sh_{avg}(bf)$	$Sh_{avg}(nf)$	% change
2×10^3	2.227	2.172	−2.47
2×10^5	8.925	8.748	−1.98

4. Conclusion

The heat and mass lines visualization techniques provide better insight for heat and solute flows inside the enclosure over isotherms and isosolutes. Overall, heat and solute transfer from higher temperature and solute wall to the fluid enhance because of the domination of buoyancy force on viscous force with the rise in Rayleigh number. Furthermore, overall heat transfer increases upto 8.02%, whereas the overall solute transfer is reduced upto 2.47% utilizing nanofluid (volume fraction of nanofluid, $\emptyset = 2\%$) because of the higher thermal conductivity of nanofluid over base fluid (water).

Nomenclature

u, v	x - and y - components of velocity
U, V	Dimensionless velocity components
x, y	Cartesian coordinates
X, Y	Dimensionless Cartesian coordinates
h	Heat function
H	Dimensionless heat function
m	Mass function
M	Dimensionless mass function
T^*	Temperature
T	Dimensionless temperature
S^*	Solute
S	Dimensionless solute
Le	Lewis number
Pr	Prandtl number
Ra	Rayleigh number
N	Buoyancy ratio
Nu	Nusselt number
Sh	Sherwood number
D	Mass diffusivity

Greek Symbols

$\emptyset$	Nanoparticle volume fraction
ρ	Fluid density
c_p	Specific heat at a constant temperature
k	Thermal conductivity

Subscripts

bf	Base fluid
nf	Nanofluid
h	Higher
l	Lower
max	Maximum
min	Minimum

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