Malaya Journal of Matematik, Vol. 9, No. 1, 914-916, 2021
https://doi.org/10.26637/MJM0901/0162

On commutative CI-algebras

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Abstract
CI-algebra is a generalization of BE-algebra. The concept of Commutative BE-algebra was first introduced by A. Walendziak. B. L. Meng applied the same definition in CI-algebras and established that any commutative CI-algebra is a BE/dual BCK-algebra. In this paper we continue to study commutative CI-algebras and try to establish some properties in some specific CI-algebras.

Keywords
CI-algebra, BE-algebra, Commutative.

AMS Subject Classification
06F35, 03G25, 08A30.

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Article History: Received **10** January **2021**; Accepted **19** March **2021**
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1. Introduction

Since the introduction of the concepts of BCK and BCI-algebras ([2,3]) by Y Imai and K.Iseki, some more concepts of similar type like BCH ([1]), BH ([4]), d ([8]), etc have developed and studied by a number of authors in the last two decades. In 2006, H. S. kim nad Y. H. Kim introduced the notion of BE-algebras ([6]) as a generalization of dual BCK-algebras ([5]). The concept of Commutative BE-algebra ([11]) was first introduced by A. Walendziak. In 2010 ([7]) B. L. Meng introduced the notion of a new algebraic structure called CI-algebras as a generalization of BE-algebras. In his paper Meng defined the commutativity property of CI-algebras alongwith many new concepts and established that any commutative CI-algebra is a BE/dual BCK-algebra. The concept of Cartesian product has been developed by us in 2013 ([9]). In 2018, we introduce some special types of CI-algebras ([10]) obtained from a given CI-algebra. In this paper we continue to study commutative CI-algebras and discuss some new properties of it in these specific CI–algebras.

2. Preliminaries

Definition 2.1 ([6]). *An algebraic system $(U;*,1)$ is called a BE-algebra if it satisfies the following axioms:*

(U1) $b*b=1$

(U2) $b*1=1$

(U3) $1*b=s$

(U4) $b*(c*d)=c*(b*d)$ *for all* $b,c,d\in A$.

Definition 2.2 ([8]). *An algebraic system (U; *, 1) is called a CI-algebra if it satisfies the following axioms:*

(A1) $b*b=1$

(A2) $1*b=b$

(A3) $b*(c*d)=c*(b*d)$ *for all* $b,c,d\in A$

Example 2.3. *Let U be a non-empty set and let $F(U)$ be the set of all function $f:U\to(0,\infty)$. For $k,f\in F(U)$, we define*

$$(k*f)(w)=\frac{f(w)}{k(w)},w\in U$$

If we put $1(w)=1$ for all $w\in U$, then $1\in F(U)$ and simple computation proves that $(F(U);*,1)$ is a CI- algebra.
In U, we can define a binary relation $\leq$ by $b\leq c$ iff $b*c=1$.

Lemma 2.4 ([8]). *In a CI-algebra $(U;*,1)$ following results are true:*

1. $b*((b*c)*c)=1$
2. $(b*c)*1=(b*1)*(c*1)$ *for all* $b,c\in U$

Theorem 2.5 ([10])**.** *Let* $(U;*,1)$ *be a system consisting of a non-empty set* U*, a binary operation* $*$ *and a distinct element 1. Let* $V=U\times U=\{(b_1,\ b_2):b_1,\ b_2\in U$*. For* $u,w\in V$ *with* $u=(b_1,b_2)\,,w=(c_1,c_2)$*, we define an operation* $\otimes$ *in* V *as*

$$u\otimes w=(b_1*c_1,\ b_2*c_2)$$

Then $(V;\otimes,(1,1))$ *is a CI-algebra iff* $(U;*,1)$ *is a CI-algebra.*

Corollary 2.6 ([10])**.** *If* $(U;*,1)$ *and* $(V;o,e)$ *are two CI-algebras, then* $W=UxV$ *is also a CI-algebra under the binary operation defined as follows: For* $u=(b_1,c_1)$ *and* $w=(b_2,c_2)$ *in R,*

$$u\otimes w=(b_1*\ b_2,c_1\circ c_2)$$

Here the distinct element of W *is* $(1,e)$*.*

Theorem 2.7 ([11])**.** *Let* $(U;*,1)$ *be a CI-algebra and let* $P(U)$ *be the class of all functions* $f:U\to U$*. Let a binary operation o be defined in* $P(U)$ *as follows:*

$$\text{For } f,t\in P(U) \text{ and } w\in U$$
$$(f\circ t)(w)=f(w)*t(w)$$

Then $(P(U);0,1^{\sim})$ *is a CI-algebra where* $1^{\sim}$ *is defined as* $1^{\sim}(w)=1$ *for all* $w\in U$*. Here two functions* $f,t\in P(U)$ *are equal iff* $f(w)=t(w)$ *for all* $w\in U$*.*

Notation 2.8 ([11])**.**
(a) *For any set* $U_1\subseteq U$*, let* $P(U_1)$ *denote the set of all functions* $f\in P(U)$ *such that* $f(w)\in U_1$ *for all* $w\in U$
(b) *For any* $u\in U$*, we consider* $f_u\in P(U)$ *defined as* $f_u(w)=u$ *for all* $w\in U$*.*

Definition 2.9 ([11])**.** *A BE-algebra* $(U;*,1)$ *is said to be commutative if* $(b*c)*c=(c*b)*b$ *for all* $b,c\in U$*.*

Example 2.10 ([11])**.** *Let* $N_0=N\cup\{0\}$ *and let the binary operation* $*$ *be defined on* N_0 *as follows:*

$$b*c=\begin{cases}0 & \text{if } b\geq c\\ c-b & \text{if } c>b\end{cases}$$

Then $(N_0;*,0)$ *is a commutative BE-algebra*

3. Commutative CI–Algebra

Definition 3.1. *A CI-algebra* $(U;*,1)$ *is said to be commutative if* $(b*c)*c.=(c*b)*b$ *for all* $b,c\in U$*.*

Example 3.2. *We consider a system* $(U;*,1)$ *where* $U=\{1,p,q,r\}$ *and the binary operation *is given by*

*	1	p	q	r
1	1	p	q	r
p	1	1	q	r
q	1	p	1	r
r	1	p	q	1

This $(U;*,1)$ *is a commutative* BE/CI *-algebra*

Theorem 3.3. *Let* U *and* V *be CI-algebras as considered in theorem (2.5). Then* V *is commutative iff* U *is commutative.*

Proof. First suppose that V is commutative. Let b and c be arbitrary elements of U. We choose $u=(b,1)$ and $w=(c,1)$. Since V is commutative, we have

$$(u\otimes w)\otimes w=(w\otimes u)\otimes u$$

This gives $((b*c)*c,1)=((c*b)*b,1)$, which in turns imply that

$$(b*c)*c=(c*b)*b$$

Hence U is commutative.
Conversely, suppose that U is commutative. Let $u=(b_1,\ b_2)$ and $w=(c_1,c_2)$ be any two arbitrary elements of V. Then

$$\begin{aligned}(u\otimes w)\otimes w&=((b_1,\ b_2)\otimes(c_1,c_2))\otimes(c_1,c_2)\\&=(b_1*c_1,\ b_2*c_2)\otimes(c_1,c_2)\\&=((b_1*c_1)*c_1,(\ b_2*c_2)*c_2)\\&=((c_1*\ b_1)*\ b_1,(c_2*\ b_2)*\ b_2)\\&=((c_1,c_2)\otimes(b_1,\ b_2))\otimes(b_1,\ b_2)\\&=(w\otimes u)\otimes u.\end{aligned}$$

Hence U is commutative. □

Corollary 3.4. *Let* U,V *and* W *be CI-algebras as considered in corollary (2.6). Then* W *is commutative iff both* U *and* V *are commutative.*

Theorem 3.5. *Let* $(P(U);0,1^{\sim})$ *be CI-algebra as considered in theorem (2.7). Then* $P(U)$ *is commutative if and only if* U *is commutative.*

Proof. First suppose that U is commutative. Then

$$(b*c)*c=(c*b)*b \text{ for all } b,c\in U$$

Let $f,t\in P(U)$. Then for any $w\in U$, we have

$$\begin{aligned}((f\circ t)\circ t)(w)&=(f(w)*t(w))*t(w)\\&=(t(w)*f(w))*f(w)\\&=((t\circ f)\circ f)(w).\end{aligned}$$

This proves that $(f\circ t)\circ t=(t\circ f)\circ f$ Hence $P(U)$ is commutative.
Conversely, let $P(U)$ be commutative. Let $u,w\in U$. We consider $f_u,f_w\in P(U)$. Then

$$(f_u\circ f_w)\circ f_w=(f_w\circ f_u)\circ f_u$$

This gives

$$((f_u\circ f_w)\circ f_w)(v)=((f_w\circ f_u)\circ f_u)(v);v\in U$$

This implies that $(u*w)*w=(w*u)*u$ (notation $(2.8)(b)$)
Hence X is commutative. □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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